SPACE-TIME

principles physics of chances

principles of probability,
information and entropy

RASTKO VUKOVIĆ

`Working version of the text!`

RASTKO VUKOVIĆ:
SPACE-TIME, PRINCIPLES PHYSICS OF CHANCES

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Preface to original

This book is a continuation of “Information of Perception” cited at the end (see: [2]). The first version “The Nature of Time” also listed in the bibliography, should only link my papers of the same subject from the previous years, but the story is abducted. It got too many pages, too much philosophy, and too much mathematics. Who would read it? That’s why this first attempt I left unfinished on the Internet (see: [1]) and make a new text called ”Space-time”. The idea is to keep some of the theorems descriptively without the proofs, but possibly referring to them. However, it did not go that simple.

Looking for, I stuck at an obscure part between speculation and science. By testing the recognized reality with an unrecognized on a somewhat incorrect way, I got the “accurate” from an “inaccurate” physics. Should such a mathematical fantasy be rejected? As we know in math, the implication “from wrong to true” is true. The physical consequences are almost the same, but which of the two assumptions is true: determinism or non-determinism? So, I accepted the second, the new version.

Seeking the support from the colleagues for such complicated and suspicious projects is a difficult job. Everybody is sorted out and “do not deal with it”. One said to me do not rush, the science should be written slowly, one at a time. The other told me in the tavern that any future emeritus is fearful of charlatanism, in the constant struggle for academic positions and he or she must be overwhelmed by learning. Beyond learning someone else’s knowledge, there is no time to develop the own skills, and even less for creative wandering, nothing for fantasy. Career does not go with great ideas, he told me with cheers and drink. Modern education is becoming more and more “successful” and it is only my problem that we get products that honestly respect the verified truths.

You missed everything – told me the third one. Solitary work in science has poor quality and is unattractive, so listen to what I say: run away! Successful one should following signs “do not wave” then “support slowly and weigh to the top” in the glory of the system and do not play the scientific bohemian that scramble up for a little surprise that his work “was not just a fiction” when the science matures. I defended myself, claiming that the lonely researchers in the affiliated environment have a special charm since there is no drawn road to the true discovery. Those with a lot of support in this game of discovery with such do not have a chance! Leonardo da Vinci, Blaze Pascal or Nikola Tesla are three of the many who defied academies. And those on the list of geniuses who have changed the world have more than successfully trained. There is a secret truth about the invincibility of despair, for which many ambitions have failed. About this wrote Vladislav Petković Dis: “And the darkness has its bright side the thought is born when luck deserted”. To succeed you need a failure.

So painfully this text emerged. To interested readers, I can only tell that there’s more.

Author, May 2017.

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Introduction

The three most important random causes of physics here presumed are called the principles of probability, information, and entropy. They define matter, space and time. The first of them starts by the knowledge that usually happens what is most likely and then continues to the theory of probability, which we don't do here too much, and to the physical consequences that modern science is insufficiently dealt with.

The second of these principles are based on an understanding of information as material things, as opposed to the uncertainty that is abstract. The ocean of uncertainty is infinitely infinite, as opposed to the very limited (amount of) information that overtakes, hardly leaks toward us, in the form of substance and space-time. The first principle generates the second: "nature gives us a minimum of information", whereby resulting information formed our "now", that is our time, space and matter. As far as generous in implementing the most likely event, nature is so stingy in creating the present. However, where is no the flow of times there is no the universe.

Note that the first principle is difficult or even impossible to seriously derogate in any way, but again it seems very unacceptable. I do not consider it as defect, but rather as my achievement. The second principle contains upgrade of the main ideas from my previous book "Information of perceptions" (see [2]), which is actually unprovable. First of all, $\ell = \mathbf{i} \times \mathbf{h}$, the formula of freedom as the (scalar) product (vectors) of the intelligence and the hierarchy. However, as the formula of kinematics $vt = s$, it cannot be challenged. So the question "why to speculate with that suspicious formulas" the answer is "find me a counter example".

Third of the rules is based on the entropy and the second law of thermodynamics: "heat spontaneously passes from the higher to the lower body temperature“. It is shown that the entropy is a part of what is sometimes called by its name. It is the amount of uncertainty of a random subset of outcomes (randomization) of physical conditions that are most likely. Because it is economical with the information, nature does not dissipate the uncertainty in the amounts the relativistic thermodynamics considered it normal, so the physics and I have split up. But, the theory remains consistent with both the classical thermodynamics and the Einstein's relativity.

The consequences of the third principle I consider really risky too and it goes to my account. But again, would that be a research if it goes only by proven routes?

1.1 Probability

What is most probably happens most often. This is the *principle of probability*. It is so obvious and simple that the mathematical probability theory just accepts it with no comment, actually does not pay attention to it. It is so apparent and indisputable. This principle still carries a deeper meaning that is worth consideration.

Under the *chance* hereinafter refer only to those of randomness that applies the principle of probability. Then we have at least two unforeseen opportunities that behave according to the rules of probability theory.

1.1.1 Objectivity of randomness

First of all, we can discuss the roots of randomness. When we have the options whose outcomes are incidental and unpredictable, but somewhere there are (one or more) causes to us unknown or inaccessible, their realizations are *subjective randomness*. But if the assumptions that there is the ultimate cause of each outcome are contradictory such randomness would be *objective randomness*.

For example, the subjective randomness is a coincidence of digits $\pi = 3.14159265359\dots$ taken successive. On a first reading it passes all tests of randomness, but the second and each next subsequent reading will show that the figures are always the same, that even in the first readings it was the *pseudo randomness*.

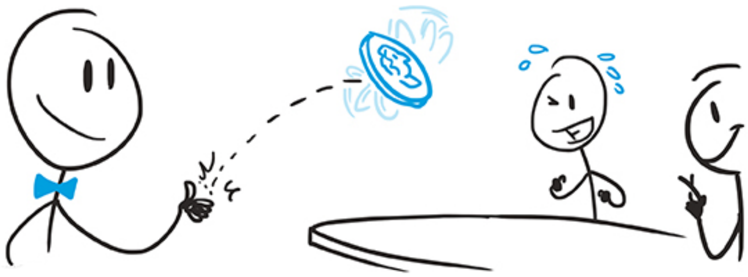


Figure 1.1: Toss-up.

In contrast to many scientific and mathematical theories, here we believe that in addition to subjective there are and objective randomness. An example of the objective chance may be a random sequence of throwing a *fair coin* (figure 1.1). I'll explain this in more detail. Random outcomes are "Head" and "Tail" with chances half and a half. In a series of 100 repeated throws, about 50 outcomes will be "Head" or "Tail". However, they will rarely be exactly 50 each. The throws are independent events, in the sense that the following does not remember the previous one, so it can happen a long series of consecutive of the same outcomes and the result is only about half and a half.

Tests of randomness predict this tolerance. Moreover, they predict not only the mean value (50) but also the mean deviation from this value (5) of falling "Head" in a row (100) throwing a fair coin. In the case of an unfair coin, or if the outcome of the throw attempts to guess a man, it can easily happen that the medium deviations are not accurate and cannot pass the test. However, the said digits of π, drawn from the row as a lottery drum, show perfectly "accidental".

Example of objective coincidence is the attitude that "there is no way that something is so organized to capture and to be controlled all the possible consequences and to exclude any unpredictability". Or: "there is no guarantee for absolute certainty in any practical assessment of natural laws". This is a step away from *Gödel's theorem* of impossibility: " however we have a great set of axioms and their consequences, there will always be accurate assertions that cannot be derived from a given set". The cause of all causes cannot exist because it would be contradictory. All the consequences of this would consist of a set of outcomes outside which would always be able to find an outcome that does not follow from a given set. In short, we proceed from the following paragraph as an axiom.

There is no *cause of all causes*. There is no a collection of all possible causes.

Gödel's theorem as a consequence can be derived from the famous Russell[1] Paradoxes of 1901, which formally write:

$$\text{if } S = \{A : A \notin A\}, \text{ is it } S \in S \text{ ?} \tag{1.1}$$

Words, let's given the set (S) of sets (A) which do not belong to itself, whether such a collection belonging to itself? The answer cannot be "yes" because from $S \in S$ and (1.1) follows $S \notin S$, which is a contradiction. The answer cannot be either "no" because from $S \notin S$ follows $S \in S$, and this is again a contradiction. There is no third because there is a theorem that every polyvalent logic ("true", "false", "maybe", ...) can be reduced to the bivalent logic ("true" or "false"). Therefore, it is impossible to set to contains all sets that do not contain themselves, so the set of all sets is impossible. If there is the cause for each event, then the cause of all causes cannot be, and this is the conclusion that formally leads us to determinism.

A little easier version of the Russell's paradox is a contradiction: "barber A shaves all those in his neighborhood who do not shave themselves". The barber A cannot exist, despite the fact that it is more reasonable to expect the sets which are not its own subsets. Even lighter version is the statement "I lie", which is a contradiction. In fact, if I lie, then I lie to lie, and if I lie it is not true what I say, therefore I speak the truth. But, if I speak the truth, then I say that I lie.

Formally these paradoxes are coming from the *algebra logic* according to which a falsehood from a truth cannot be obtained. All mathematics is based on the simple logical statement that $\top \Rightarrow \bot$ is a contradiction, which is sometimes difficult to mind. Today we know that all attempts to challenge that fact failed.

Demonstrating the impossibility of something is a common topic in the mathematics itself. For example, if I am at place A, then I'm not in B if $A \neq B$. The inability to exist at place B would be incalculable. Another example, there is no even number whose square is an odd number.

Unlike $\top \Rightarrow \bot$, the statement $\bot \Rightarrow \top$ is not a contradiction. From the false, it can be obtained true. This is testified to by many laws of physics that move by small or large jumps from less to more precise ones. An example of this is the mathematical theory of deterministic chaos, and whose attitudes will be used here. The very principle of equality of contemporary democracies is an example of the occasional use of inaccuracy, and as such has been the cornerstone of successful states and modern legal systems for years. Finally, the truthfulness of the implication $\bot \Rightarrow \top$ allows the realization of objective coincidence, precisely because it allows its unrealization.

Let us try to understand the advantages that the universe based on objective substance has by Arrow's[2] impossibility theorem. Arrow using mathematical game theory explored

[1]Bertrand Russell (1872-1970), British mathematician.
[2]Kenneth Arrow (1921-2017), American economics.

the democratic system of elections, seeking the best way and according to its result from 1950, there is no ideal method. For this, he received the Nobel Prize in 1972.

For example, suppose that the voters have three candidates A, B and C which were elected as follows:

- 45 ballot $A > B > C$ (45 put A before B and B before C)
- 40 ballot $B > C > A$ (40 put B before C and C before A)
- 30 ballot $C > A > B$ (30 put C before A and A before B)

Candidate A was the most voted and should be the winner. However, if B withdraws, the winner will be C, because then C has 70 ballot, and A has 45 only.

This is a simple demonstration of Arrow's theorem that in democratic communities had made a shock why is suppressed from the public. Briefly, for the common man, this theorem (Arrow and Gibbard–Satterthwaite theorem), and the whole story behind are saying that for the good choice, for example, should be set a very high criterion, and then from the selected choose one randomly.

I'm paraphrasing; the imposition of rules in democratic elections corrupts efficiency. In the best of intentions with seemingly ideal of fair procedures, and also because of manipulation, we may get the worst results. On the other hand, what would our good "intention" spoiled by excessive regulation, giving the final decision to mere coincidence the long-term objective can improve.

1.1.2 Kolmogorov's Axioms

When you flip a fair dice you have 6 equal opportunities, all six with the same probability $\frac{1}{6}$. If you throw an unfair dice, the probabilities[3] of the resolution may be different numbers $\Pr(1),\dots,\Pr(6)$ which sum is 1. If the probability to fall "one" or "two" is $\Pr(1)+\Pr(2)$, it is said that "one" and "two" are mutually *independent* random events. Thus, the chance of the lottery game are $n = 2,3,4,\dots$ times with n paid combinations than with one.

In general, let are given random events $\omega_k \subset \Omega$ (the dice have $k = 1,2,\dots,6$ outcomes) in the area of random events Ω (the set of all $n = 6$ dice outcomes), with finite $n \in \mathbb{N}$ or infinite $n = \infty$ elements (random events). Therefore, the union is a collection of random events

$$\Omega = \bigcup_{k=1}^{n} \omega_k. \tag{1.2}$$

Number $P(\omega_k) = \Pr(\omega_k)$ is the *probability* of event ω_k. Than the axioms are valid:

1. $0 \le P(\omega_k) \le 1$;
2. $P(\Omega) = 1$;
3. $P(\omega_i \cup \omega_j) = P(\omega_i) + P(\omega_j)$, if $\omega_i \cap \omega_j = \varnothing$.

These are the *probability axioms* of the probability theory. From the nature of these axioms we see that the union of the sets can be written as the sum ($A \cup B$ as $A + B$), and the intersection as product ($A \cap B$ as AB).

[3]The probability of a random event ω is a real number $\Pr(\omega)$ from zero to one.

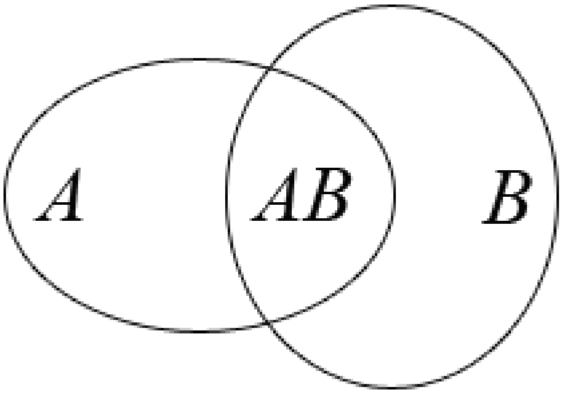


Figure 1.2: The intersection of the sets.

To this axioms is added and the axiom of additivity *countable sets*

$$P(\sum_{k=1}^{n}\omega_k) = \sum_{k=1}^{n}P(\omega_k), \tag{1.3}$$

when $\omega_1, \omega_2,\ dots, \omega_n$ are all mutually exclusive events, ie. when the $\omega_i \cap \omega_j = \varnothing$ for each pair of different indices $i, j = 1, 2, \dots, n$. Appropriate axioms denumerable sets would be

$$P(S) = \int_{\omega\subseteq S} \rho(\omega)\, d\omega, \tag{1.4}$$

wherein $S \subseteq \Omega$ subspace of *continual space* all places ω of random events from the spac Ω, and $\rho(\omega)$ is the probability density at the site of ω.

The event $\omega' \subseteq \Omega$ is complementary (opposite) to the event ω if stays both $\omega \cup \omega' = \Omega$ and $\omega \cap \omega' = \varnothing$, which we write briefly as difference of sets $\omega' = \Omega \backslash \omega$. When the ω' happened then is not happened the ω and vice versa, so we intuitively concluded

$$P(\omega') = 1 - P(\omega). \tag{1.5}$$

This equality can be proven by Kolmogorov axioms too, namely:

$$P(\omega + \omega') = P(\Omega),$$

$$P(\omega) + P(\omega') = 1,$$

and from there (1.5).

Intuitively we see that the probability of a random event increases from 0 to 1, which means that the empty set has zero probability and write:

$$P(\varnothing) = 0. \tag{1.6}$$

The same result follows from the axioms, because of $\Omega' = \varnothing$ and $P(\varnothing) = 1 - P(\Omega) = 0$.

Example 1.1.1. *Using the axioms of Kolmogorov prove:*

1. *if $A \subseteq B$ then $P(A) \leq P(B)$;*

2. for all $A \subseteq \Omega$ is valid $P(A) \leq 1$.

Proof. 1. Because $A \subseteq B$ it is $B = A \cup (B\backslash A)$ and $A \cap (B\backslash A) = \varnothing$. Hence

$$P(B) = P(A) + P(B\backslash A).$$

But, $P(B\backslash A) \geq 0$, so $P(B) \geq P(A)$.

2. From $A \subset \Omega$ follows $P(A) \leq P(\Omega) = 1$. □

Using the axioms can be proved and (1.3), and then a version of the integral (1.4). In addition, it is interesting to us, and identity

$$P(A \cup B) = P(A) + P(B) - P(A \cap B). \tag{1.7}$$

For proof, notice that the random events, sets $A\backslash B$, $B\backslash A$ and $A \cap B$ are *disjoint*, ie. they have no common elements, and its union is $A \cup B$. Then:

$$P(A \cup B) = P(A\backslash B) + P(B\backslash A) + P(A \cap B) =$$

$$= [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)] + P(A \cap B),$$

and thence (1.7).

1.1.3 Quantum entanglement

From the standpoint of theoretical physics, the confirmation of objective randomness is *Bell's theorem* (see [4]) 1964. It finds, there are no hidden parameters in quantum mechanics that could be behind the "spooky action on distance" discovered by Einstein, Podolsky and Rosen in 1935 which is now known as *quantum entanglement*. However, this theorem proves that there is no something or someone (people certainly are not) that might predict random events in micro-world. This meaning of Bell's theorem the physics has not yet recognized (I believe), but the following is.

Quantum entanglement is resulting in mutually dependent random events. If we measure the momentum p and the position q of a particle (such as electrons) along the same coordinates, we cannot determine the both exactly. Measurement errors are said Δp and Δq. Heisenberg found, if it is more precisely determined the momentum (position) then position (momentum) remain more indefinite, so that the product of the uncertainty is greater than *Planck's constant* $h = 6,626 \times 10^{-34}$ Js, namely

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}, \quad \Delta p_y \Delta y \geq \frac{\hbar}{2}, \quad \Delta p_z \Delta z \geq \frac{\hbar}{2}, \quad \Delta E \Delta t \geq \frac{\hbar}{2}, \tag{1.8}$$

where $\hbar = \frac{2\pi}{h} = 1.055 \times 10^{-34}$ Js reduced Planck constant. Here is used *rectangular Cartesian* coordinate system $Oxyz$ (abscissa, ordinate, applicate), but the uncertainty of momentum and the position along the two orthogonal coordinates (for example, the abscissa and ordinate) can be infinitely reduced. This is the meaning of the famous Heisenberg *uncertainty relations* for which he won the Nobel Prize. The momentum and position (along the same coordinates) are *dependent random events.*

These relations can be seen in the example of observation of the electron under the so-called Heisenberg microscope, sketched in Figure 1.3. To get the highest possible accuracy Δx for the position of the electron we use electromagnetic waves of shorter wavelengths up to hard γ rays. By collision the beam with an electron it is revealed the position of the

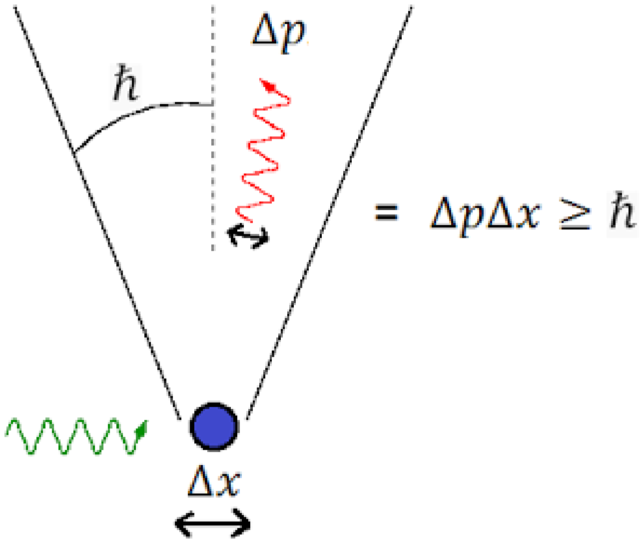


Figure 1.3: Heisenberg microscope.

electron up to the wavelength of $\lambda = \Delta x$. However, the beam with a less wavelength has greater momentum, for $p = h/\lambda$, and due to the law of conservation of the total momentum in the collision, the product of the uncertainties $\Delta p \Delta x$ should be proportional to the $p\lambda = h$. Hence (1.8). This proportionality we don't have if the beam goes by one axis and the momentum by another.

The probability of two random events is reduced by (1.7) if the events are more dependent, more untangled. It looks as nature, protecting the principle of probability, is trying to show as little as possible of unrealized connection. Also, if the probability of the random event is higher it is going to be realized sooner, so we got the impression that nature is trying to better hide its greater uncertainty.

Applied to the explanation of Heisenberg microscope, we get a new interpretation. Decreasing the wavelength of the light by reducing the uncertainty of the location of the (observed) electron, means increase the probability of finding said location. Simply, less Δx means higher $P(\Delta x)$. Accordingly, Δp_x from (1.8) is proportional to $P(\Delta x)$, and vice versa. Heisenberg uncertainty relations can be written in the form

$$a_x P(\Delta x)\Delta x + a_y P(\Delta y)\Delta y + a_z P(\Delta z)\Delta z + a_t P(\Delta t)\Delta t \geq \ell, \tag{1.9}$$

where a_x, a_y, hr, a_t are arbitrary constants of the sum $2\ell/\hbar$. Number ℓ is (generalized) constant dot product of a vector whose components are the probability, and a vector whose components are the uncertainty, here the position and time (event 4-dim space-time). The constancy of the number indicates to law of conservation of total "amounts" unrealized and realized coincidence. We will discuss this later, along with the information.

Another example of quantum entanglement is the typical experiment which proves it today. A particle with the total spin zero is emitting two particles, for example, two electrons spin $\pm\frac{1}{2}$ or instead of them two photons spin ± 1, on the two opposite sides, left and right as shown in Figure 1.4. It is impossible to know in advance which will go left, but it can be found out by passing the incoming particles through a magnetic field when it will reveal its spin (plus or minus). Almost simultaneously is measured another emitted particles spin,

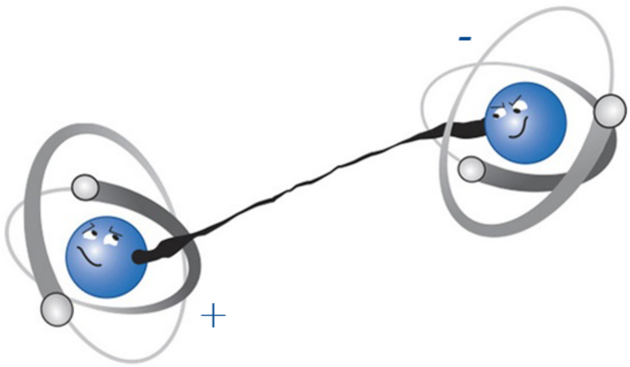


Figure 1.4: Quantum entanglement.

went to the right. The distance between the measured particles is too large, a time between measurements is too short to anything (not faster than light) came from one measurement to another. Let's say, somewhere in the Pacific is emission, and the emitted particles are captured at the two ends of the ocean.

Experiments show that spins to the left make up a random set of (positive and negative), which is quite expected. However, the spins of measured particles corresponding to the right are always paired with the left, which is not expected. Whenever a positive spin is on the left particle the other has negative and vice versa. They both follow the law of conservation (zero) the total spin of the system thus proving "spooky actions at the distance", another term for quantum entanglement.

The third example of the quantum entanglement, which will be mentioned here, comes from a well-known physics of the *particle collisions*, which is still not well understood. Consider a particle C, between two of its consecutive interactions A and B. Let A is separation the photon C from electron (for example, which drops from one to the other atom orbits) and B a collision of the emitted photon with the next electron. On the way to the second electron the photon C has no interaction and until then there is no information about the direction of its departure. How the randomness is objective, so is objective the unpredictability of event B, and until then the appropriate amount of unpredictability has the event A.

Only after the event B will be determined the event A. Only after the declaration of a future event B can happen *fixing the past* in accordance with the conservation laws (of mass, energy, pulse, spin). If the event B never happens, then the behavioral change of A should not happen too.

Accordingly, the current time is not a causal series of events as we usually imagine it. The follows of the previous deduction for the events is not inevitable, but they are two phenomena in a series of random realization that might be different. Consistent to the principle of probability, something is happening "now" and not "then" simply because from the standpoint of a given subject is so more probable. In another point of time, the same subject will more likely be its new present. That is the time, relative tank of events. The highest probability always has immediate future; a further has lower probability, as the past. Stable past is talking to us about some form of conservation law of implemented coincidence. What was once completed, for the given entity remains unchangeable beyond the scope of initial uncertainty causes.

The similar is with space. We are "here" and are not "there" because it is so more probable. By acting in our environment we change the probability so the result may be that we are somewhere there, but wherever that is, then that's the most likely place for us. Any distant place is less likely. New definitions of space and time are symmetric and both result from the principle of probability. It will be shown that the movement of a given subject from the past to the future is due to the relatively higher probability of his future than the past, as the perception of the movement of a given body along a given path will be reflected by the relatively higher probability of his future than past positions.

We will see that such conceptions of space are in accordance with the theory of relativity. Moreover, the principle of relativity will be extended to the principle of probability. Namely, if I'm "here" and I'm not "there", and vice versa, "there" is not "here", then we are not in the same place and our relative observations of probability are not the same. Secondly, such an understanding of space and time is consistent with quantum entanglement. First of all, because quantum-entangled events represent mutually dependent random outcomes, it is not necessary to first realize it and to send the signal to others. There is no "spooky" travel of material signals at speeds greater than light, and the principle of relativity stay. However, there is no more identification of interaction with signal transmission. Occurrences can occur (and have consequences) without the transmission of information. These are the properties of the material world based on objective coincidence.

Note that the aforementioned feature has a light which the source is moving. What is called the *Doppler effect* of electromagnetic waves can now be seen as relative qualities of space probabilities. The source which comes to us we see with greater *frequency* (f) and shorter *wave lengths* (λ), and the source that is going from us send the waves with lower frequencies and the longer wavelengths, so

$$\lambda f = c, \tag{1.10}$$

where $c \approx 3\times 10^8$ m/s is speed of light in vacuum. However, the greater (smaller) wavelength photons representing such a smearing of photon position, which means a smaller (larger) the probability of finding its location. The movement of the photon sources we consider as the movement of the coordinate system in which the source is motionless.

1.1.4 Born probability

The dualism of determinism and randomness is reflected in the extreme nature's consistency in terms of the abstract law, and on the other side, of the objective uncertainty of substances. All matter, space and time, constantly arise from vagueness forming our present and our current time on the objectively random way together with unconditional subjecting everything else, under cold, immutable logic. How strange so it is also useful observation. Namely, if the randomness is objective, and we suppose it is not, then a contradiction must break sometime and there is assorted evidence. That is the idea we are followed here.

Properties of the particles-wave, as well as all the other states of quantum mechanics is determined by *wave function* of position $\mathbf{r} = \mathbf{r}(x,y,z)$ and time t

$$\psi = \psi(\mathbf{r}, t). \tag{1.11}$$

If the number ψ would represent an absolutely exact value of the material properties, which is not, with premise $\psi \in \mathbb{R}$, then the algebra of the quantum mechanics would have difficulties. Therefore, we write

$$\psi = \Re(\psi) + i\Im(\psi). \tag{1.12}$$

The real $\Re(\psi)$ and imaginary $\Im(\psi)$ part of ψ are real numbers (from set $\mathbb{R}$), and $i^2 = -1$ is square of *imaginary unit*. The number (1.12) is said to be *complex*. When the imaginary part does not disappear, $\Im(\psi) \neq 0$, then there is no exact (measurable) value of the state described by ψ, and then the *modulus* of the complex number ψ

$$|\psi| = \sqrt{\Re^2(\psi) + \Im^2(\psi)} \tag{1.13}$$

is greater than its real part.

It is clear that the quantum properties of ψ can be considered as arguments of (those variables that define) the probability $P(\psi)$. The question is can we request that ψ is a random event? Then the wave function would be the elements from a "reservoir" Ω of random events which in a given situation include all possible outcomes and for them would be valid probability axioms. The first and second axioms, of course, are not in dispute:

$$0 \le P(\psi) \le 1, \quad P(\Omega) = 1, \tag{1.14}$$

but the third asks for clarification. Two random events ψ_1 and ψ_2 are independent if the quantum entanglement cannot happen to them, and on the other hand, if the sum (union) of these properties, $\psi = \psi_1 + \psi_2$, is random event that contains all the elements of the summands. However, at the *interference* of photons we see that the both requirements cannot be achieved.

On the figure 1.5 we see the wave of light that encounters an obstacle with two narrow slits through which flow two waves that interfere and fall on the screen, the photo-plate, forming on it the concentric bright and dark stripes[4]. This famous experiment *two slits* can be performed with electrons or any other particles, which shows that they all are actually waves.

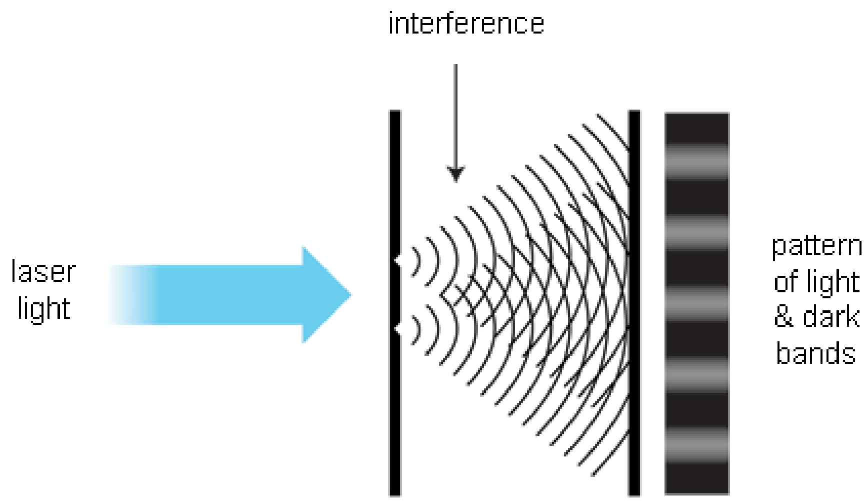


Figure 1.5: Interference of light.

We said that by passing light through the first and second openings can be defined by complex numbers ψ_1 and ψ_2, but the question is whether the same can represent the

[4]see http://www.bbc.co.uk/schools/gcsebitesize/science/add_ocr_pre_2011/wave_model/

corresponding random events too. They would be independent random events if passing the light through only one hole and then through the second is produced the sum that is the same result as if the light passing through both holes at once. As the result of throwing the dice one by one is the same as throwing both at once. However, the experiment with the two holes shows that such a result is not obtained.

It is understood that passing only the light through the first opening with the second closed cannot produce the interference, and that's the same inversely, by passing the light only through the second opening. It is quite understandable that the accumulated brightness on the screen after so selective the first passing then the second will not give the same picture as the one passing the light through both openings at once. It is what the experiment will verify, which is why we must reject the assumption that $\psi = \psi_1 + \psi_2$ well represented the random event of summands.

Further we guess. Let $P(\psi_1)$ is probability of falling photons at a given point of the screen after passing through only the first opening, $P(\psi_2)$ is probability of falling of a photon at the same point after passing through only the second opening, and let P_{12} is probability of photons at that point due to interference. Then

$$P(\psi_1 + \psi_2) = P(\psi_1) + P(\psi_2) + P_{12}. \tag{1.15}$$

The question is how to define the probability $P(\psi)$ to be satisfied this requirement and to stay the undisputed (1.14) for the functions of quantum states (1.11)? The answer is not unique, but one of the simplest of such functions is already accepted in quantum mechanics. It is a square module, called *Born probability*

$$P(\psi) = |\psi|^2, \tag{1.16}$$

proposed by Born[5] 1926. He got the Nobel Prize 1954 in physics for "fundamental research in quantum mechanics, especially in statistical interpretation of the wave function".

Recall that $\psi^* = \Re(\psi) - i\Im(\psi)$ is conjugated to complex number ψ and that the product of conjugated complex numbers is equal to the square of the module (1.13), ie.

$$\psi^*\psi = \Re^2(\psi) + \Im^2(\psi) = |\psi|^2. \tag{1.17}$$

That's why:

$$P(\psi_1 + \psi_2) = |\psi_1 + \psi_2|^2 = (\psi_1 + \psi_2)^*(\psi_1 + \psi_2) = (\psi_1^* + \psi_2^*)(\psi_1 + \psi_2) =$$

$$= \psi_1^*\psi_1 + \psi_2^*\psi_2 + \psi_1^*\psi_2 + \psi_2^*\psi_1 = \psi_1^*\psi_1 + \psi_2^*\psi_2 + \psi_1^*\psi_2 + (\psi_1^*\psi_2)^*$$

$$= |\psi_1|^2 + |\psi_2|^2 + 2\Re(\psi_1^*\psi_2)$$

so

$$P(\psi_1 + \psi_2) = P(\psi_1) + P(\psi_2) + 2\Re(\psi_1^*\psi_2). \tag{1.18}$$

Compared with (1.15) we see the interference addend

$$P_{12} = 2\Re(\psi_1^*\psi_2) = \psi_1^*\psi_2 + \psi_2^*\psi_1. \tag{1.19}$$

It can be calculated that is also $P_{12} = 2|\psi_1||\psi_2|\cos\theta$, where the angle θ is between complex numbers ψ_1 and ψ_2 as vectors in complex plane.

[5] Max Born (1882-1970), German physicist and mathematician.

When the angle $\theta = 0$ then the complex numbers[6] are parallel, $\psi_1 || \psi_2$, the quantum properties are consistent and $|\psi_1 + \psi_2| = |\psi_1| + |\psi_2|$. Modulus of aggregate properties is the sum of the individual modules. On the other hand, the maximum difference between the module of the sum and the sum of the module gives the angle $\theta = 90°$ when the complex numbers are orthogonal, $\psi_1 \perp \psi_2$, and when there is no interference, $P_{12} = 0$. Born probability, formula (1.16), includes all of these cases equally, and when there is no interference gives the least likely. The absence of interference means the presence of the quantum entanglement; it is the mutual dependence probability of the wave function, which has to be in accordance with (1.7).

We came to the strange conclusion that the interference is a sign of independence. It's a bit unexpected but not incomprehensible. If two particles can collide repel or attract each other, they are not independent of each other, contrary to for example the two photons that are mutually interfering and are ignored. Logical? Consider this in more detail.

Why do particles repel? The first contemporary example you see on the picture 1.6 of *Feynman diagram* rejection two electrons (e^-) by means of a virtual photon (γ), to which we add the ideas expressed in these book.

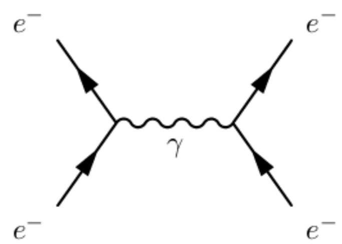


Figure 1.6: Feynman diagram.

Electron left constantly periodically emitted virtual photons (γ), which sometimes affects the second electron (e^-) to the right, see the image 1.6. Such virtual photons are an integral part of the electromagnetic field around each electron. When a photon hits an electron on right, because of the law of conservation of momentum for the closed physical system (in this trio: electron, photon, electron), then left electron repels to the left and right to the right. If the spin of electrons left was $+\frac{1}{2}$, right $-\frac{1}{2}$, then the spin of the photon could be +1 and rejection process is in accordance with the law of conservation of spin. After the exchange, the total spin of photon and two electrons again is the same (zero). Therefore it is not possible to exchange a virtual photon between two electron with equal spin (spin electrons can only have two values $\pm\frac{1}{2}$, spin photons only two values ± 1).

It is not possible to change the momentum, spin, energy, etc. of the first electron after the departure of virtual photon until happens the collision with the second. I just repeat and stress it now. Our world is made up of a large abstract (infinitely infinite) contingent of uncertainty from which our nature because the principles of probability in the smallest possible portions as on a spoon is leaking the matter, space and time. The substance arises from the huge uncertainty, which is intangible, spaceless and timeless. As such, they neither know our terms "now", "tomorrow" or "yesterday", nor they care if the two events are to us even millions of years away. Therefore, the *virtual photon* after the collision with another electron will "tune" momentum, spin, energy, etc. of the first, even if between the first and second interaction lay millions of years, from the standpoint of us, which are material, from

[6] We mean: vectors represented by complex numbers.

this side of the coincidences. So, we are working on the theory in which the future affects the past, I guess, on a lesser extent than the past affects the future. This finding should not be inherently repulsive if there is no contradiction.

From the standpoint of classical physics, when there is no objective randomness, where is deterministic future, the virtual photon leaving the first electron could know exactly where to go to collide with the second electron, and it goes there. Then there is no contradiction again, and there is no need for the future to affects the past. However, it opposes here the required objective randomness.

In both theories, deterministic or not, it makes sense to talk about reverse flow of the time, each in its own way. In the new theory, the prior and the next probabilities in the time flow are both (almost) greatest possible, so if you go backward in time, you again returning back into greatest probability, so the principle of probability stays for both directions of time. It seems absurd, but it means that the perception of the reality of the two observers must be (slightly) different. This is not a vision of the film released backward. That's why in this theory makes sense to talk about the *positrons* which direction of time is opposite to electron.

So how do these two, whose time flows in opposite directions, can communicate? The question the first in his own time asks the second expecting the answer later, the other received before the answer. They would be like two old friends who can in advance to answer, knowing exactly what will be asked. It is also not contradictory if we consider how the elementary particles are "stupid" and how they do not have to act organized.

Now finish the story about the experiment with two slits. When we let pass one by one electron through both holes open, no matter how long intervals between successive release where, it will not be important to uncertainty (it is timeless). Electrons can *interfere* just because the interference means independence of the random events. Dark and bright strips interference on the screen will be even clearer than when all the electrons move toward those openings at the same time when the repulsive forces among electrons intervene and spoil the mutual independence.

1.1.5 The universe and chaos

The Cosmos or *universe* were roughly speaking synonyms for "unlimited" expanded matter, space and time that surrounds us. The universe comprises various celestial bodies (figure 1.7), such as the galaxy, star, planet, satellite, comet, in addition to space and time in which these bodies are. Its visible part of us has a diameter of about 93 billion *light years* (the length that light travels in a year), and is considered to begin with a great explosion, the *Big Bang*, before about 13.8 billion years. In a world that is based on the principle of probability, the physical limits of the universe are where stop uncertainty. These are the random states of so small probabilities that are almost never realized or are the places of such large probability that, because of the law of large numbers, becomes almost deterministic events.

The principle of probability forces the nature onto the continuous release of uncertainty, the resolve first it's most likely random states, and if it must, then those less likely. How the more likely events are less uncertain, nature acts as it is saving its vagueness. In front of us, it is constantly creating the matter, space and time, grudgingly, in small doses. With this notice, it is easier to understand another paradoxical *dualism* of coincidence and causality: the created world of substance is much smaller than its opposite abstract worlds. It is the spark of mathematical rules that are infinitely infinite.

Figure 1.7: The Universe.

From the above text, it is clear that our universe is much more than just matter, but we guess that it can be and a part of an even larger universe. The question is, how larger and of what? In other words, why should so many mathematics be needed to matter and where it is hiding? Let's look at part of the possible answers through the following speculations.

The *realization of uncertainty* arises our present (time, space and matter). Piled present in order of how they accrue make our past that we leave behind, moving constantly into the future. We have already noticed that the movement of a subject from our future into our past, which is going from its past to its future, is not in contradiction with the principle of probability. Even if it went right to our steps backward, it would always be in the most likely the existing situation. What has to be realized uncertainty for us, it is unrealized cause to the such. Our cause is its consequence. The laws of mechanics by changing the sign of the times remain the same; the speed of light for it is the same as to us.

Not only are the principles of the theory of relativity there as here applied, but say and the Heisenberg uncertainty relation. If the consequence of a subject whose time is going backward in relation to us was absolutely precisely defined, then the cause would for us be like that, and it would be contrary to the assumed objective randomness. I'm not saying that every law of physics that applies to us applies to the entity whose *time* is going backward, but I would be surprised with some general asymmetry, inequality of such sudden perception of the universe. These include the possibilities of other directions of time.

The differences between these deterministic theories are starting with a small probability that the subject could go back through the same events that we've been through. The consequence is again a need for (extended) *relativity theory.* Nature could a part of such diversity cover up treating all elementary particles (electrons) identical, almost as if in the world there is only a single electron, but then arises the question of complex bodies. Misalignment of large quantities of materials can also be justified by laws of probability. For example, let us try to understand the difference between the total amount of matter-antimatter in the universe by *paradox lifts* (see [5]).

In a tall office building employees have the habit of using the elevator to jump from the office to the office, usually going from a floor to floor. The frequency of such transitions is reduced to the ends of the building, in the ground floor and the roof. A passenger waiting for an elevator near these areas (ends) is likely to go to the central part of the building. The paradox is that such a traveler would likely see an elevator in coming that goes to him in an undesirable direction, from the center of the building to the end.

This accident of elevator passengers becomes occasions in the case of entities which time flows in opposite directions. Anywhere that we are near the end of the universe is likely that several entities have the same direction as the flow of time identical to me, then the opposite. Note that in this way we can justify many other different time directions, on which we will work later.

From the objective uncertainty arises the matter, space and time, and we cannot only see direct cause of creation "on the other side", but these ultimate objective causes do not exist. Any entity in the universe perceives its own part of the universe, we can assume, always with the same laws, but never the whole picture. It others "own perceptions" see relatively and little differently. In short, the generalization of Einstein's relativity to the probability, we can preserve the unity and equality of each entity in the universe, and material consequences include much more abstract parts than by one subject.

When you discuss the implementation of the initial problems caused by coincidence, the question arises how far we can take diversity. Basically, we have two kinds of difference, one that *converge* as increasingly shorter pendulum swings toward the equilibrium position and those who *diverge* as the fate of a person and the environment after a small movement of guns trigger, for example, because of the murder of constructor some later important buildings or rather the terrorists. These are the dilemmas of the deterministic *chaos theory*.

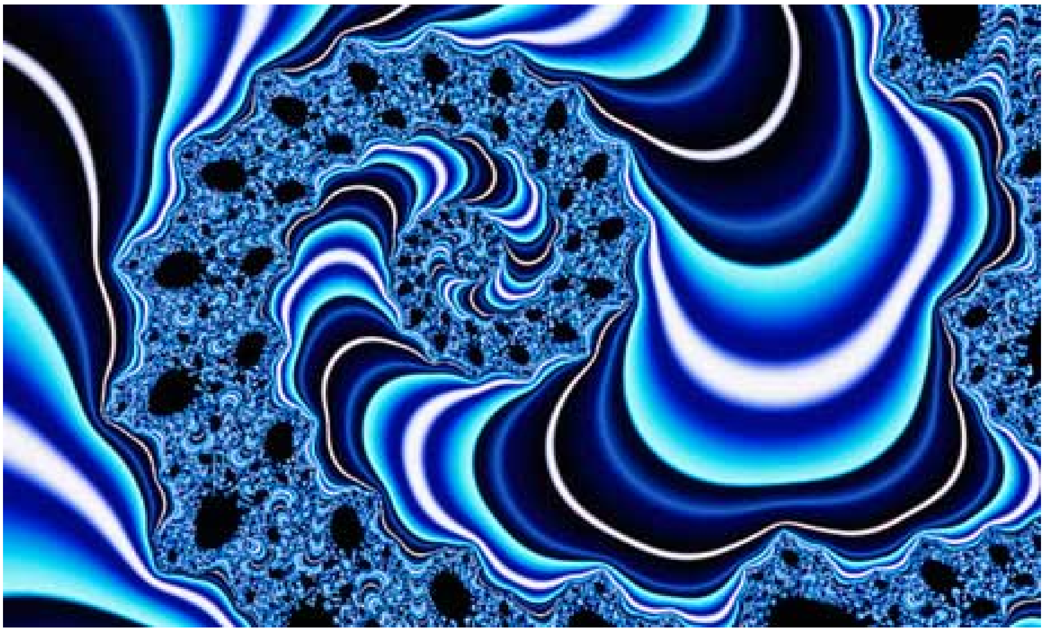


Figure 1.8: Chaos – order hidden by disorder.

Chaos theory[7] is a branch of mathematics which deals with the behavior of dynamic systems, which are very sensitive to initial conditions. In this context "chaos" could be considered as something that looks random but what it essentially is not, because somewhere inside are hiding the rules, feedback loops, repetition, self-similarity, fractals and self-organizing, that by small changes in the initial values lead us to a big difference in the final. In this sense, it is written (see [8]) today famous sentence on the *butterfly effect*

[7]It is understood: the theory of deterministic chaos.

whose move of wings in Brazil may be the cause of the tornado in Texas. The pioneer of this theory Lorenz[8] defined the chaos (see [7]) by following sentence: Chaos is when the present determines the future, but the approximate present does not define the approximate future. According to all this, chaos theory is not correct basis for this book.

The mathematical theory of chaos, the same as probability, does not address the ultimate causes. The both stops when it becomes important if the randomness is objective or it is merely subjective. It is enough for them that the forms which they are dealing with are applicable laws of probability and general logic of mathematics. We have said that it allows them the accuracy of the implications $\perp \Rightarrow \top$, and this is because beyond a group of truths there are more truths. On the other hand, chaos theory reveals useful rules where we do not expect it, that the artist tried to show in figure 1.8.

An example of such beneficial results is *Ramsey's theorem*, paraphrasing: it is always possible to find some cliché no matter how hard you try to keep things blur. Thus, if enough random sky clouds passed, it will always be possible to see in them a pre-given face. Generating a random word in a book, when the book becomes sufficiently extensive, it will contain and meaningful sentences. But here, because the multitude of coincidences always have elements of necessity, we will not make a logical fallacy to believe that all the coincidences are composed only of necessity.

Ramsey's theorem was discovered before the chaos theory. It was part of the *combinatorics* (the theory of scheduling) and the result of Dirichlet[9] principles (if you have $n = 1, 2, 3, \dots$ pigeon cages and $n+1$ pigeons, then at least one of the openings must have at least two pigeons), and today is a common theme in competitions of students in mathematics. Since many are understood the combinatorics as the part of the probability, I believe that Ramsey's theorem will be similarly regarded as part of quantum mechanics.

Chaos theory is built on a more intense search for the correctness then Ramsey's. It reveals the interesting cyclical stabilities (equilibrium) in the instability. Stability as well as sets of values to which the systems evolve with a wide range of initial conditions is referred to as *attractor*. Roughly classified, all these kinds of balance are divided into static and dynamic. When the systems are developed in such equilibrium, the first we call attractors, the second *strange attractors*.

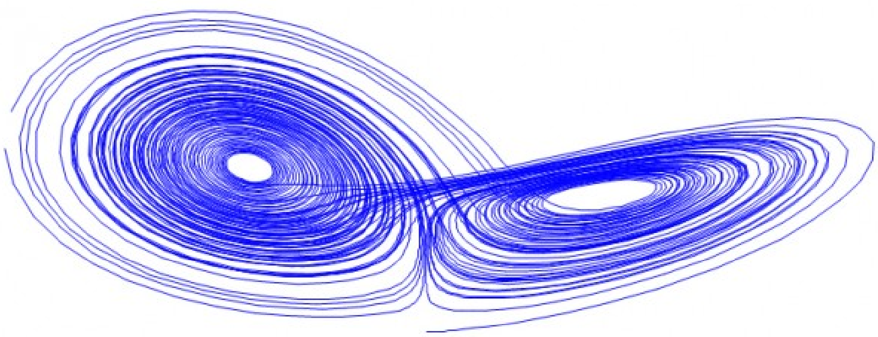


Figure 1.9: Strange attractor.

For example, imagine a town with 10 000 inhabitants. For the accommodation of its

[8]Edward Norton Lorenz (1917-2008), American mathematician.
[9]Peter Gustav Lejeune Dirichlet (1805-1859), German mathematician.

inhabitants, the town has a store, a kindergarten, a school, a library and three churches. The infrastructure is enough to them and people are living in balance. However, some companies decide to open a factory in the suburb that could employ 10 000 people, and rapidly to develop the city to accommodate 20 000 people. Therefore, they work on the opening of another department store after another kindergartens, schools and libraries and three other places of worship. Investors are targeting the new balance that is called an attractor.

Imagine further that instead of the arrival of 10 000 people in the existing 10 000, leave the city 3 000 and it remains only 7 000 inhabitants. Heads of the department store estimate that their shops can only survive with at least 8 000 customers, so they start to close. Meanwhile, demand rises and another company decides to build a supermarket, hoping that it will attract new people. It happens, but many already moving out and a new supermarket does not change their intentions. The company keeps the stores open for another year and then came to the conclusion that there are not enough buyers and close them again. People moving out, but demand is growing again. Someone opened the third shopping center. People begin to return but not enough. The shops are closed again. The process of opening and closing runs even further, allowing the process itself to becomes a steady state we call strange attractor (see figure 1.9).

The difference between ordinary and foreign attractors is that the first is a condition in which the system is finally placed, and the second represents the type of which the system goes from situation to situation without a final pacification. *Poincaré-Bendixson Theorem* says that strange attractor can only exist in systems with three or more dimensions or two dimensions if space is not Euclidean.

Because of space has three dimensions (length, width, height) physical systems can strive to a dynamic equilibrium. In the two-dimensional form, attractors become static. For example, the evolution of life on Earth may be in dynamic equilibrium, and so the development of the universe itself. On the contrary, flakes of snow are developing in the static 2-dim[10] crystals. A type of attractor is also the *wave function* which represents the waves of matter in quantum mechanics.

When a large number of particles form a pattern that is almost equal to starting options of an individual particle, then we are talking about the *self-similarity*. It may also be a static (as a snowflake) or dynamic (as the atom), expressed in the repetition of the shape or behavior at various levels of complexity. Not as identically copy, but as variations of the same base, such as *fractals*[11] on figure 1.10.

Fractals are a kind of repetition of the depth of the micro-macro world. Like other attractors, they ignore the ultimate causes of the accident and lead us into the illusion that the corresponding cycles are present everywhere and always is adjustable. It reminds me of rational numbers in a set of all real numbers. If from the set of real numbers thrown out the irrational, the remaining rational numbers (fractions) written in decimal format will always have a finite set of digits that are endlessly repeated. For example, the decimal number 0.131313... is equal to 13/99. Unlike them, irrational numbers are those that are not periodic.

Analogously, the endless repetition of fractals are impossible because of the limited universe, say once again, because of the law of large numbers: "repeating in the same conditions, the frequency of random outcomes is becoming equal to the probability of the

[10] 2-dim is short for "two-dimensional"
[11] Fractal: `http://mathworld.wolfram.com/Fractal.html`

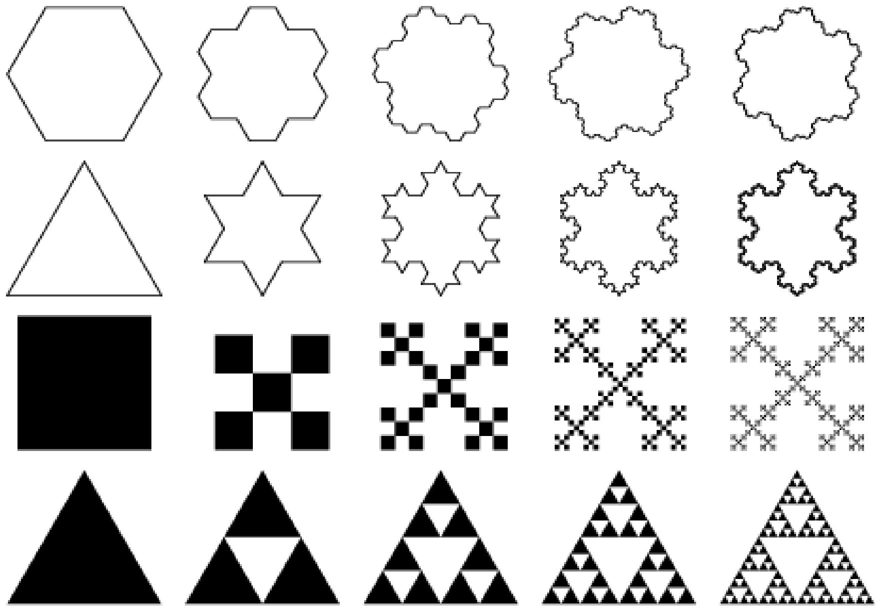


Figure 1.10: Fractals.

outcome". For example, in the 18th and 19th centuries, it was observed that a large sample ratio of male and female newborns is constant. At the same time, the number of daily born male and female children is not uniform, but that relationship begins to stabilize only over a longer observation.

Because of the law of large numbers, the big things in the universe from a completely random are growing into completely non-random, or vice versa, reducing into the micro-world has its end in the objective randomness. Because of this law, and because we do not live in the micro-world, we have the illusion of determinism. Usually, mistakenly we believe that the matter that surrounds us and the space at all times of present have only one possible continuation into the future.

1.1.6 Dimensions

Even our everyday life and our macro-world sometimes show that the exact prediction of the future is not reliable. Weighing the arguments for-and-against absolute predestination sooner or later we must come to the toughest stronghold of determinism in science in general, and that is the classical mechanics, Newtonian[12] or Einstein's. Doubts of the items were the crucial reason in the book [2] I revived Urysohn's definition of dimensions.

According to the definition of the dimensions by Urysohn[13], the number of dimensions of a finite set of points is zero ($n = 0$). A set of points has the dimension $n+1$ if and only if it can be split up by subset having at least dimensions $n = 0, 1, 2, \dots$. If such a separation is not possible, given a set of dimension $n+2$ or more.

For example, a straight line forming a single point can cut off the two parts (two rays), and this figure has the smallest dimension which can execute such division. Therefore, the line has one dimension, because the point has dimension zero. A plane can be halved (at

[12]Isaac Newton (1642-1727), English mathematician and physicist.
[13]Urysohn's definition is slightly different, but the topological and inductive.

least) by one line (on the two half plane), so the plane has two dimensions. Space (length, width, and height) has three dimensions because it can be divided by a single plane and cannot be divided by figure with less than two dimensions. If the *space-time* had "only" four dimensions (three spatial and one temporal), as is the case in the Minkowski[14] model made for the theory of relativity, then present that contains all of the (three-dimensional) space of the universe at the moment "now" could divide the universe to its past and future. Something like that is possible only in inertial systems.

In inertial system clocks can be synchronized, showed Einstein already in it's the first work ([3]) in 1905. The *synchronization of cloks* means such establishment of measuring the time at the area which will change a little-by-little, with continuously changing the position. The resulting isolated space-time continuum is present or "now" of the given space-time. It is "frozen" clip without the time of space-time. Consistently, to the fact that the clocks in inertial systems can be synchronized and Urysohn's definitions of dimensions, it follows that the present of those systems is located in the four-dimensional space-time. That "now" extends along the new coordinates - duration.

Inertial systems are those coordinates which are related to the (stationary in relation to) the body in a uniform rectilinear motion. These are bodies in the *inertial motion* on which, we can consider that, are not acting any external forces and that therefore do not feel the acceleration. Hence we can say that (about) inertial system is also an attached to the body that falls freely in the (weak) gravitational field. It's just about "inertial system" because gravity is pulling the body of one axis squeezing it laterally. The gravity of massive stars elongates the body by sucking it so the free fall towards it can no longer be treated as the inertial system, at least not in the sense that this body does not feel the external forces.

So, inertial systems of space-time have four dimensions. If there would be only one inertial system then the universe would be ruled by *determinism*. All events would be predestined because there was only one direction to change. There would be only one timeline the present would be developed, and that "now" would again at any time selected the past from the future. To rule the deterministic universe there is no need for the principle of probability. That principle would be illusory, temporary or meaningless. However, in the world we exist there are and the non-inertial systems too, and such is impossible to put in only four dimensions.

For example, in a system that *rotates* the clock's synchronization is not possible. While the Cartesian coordinate system $Oxyz$ rotates around the z-axis, the further points of this axis are moving faster, each on its circle of rotation, with increasingly slower time (relative to a fixed point on z-axis). When you synchronize the clocks going in one direction of rotation of the circle, it will happen a big time jump after touring the whole circle and arrival at the start. This jump will be greater with a larger radius of the circle, but also with a higher angular velocity of the rotation system. This known impossibility of synchronization of clocks now reveals the impossibility of defining the 3-dim[15] present from all the future and past events for the viewer. In other words, according to Urysohn's dimensions, a system that rotates cannot fit into the space-time with four dimensions.

Also, even the inertial systems that move with different relative speeds (each in uniform rectilinear motion) and would, therefore, have a different speed of time flow would not fit in the four dimensions of space-time, but it is not as obvious as in the case of rotation.

[14] Hermann Minkowski (1864 to 1909), German- Jewish mathematician.

[15] 3-dim is short for "three-dimensional"

That is a little clearer in the case of centrally symmetric gravity, which creates a lonely planet or star. Radially attraction towards the center of gravity (planets), which increases with the approach to the center, makes a growing deviation of time axis from the vertical line (which coincides with the vertical only in the absence of both, the gravity and motion). These remove of directions of the timelines are different at different points around the center and all are directed toward the center and cannot be deployed without the (at least) three dimensions of time.

Incidentally, note that the extra dimensions of the time we are talking about here are not those extra dimensions of space known within string theory[16]. Secondly, to say that we (must) define the additional dimension of time still does not mean that these dimensions have strong links with the daily uncertainty. To fill these logical gaps see the next explanation that connects uncertainty, erosion and Urysohn's dimensions. Please note that the *erosion* as a result of chance may diverge (as in the Butterfly Effect of the chaos theory), and converge (to the steady state), but on average it is destructive.

Let's imagine that part of space-time within an atmosphere is surrounded by walls that completely separate the interior (of the prison) from the outside. Brick, stone or other used construction material has three dimensions of space and also has duration, meaning that it has at least four dimensions. Assume that these walls will not last indefinitely, but that due to erosion sooner or later must relent. Then these four dimensions cannot separate space-time on two parts, which means that it has more than five dimensions.

If we assume the contrary that the said walls will endure indefinitely, that they will remain unchanged due to the absence of erosion or other factors of destruction, then again according to the Urysohn's definition the space-time must not have more than four dimensions, how many the walls of the prison have. Thus, the erosion as a series of random events, which undermined the structure of the material, requires more than four dimensions of space-time.

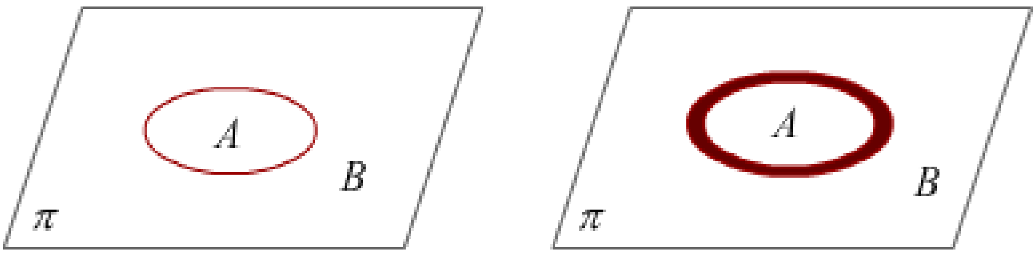


Figure 1.11: A closed circular line and the ring in the plane π.

Figure 1.11 shows something analogous to 2-dim plane π. On the left is a closed circular line (one dimension) which will completely separate area A from B as well as the ring (two dimensions) on the right plane. However, the figure of the left line is of the smaller dimension that can separate area A from B on plane π. Ring on the right corresponds to 4-dim walls of the mentioned prison, which in the case that does not last enough, indicates the existence of more than five dimensions of space-time.

The three dimensions of time, along with the three dimensions of space, are not just the consequence, but also the presupposition of the existence of objective randomness. As we have seen, they are in some sense a result of the principles of probability. Because we learn from them by using mathematical deduction, what we call the universe we must be viewed

[16] String Theory: `http://superstringtheory.com/`

using unpredictability, and from the other side by the lows of logic to which is subjected all the rest. Our universe is just one part of the universes.

1.1.7 Special relativity

Because of the great importance for us of *special relativity* (see [3]), which Einstein published in 1905 in German, we'll look at the main results of this theory from the perspective of principles probability and so far the observed effects. The first is a new vision of space. The given body is where it is because it is the most probably position. It has a different perception of the places around then all other bodies, and vice versa, and one of those differences is known for bodies in the inertial movement.

The usual assumption that we presume is the following. One body is at rest in the inertial coordinate system K, and the other in the system K'. The second system is moving uniformly in a straight line by speed v relative to the first, and both are represented by rectangular *Cartesian coordinates* $Oxyz$ and $O'x'y'z'$ respectively. We assume that in the initial moment $t = t' = 0$ the axes of the systems match, $K \equiv K'$, and that the movement occurs along the abscissa (x and x' axis). The *own observer* is the one that the phenomenon was seen in the standby mode, the *relative observer* sees it in the motion. In the literature, it is often used term 'proper' instead the 'own'.

Further, we work abnormally. The probability of the position of the space-box we'll estimate by a wavelength of arbitrarily selected photons (the electromagnetic wave). When the photon source is located (at rest) in the system K', by its own wavelength $\lambda_0 = \lambda'$, the relative, as seen from the system K is λ. From the above considerations it is clear that observed *wavelength* (λ) may also depend on the speed (v), and the direction of the source velocity at the time of observation. Then assume that there is a function $\gamma > 1$, which depends on the speed v such that the *average wavelength* is

$$\lambda = \gamma\lambda_0 \tag{1.20}$$

and that $\gamma = 1$ when $v = 0$. Namely, the larger the wavelength means the longer smearing of photons position; the less probable its position. The first observer stays in its system (K) and sees the other positions (K') as it is in a lower probability.

If the probability of particles from the K to stay in the system is P, then its probability is $P' = P/\gamma$ to be in the K'. The probability for all $n \to \infty$ particles to jump from K into K' is proportional to $(1/\gamma)^n \to 0$, so that becomes equivalent of *impossible event*. That is why holds Newton's *law of inertia*: each body remains stationary or in uniform rectilinear motion until another body or force influence on it.

Generally, for all wave phenomena (sea waves, sound, light) the product of the wavelength and the *frequency* is equal to *wave speed*, so in this case

$$\lambda f = c, \tag{1.21}$$

where $c \approx 3 \times 10^8$ m/s *speed of light* in vacuum. From relativity theory, we know that the speed of light (electromagnetic waves) does not depend on the speed v of the light source. Replacing (1.20) we get

$$f = f_0/\gamma. \tag{1.22}$$

The average relative frequency (f) of the photon is lower than the own (f_0).

As the frequency is the number of flashes per time, we come to the conclusion that the relative time is slower than the own (proper). The observed relative time interval Δt is

$$\Delta t = \gamma \Delta t_0, \tag{1.23}$$

if Δt_0 the appropriate own. We've well-known formula for the relativistic *time dilation*, where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{1.24}$$

Recall that:

$$\gamma \approx 1 + \frac{1}{2}\frac{v^2}{c^2} + \dots, \quad \frac{1}{\gamma} \approx 1 - \frac{1}{2}\frac{v^2}{c^2} + \dots, \tag{1.25}$$

where we do not write the terms of fourth and higher degrees, because the number $\frac{v}{c} \to 0$ are negligible. These are developments into Maclaurin Series.

These conclusions are at first sight so convincing that it is very difficult to contest, but they are in the physics completely new and we should analyze them from different sides. Below, I proceed from observations on quantum entanglement, here initiated the Heisenberg Uncertainty (1.8), and then further on a new way. The scalar product ℓ, the minimum value of the uncertainty (1.9), now write

$$\ell = \Delta p_x \Delta x + \Delta p_y \Delta y + \Delta p_z \Delta z - \Delta E \Delta t, \tag{1.26}$$

with the same meaning as there. It is clear that these "generalized uncertainty" ℓ is not dependent on the relative speed v of system K' in comparison to the system K, and we know that the product of orthogonal uncertainty momentum and the position is zero.

Example 1.1.2 (Lorentz transformation)**.** *Show that*

$$\begin{cases} \Delta p_x' = \gamma(\Delta p_x - \beta \Delta p_t) \\ \Delta p_t' = \gamma(\Delta p_t - \beta \Delta p_x), \end{cases} \quad \begin{cases} \Delta x' = \gamma(\Delta x - \beta \Delta ct) \\ \Delta ct' = \gamma(\Delta ct - \beta \Delta x), \end{cases} \tag{1.27}$$

where $\gamma^2(1-\beta^2) = 1$. With $\beta = \frac{v}{c}$ this becomes the Lorentyz transformation.

Proof. Uncertainty (1.26) is the same in both systems K and K', we have:

$$\ell' = \Delta p_x' \Delta x' - \Delta p_t' \Delta ct' =$$

$$= \gamma(\Delta p_x - \beta \Delta p_t) \cdot \gamma(\Delta x - \beta \Delta ct) - \gamma(\Delta p_t - \beta \Delta p_x) \cdot \gamma(\Delta ct - \beta \Delta x)$$
$$= \gamma^2(1-\beta^2)(\Delta p_x \Delta x - \Delta p_t \Delta ct) = \ell,$$

because the $\Delta p_x \Delta ct = \Delta p_t \Delta x = 0$. From invariance, $\ell' \equiv \ell$, follows $\gamma^2(1-\beta^2) = 1$, which is supposed to prove. Interpretation of the Lorentz's movement gives $\beta = \frac{v}{c}$. □

Note that in the example neither we give the particular value to relativistic gamma coefficient (1.24), nor use the previous pair of results, but assuming *symmetry* in the transformation of the uncertainty of momentum and position of (1.27). We do this so the coefficient γ can be treated in a similar way in the analysis of the general theory of relativity.

Another important note is that the previous interval Δx and further, are not commonly used distances in dynamics (theory of relativity), but the uncertainties drawn from quantum mechanics. Yet, we know that Lorentz transformations apply to distance and the same rectangular Cartesian coordinates:

$$x' = \gamma(x - \beta ct), \quad y' = y, \quad z' = z, \quad ct' = \gamma(ct - \beta x), \tag{1.28}$$

assumed system K and K'. Hence, in a known manner we obtain the contraction of length for the rod in the movement.

Example 1.1.3 (Length contraction)**.** *Show that the relative distance parallel and perpendicular to the velocity v, respectively:*

$$\Delta r_{\|} = \Delta r_0/\gamma, \quad \Delta r_{\perp} = \Delta r_0, \tag{1.29}$$

where Δr_0 is appropriate own length.

Solution. In the Lorentz transformation (1.28) choose two places on the abscissa and form the own distance $\Delta r_0 = x'_2 - x'_1$ and relative distance $\Delta r_{\|} = x_2 - x_1$ simultaneously, while the time $\Delta t = t_2 - t_1 = 0$. Hence, the first equality. The second get analogously, choosing the length along the ordinate (y-axes), or the applicate (z-axes). □

Relative and own observations of probabilities, flows of time, length in the direction of movement, including the movement itself, are different in the two systems. Yet the differences are too small to give us the necessary new dimension of time. Lorentz transformations apply only to inertial systems in which we can ignore the external forces, and without forces there is no need for additional time dimensions. But then we are at the step on other phenomena that go along with the forces.

If we understand Newton's inertia literally as *body laziness*, then we expect that the relative mass m of the body increases with time slowing down, or that

$$m = \gamma m_0, \tag{1.30}$$

where m_0 is own weight (laziness) of the body. Momentum of the body, $p = mv$, is the product of its mass and velocity. This increase in weight we can understand by the increasing energy of the body given to it in the process of increasing the speed. Thus, the total (relative) energy is

$$E = \gamma E_0, \tag{1.31}$$

where E_0 is energy that the body had at rest. Comparing this with what we know from classical physics, the kinetic energy of the body (E_k) is equal to the difference of the total energy that the body has in translation by speed v and the energy of the body at rest:

$$E_k = E - E_0 = (\gamma - 1)E_0 \approx \frac{1}{2}\frac{v^2}{c^2}E_0, \tag{1.32}$$

where we use the approximation (1.25), followed by $E_k = \frac{1}{2}m_0v^2$, we get $E_0 = m_0c^2$ for own, and then in general:

$$E = mc^2, \quad p^2c^2 = E^2 - (m_0c^2)^2. \tag{1.33}$$

The first is known relativistic formula that links the energy of the body concerned by its mass and speed of light, and the other is derived from it.

Example 1.1.4 (Energy by momentum)**.** *Show that the second formula (1.33) is true.*

Solution. It follows from ($p = \gamma v_0 v$):

$$p^2c^2 = \frac{m_0^2 v^2\, c^2}{1-\frac{v^2}{c^2}} = \frac{m_0^2 \frac{v^2}{c^2}\, c^4}{1-\frac{v^2}{c^2}} = \frac{m_0\left(\frac{v^2}{c^2}-1\right)c^4 + m_0^2c^4}{1-\frac{v^2}{c^2}},$$

$$p^2c^2 = -m_0^2c^4 + (mc^2)^2,$$

and hence the asked result. □

It is well known the more accurate (and more extensively) the execution of these formulas in the special theory of relativity, and so I do not cite them. What is here more needed is their connections with randomness. For example, we mentioned two types of boundaries of the universe. One is in very large bodies (the law of large numbers and the beginning of determinism), and the other is very small (the cessation of any certainty). But now we see that both, the first and second border have (almost) infinite energy. The first example is the event horizon of a black hole, and the other is a very large energy required to reach a very low, quantum value. It looks as our universe is an energy gap, *vacuum* in something.

1.1.8 Force

When the body feels a force it is not moving inertial, and we[17] Lorentz transformation of forces considered unreliable. Special Theory of Relativity with the Lorentz transformation is limited to inertial motion in which the body does not feel the effect of the force and, to say the least, their use in a wider context must be carefully considered. The famous paradoxes of special relativity arise, we stress, due to mixing forces with inertial motion.

Twin paradox is one such example. Here we have one-fold mixing forces with the effects of time dilation of special relativity. The first of the twin brothers remain on Earth and other goes by spacecraft in a straight line uniformly to some distant point in the universe, then goes also inertial back to Earth. While moving forth and back in a constant speed compared to the first brother, he is slowly getting older in accordance with dilatation time (1.23). However, according to principles of relativity, it doesn't matter which of the two is moving at a uniform rate, so the first brother should be younger than the second in accordance with the same dilatation. This is the paradox, as at the end the travel the both brothers are on Earth with seemingly contradictory interpretations.

In this paradox, the force occurs once needed to rotate the direction of the velocity of the second brother, and then his movement was not inertial. While the second brother departing from Earth, his time runs slower relatively to the first, and he is increasingly lagging behind in the past of the first brother. In the reverse direction, as he approached the Earth, again, his time ran slower, but he was in the future of the first brother, approaching to its present as he come closer to the time of their arrival on Earth when the both presents equalize. This means that in the one-fold action of the force, to rotate the direction of the speed for 180°, a period of life of the second brother shortened, relative to the first.

A force is a vector quantity $\mathbf{F}$. We must describe it with both, the magnitude (size or numerical value) and the direction. It can be used for the (infinitesimal) useful work dE along the (infinitesimal) path $d\mathbf{r}$, by relation

$$dE = \mathbf{F}\cdot d\mathbf{r}. \tag{1.34}$$

[17] Here, but not the official physics.

The operation is the *inner product* (also called "dot product" or "scalar product") of the vectors of force and path. This relationship is an expression of the law of conservation of energy and in this sense is more general than Lorentz transformations. These transformations we ignore for now, but still, use tags K and K' for two inertial systems moving in mutual speed v.

Component relative force $F_{||}$ parallel to the velocity vector $\mathbf{v}$ must be in accordance with the relative length $dr_{||}$ and energy dE of formula (1.34). Hence, the first of the formulas:

$$F_{||} = \gamma^2 F_1, \quad F_\perp = F_2. \tag{1.35}$$

The second equation gives the relative component $F_\perp$, perpendicular to the direction of movement. The respective components of the own force are F_1 and F_2. This is the interpretation that we promote here, although it is quite the reverse of the known physics today.

For example, let the body mass m_1 and m_2 are at a fixed distance r from each other. From Newton's mechanics is known that they are mutually attracted by *gravitational force*

$$F = G\frac{m_1 m_2}{r^2}, \tag{1.36}$$

where $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2} is the *gravitational constant.* If those two bodies are in the system K', there are two extreme observations from K. The first, when the bodies are lined in the direction of the velocity vector, and the second, when they are perpendicular to the axis. Due to contraction of the length in the direction of speed only (1.29), we obtain for the corresponding components of the gravitational force agreement with (1.35). It is obvious that the same conclusion applies to all attractive or repulsive forces that decrease with the square of the distance. Deviations from this will be considered as a consequence of our lack of knowledge of the nature of power, places where the physics has to be repaired.

Consider a similar result, now for the product of mass and acceleration ($\mathbf{F}_0 = m\mathbf{a}$) relative to the momentum change over time ($\mathbf{F} = d\mathbf{p}/dt$). For a particle of mass m:

$$m\mathbf{a} = m\frac{d\mathbf{v}}{dt} = m\frac{d}{dt}\frac{\mathbf{p}}{m} = m\left(\frac{1}{m}\frac{d\mathbf{p}}{dt} - \frac{\mathbf{p}}{m^2}\frac{dm}{dt}\right) = \frac{d\mathbf{p}}{dt} - \frac{\mathbf{p}}{m}\frac{dm}{dt}. \tag{1.37}$$

We took into account that the mass of the accelerated particles changes, and the acceleration vector, $\mathbf{a} = d\mathbf{v}/dt$, is the ratio of the velocity vectors and time. In addition, the vector of the momentum is the product of mass and velocity, $\mathbf{p} = m\mathbf{v}$. Accordingly, the force defined as the product of mass and acceleration is not the same as the force $\mathbf{F} = d\mathbf{p}/dt$ as a change of momentum over time. Below, the first is treated as "own" and the second as "relative".

We use the relativistic relation (1.33) in which the speed does not appear (explicitly). From the second we have $E^2 - p^2c^2 = \text{const}$, where we take derivative:

$$2E\frac{dE}{dt} - 2\mathbf{p}c \cdot \frac{d\mathbf{p}c}{dt} = 0, \quad \frac{dE}{dt} = \frac{\mathbf{p}}{E}\cdot\frac{d\mathbf{p}}{dt}c^2 = \frac{\mathbf{p}}{E}\cdot\mathbf{F}c^2.$$

Include (1.37) and we get:

$$m\mathbf{a} = \mathbf{F} - \frac{\mathbf{p}c}{E}\frac{\mathbf{p}c}{E}\cdot\mathbf{F} = \mathbf{F} - \frac{\mathbf{v}}{c}\left(\frac{\mathbf{v}}{c}\cdot\mathbf{F}\right). \tag{1.38}$$

Forces $\mathbf{F}_0 = m\mathbf{a}$ and $\mathbf{F} = \frac{d}{dt}\mathbf{p}$ are not the same. Each of them may be separated into two components, parallel and perpendicular to the velocity. Finally, from

$$m\mathbf{a} = \left(1 - \frac{v^2}{c^2}\right)\mathbf{F}_{||} + \mathbf{F}_\perp, \tag{1.39}$$

is obtained consent with (1.35). The relative speed, a force component parallel to **v** increases proportionally with γ^2, until the relative component normal to velocity remain unchanged, equal to "own". This is a reverse of the treatment in modern physics.

The force gives to mass acceleration, so from different directions viewed forces in motion can produce significantly different relative realities which cannot be achieved in terms of the Lorentz transformation. Twins paradox is an example that this force can be a single, strong, but there are examples where these forces are mild (and persistent), then again effective in changing the perception of reality.

A typical example (locally) of widespread mild force is a *cosmological repulsive* force, which is still unexplained in physics. Now let's try to understand it using the principles of probability and objective randomness, which is said, would not make sense if nature has no force. We assume that the facts about the expansion of the universe are not yet sufficiently reliable, so we continue to treat the following as the test of previous results.

Among the first for us major cosmological discoveries is the Hubble law which is considered the basis for observation of the universe expansion and today is one of the best supported for the spread model (Big Bang model), shown in the Fig 1.12. Although the discovery of the expansion of the universe mainly attributed to Hubble[18], this phenomenon was predicted earlier, as the contribution to the general theory of relativity by Lemaitre[19] 1927, in which he suggested not only the expansion of the universe, but and estimates of the speed of this expansion, now known as *Hubble constant.*

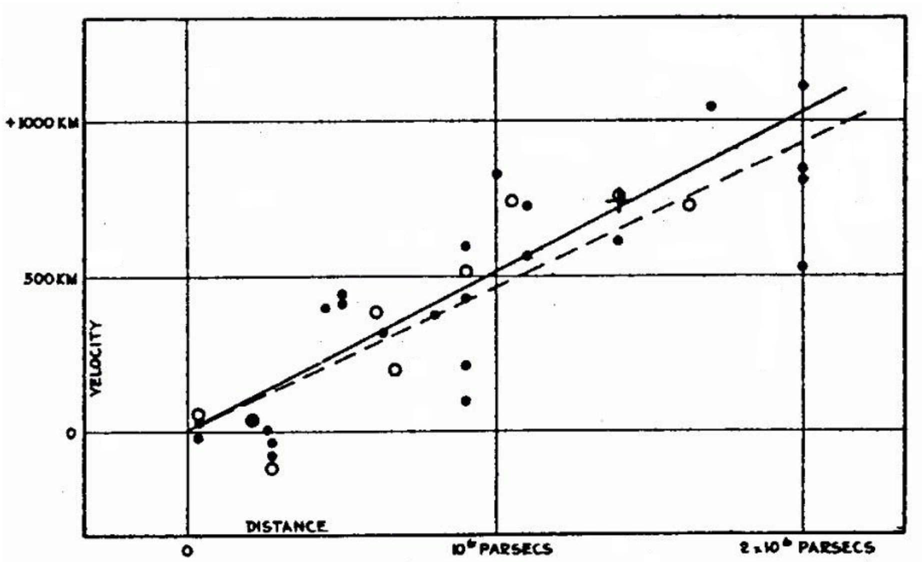


Figure 1.12: Speed with distance, according to Hubble in 1929.

According to Hubble's Law is the average speed of galaxies moving away from us

$$v = H_0\, r, \tag{1.40}$$

where H_0 = 72 km/s/Mpc constant of proportionality (Hubble constant, the image 1.13, r is our "own distance" from the distant galaxies to us (which can be changed over time) in kilometers and v is the speed of removal in km/s. A megaparsec is a million parsecs (mega-

[18]Edwin Hubble (1889-1953), American astronomer.

[19]Georges Lemaître (1894-1966), a Belgian physicist and the priest.

is a prefix meaning million; think of megabyte, or megapixel), and as there are about 3.3 light-years to a parsec, a megaparsec is rather a long way. The standard abbreviation is Mpc. The reciprocal value $1/H_0$ is called *Hubble time*.

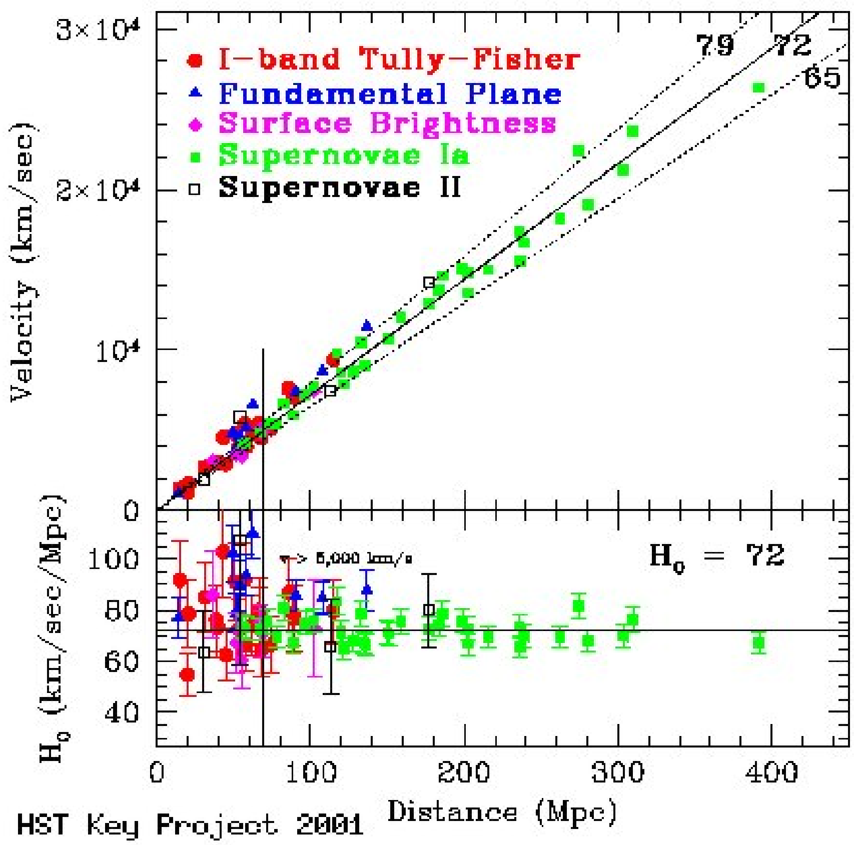


Figure 1.13: Modern estimates of removal of galaxies.

For example, *Andromeda Galaxy* (M31), which is from the dark places visible by the naked eye, is about 0.89 Mpc from us (2.9 million light-years), and it is just one of the nearly 50 known from Local cluster of galaxies located about 2 Mpc from *Milky Way*, our galaxy. Groups consisting of galaxies have typically 1-10 Mpc in diameter, while the super group (eg. Virgo Supercluster) with a diameter of about 100 Mpc.

From the standpoint of our previous considerations, in a homogeneous isotropic universe is logical to assume the existence of some ubiquitous constant force $F_0 \ne 0$, because otherwise we would have had determinism. Note that this is different from the Hubble and Lemaître hypothesis that the universe was created in one bang, and then spread without a functioning of constant repulsive force F_0.

Compare this with the Hubble law, using the formula for relative energy (1.31) and the definition of the force $F_{\|} = \frac{dp}{dt} = \frac{dE}{dct} = \frac{dE}{dr}$. Note that:

$$F_{\|} = \frac{dE}{dr} = E_0 \frac{d}{dr} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{E_0 H_0 v}{c^2 \left(1 - \frac{v^2}{c^2}\right)^{3/2}} = \frac{E_0 H_0^2}{c^2} r_0 \gamma^2, \tag{1.41}$$

where r_0 is own length when viewed from the stars. Now we can write:

$$F_{||} = F_0 r_0 \gamma^2, \quad F_\perp = F_0, \tag{1.42}$$

where $F_0 = E_0 H_0^2/c^2$ is the force produced by the particle with its own energy E_0, which gives the speed of the galaxy, equal to orthogonal force $F_\perp = \text{const}$, as seen from the Earth. This brings us to a constant force ($F_0 = \text{const.}$) of the universe, which at all places is seen as own, not a relative. In the direction of expansion at the site of given galaxy, this force is seen increased proportionately to relativistic factor (γ^2) and the own distance (r_0). The account used that the own distance is proportional to the own time ($t_0 = r_0/c$). These conclusions are still unknown to modern physics.

The small constant of the force (F_0) of the universe with a default objective randomness makes that from the different places observed the same galaxy has a variety of real developments. The first difference even if they start with very small steps, because "butterfly effect" of chaos theory, may eventually escalate into a substantially different time lines.

1.1.9 Compton Effect

Compton[20] have by experiment in 1923 given the most convincing confirmation of the particle nature of radiation. Scattering X-ray free photons, he has found that the wavelength λ' of scattered radiation are greater than the wavelength λ of incident radiation, shown in Figure 1.14. The scientific community has accepted the opinion that this can be explained only by the assumption that X-ray photons behave as particles causing the Compton received the Nobel Prize in Physics in 1927.

An incident photon (γ) comes from the left side in the figure, with the energy $E_\gamma = h\nu$ and momentum $|\mathbf{p}_\gamma| = h\nu/c$. It is in a collision with an electron (e^-) which is at rest (momentum $\mathbf{p}_e = 0$). After the collision, the photon turns to the angle θ and go with the momentum $|\mathbf{p}'_\gamma| = h\nu'/c$, while the electron bounce (in the direction of the arrow) with momentum $\mathbf{p}'_e$.

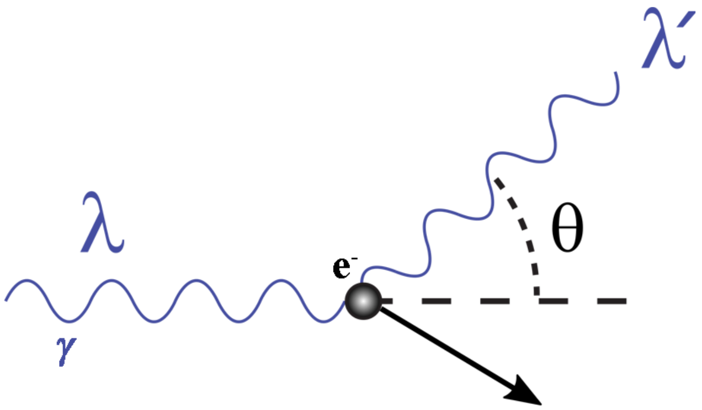


Figure 1.14: Compton Effect.

From the law of conservation of energy and momentum follows, respectively:

$$E_\gamma + E_e = E'_\gamma + E'_e, \quad \mathbf{p}_\gamma + \mathbf{p}_e = \mathbf{p}'_\gamma + \mathbf{p}'_e, \tag{1.43}$$

[20]Arthur Compton (1892-1962), an American physicist.

$$\frac{hc}{\lambda} + m_e c^2 = \frac{hc}{\lambda'} + \sqrt{(p'_e c)^2 + (m_e c^2)^2}, \quad p'^2_e = p^2_\gamma + p'^2_\gamma - 2p_\gamma p'_\gamma \cos\theta, \tag{1.44}$$

$$\begin{cases} (p'_e c)^2 = (\frac{hc}{\lambda})^2 + (\frac{hc}{\lambda'})^2 + (\frac{1}{\lambda} - \frac{1}{\lambda'})2hcm_e c^2 - \frac{2h^2c^2}{\lambda\lambda'}, \\ (p'_e c)^2 = (\frac{hc}{\lambda})^2 + (\frac{hc}{\lambda'})^2 - \frac{2h^2c^2\cos\theta}{\lambda\lambda'}. \end{cases} \tag{1.45}$$

By comparing the two results is obtained

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta), \tag{1.46}$$

and that the change in wavelength of the scattered photons in relation to the incident that is called the *Compton shift*.

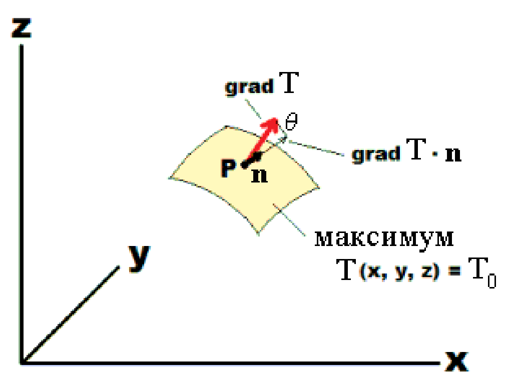
When $\theta = 0$, then $\Delta\lambda = 0$, which means that there is no change in the wavelength of the photon and no change of the photon energy, so there was no a collision of electrons on that path. When the $\theta = 180°$, then the incoming photon is reflected back, change of the wavelength is the maximal and contains the maximum energy that an electron can be gained from such a collision. That was Compton's explanation that was soon confirmed and accepted.

To these, we now add a new explanation based on the principle of probability. According to this principle, the photon has to go on to the most probable direction. Indeed, as the probability of finding a photon in a given place is greater when its wavelength is shorter, so the lesser probable state corresponds to the increase of the wavelength. This change in the trajectory of (relatively) less probable state is accompanied by a collision or the influence of a force because a change of walk along the most likely path is possible only in a non-spontaneous way. This creates the possibility of extending the concept of "gradient" with further understanding of the wave function of quantum mechanics.

Let us imagine that we have a 3-dim space $Oxyz$ and a real function, $T = T(x, y, z)$, such as the temperature in the oven, which is slowly changing from point to point. When looking for the direction of the temperature rise, observers consider all three coordinates and use them to form a vector

$$\text{grad}\, T = \frac{\partial T}{\partial x}\mathbf{i} + \frac{\partial T}{\partial y}\mathbf{j} + \frac{\partial T}{\partial z}\mathbf{k} = \nabla T, \tag{1.47}$$

referred to as *gradient*. Cartesian unit vectors of abscissa, ordinate and the applicate were respectively $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. The gradient has a direction of the fastest growth temperature.



At various points the temperature takes various values, defining a surface $T = T(x, y, z)$ as in the left figure. The gradient of T at a point is a vector pointing in the direction of the steepest slope or grade at that point. A level surface or *isosurface* is the set of all points where some function has a given value. The level surface is forming the points of constant temperature levels and we can refer to as the *geodesics* lines of the temperature. For points with the same level of the temperature difference is zero, however, the increase of temperature along such line was zero, and their gradient would be the zero vector which is perpendicular to any vector, including the one parallel to the level surface.

Dot product of vectors, in general $\mathbf{x} = (x_1, x_2, \dots, x_n)$ and $\mathbf{y} = (y_1, y_2, \dots, y_n)$, equals to the product of their intensity $|\mathbf{x}| = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$, the similar for $|\mathbf{y}|$, and the cosine of the angle $\theta = \angle(\mathbf{x}, \mathbf{y})$ between them, is

$$\mathbf{x} \cdot \mathbf{y} = |\mathbf{x}||\mathbf{y}| \cos\theta. \tag{1.48}$$

Especially in the *orthonormal system* the dot product becomes

$$\mathbf{x} \cdot \mathbf{y} = x_1 y_1 + x_2 y_2 + \dots + x_n y_n. \tag{1.49}$$

Hence and because $\cos\theta \le 1$, we have *Cauchy–Schwarz* inequality

$$x_1 y_1 + x_2 y_2 + \dots + x_k y_k \le \sqrt{x_1^2 + x_2^2 + \dots + x_k^2}\sqrt{y_1^2 + y_2^2 + \dots + y_k^2}. \tag{1.50}$$

which is valid for all $x_k, y_k \in \mathbb{R}$, where equality holds if and only if there is a real number λ such that $x_k = \lambda y_k$ for all $k = 1, 2, \dots, n$.

From the following, we can see that gradient has the direction of the fastest rising temperatures. Let $\mathbf{n} = \mathbf{n}(x, y, z)$ is the unit normal vector perpendicular to the surface formed by the same temperature at a given point. The sum of the square of the normal components n_x, n_y and n_z is one, and θ is an angle between the gradient and the normal. Multiplied in Cartesian coordinates, we get:

$$\operatorname{grad} T \cdot \mathbf{n} = \frac{\partial T}{\partial x} n_x + \frac{\partial T}{\partial y} n_y + \frac{\partial T}{\partial z} n_z = |\operatorname{grad} T| \cos\theta. \tag{1.51}$$

This is the scalar (number) which has the largest value when the angle between the gradient and the normal is zero, $\theta = 0$, since the maximum of the cosine, $\cos 0 = 1$. In the point T_0 is local extreme, all three derivatives are zero, so the gradient and the number (1.51) are zero. Conversely, the number of (1.51) is maximized when and the term of the Cauchy-Schwartz inequality (1.50), and this is when there is one λ in all three of equality $\partial_\xi T = \lambda n_\xi$, for all three coordinates $\xi \in \{x, y, z\}$. Hence $\operatorname{grad} T \| \mathbf{n}$.

Thus we proved that the gradient (1.47) has the direction of the fastest growth temperature fields. The intensity of gradient is the intensity of that growth. The same will happen when word "temperature" is changed by "potential" or any other name for the values assigned to the points of space, if such is gradually (continuously) changing from place to place, forming a smooth level surfaces intensity. In this regard, we are particularly interesting for the Born's probability wave function.

For example, the wave function of the *free particle* is

$$\psi(\mathbf{r}, t) = a e^{i(\mathbf{p}\cdot\mathbf{r} - Et)/\hbar}, \tag{1.52}$$

with constant amplitude a. Here $\mathbf{p} = p_x \mathbf{i} + p_y \mathbf{j} + p_z \mathbf{k}$ and $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ are vectors of the momentum and location of particle in a rectangular Cartesian coordinates with the unit vector of axis $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$, then E and t the energy and time when the particle is at the given site. The gradient of the wave function is

$$\operatorname{grad} \psi = \frac{i}{\hbar} \mathbf{p} \psi. \tag{1.53}$$

When the wave function is normalized, the intensities of the gradient and momentum are equal. Moreover, the *imaginary gradient* has the same direction as momentum, because of the particle grips through the imaginary space holding the maximum probability. The space of quantum states, the particles, is actually a space of complex wavefunctions and the corresponding Born's probability in the way they are seen by these particles.

With a close look at the expression (1.53) we see that the momentum $\mathbf{p}$ can be substituted by the *momentum operator*

$$\hat{\mathbf{p}} = -i\hbar\nabla = -i\hbar\left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right), \tag{1.54}$$

and on the right side of equality to have a gradient of ψ too. This substitution is well known in quantum mechanics, but now we see that it is possible because of the principles of probability, that a particle moves along most likely paths, at least as far as it is its own vision.

It is interesting to note that the momentum and position in physics are often symmetrical and interchangeable. So it is in the quantum mechanics. When a gradient of (1.47) is defined by derivatives of the momentum p_x, p_y and p_z, and not by the positions x, y and z, then the previous analysis remains the same up to (1.53). Then we continue replacing the vector $\mathbf{p}$ by vector $\mathbf{r}$, so the *position operator* is replaced by the momentum operator:

$$\hat{\mathbf{r}} = i\hbar\nabla = i\hbar\left(\frac{\partial}{\partial p_x}\mathbf{i} + \frac{\partial}{\partial p_y}\mathbf{j} + \frac{\partial}{\partial p_z}\mathbf{k}\right) \tag{1.55}$$

Thus, the particle traces the most likely momentum as well as the most likely positions.

1.2 Information

Here we consider Hartley's proposal for information, as a logarithm of equal opportunities, then Shannon's as the mean value of Hartley's logarithms, and a new one. The new is a generalized definition of information, which can be reduced to Hartley and Shannon's definition, and one day will be called the "amount of uncertainty", but for now it is irrelevant because the distinction between uncertainty and information we still do not notice.

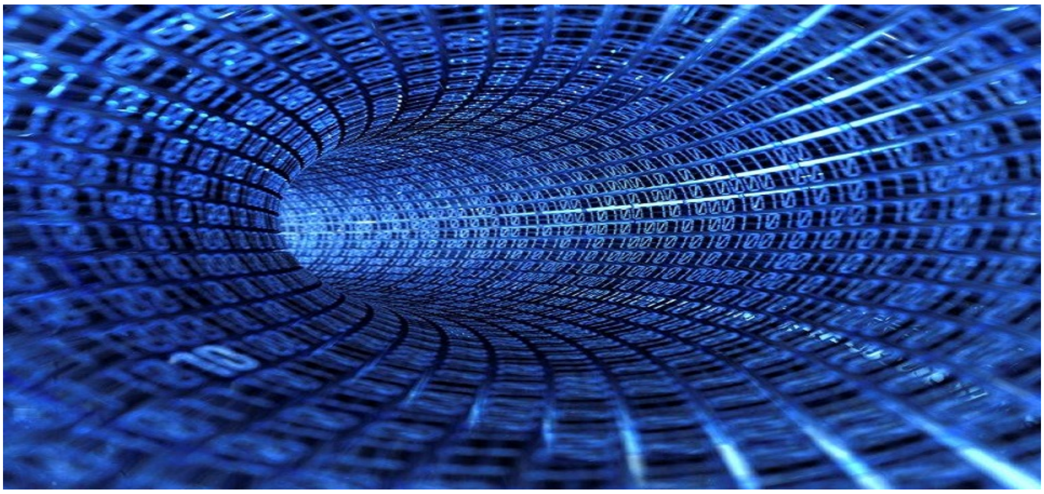


Figure 1.15: Information.

The ultimate goal of this section is to understand the substance using the information to explain the mechanics, dynamics and gravity, today the hardest strongholds for determinism. These are the places where we least expect the accidents as causes of the theory and where one do not hope to discover an information as a physical reality.

1.2.1 Hartley's definition

The information as a number firstly successfully defined by Nykvist[21] and Hartley[22] 1928. They found that the information as the outcome of equally likely options is proportional to the logarithm of the number of the options. Working for the Bell phone company, they developed the idea of information as a "quantity of news" inspired by journalism. The higher is news that "man has bitten a dog" than news "dog has bitten a man" because the first is less likely, but they did not naively run to define information simply as the number of options.

Information can be defined as the smallest number of questions (number $H = 1, 2, 3, \dots$) required by *binary searching* to find one from (2^H) the given answers. Binary process divides set of possible answers into two parts. When you find out in which of the two subsets is the looking answer, that subset is divided into two parts and continue the search. The number of searched elements $N = 2^H$ is an exponential function of the number of divisions H, which we call information. Thus, the information is inverse of exponential function, so it is the

[21] Harry Theodor Nyqvist (1889-1976), American engineer of Swedish origin.
[22] Ralph Vinton Lyon Hartley (1888-1970), an American engineer.

logarithm of possibilities, that is the negative logarithm of the probability of the outcome

$$H = \log_2 N, \quad H = -\log_2 \frac{1}{N}. \tag{1.56}$$

Number H we call *Hartley information*, or information that contains N equally likely possibilities, and the number $P = \frac{1}{N}$ probability of outcomes. It turns out that this choice was a big hit.

For example, for detection a supposed (number five) of the first eight positive integers ($N = 8$) requires three binary questions ($H = 3$). The first question "whether the supposed number is less than five", we expect a response "yes" or "no", the answer is "no". On the second question "whether the supposed number is less than seven", the answer is "yes". The third question is "whether the supposed number is less than six", the answer is "yes". Thus we find that the required number is five by *binary search*.

When the options are multiplied then the information is added, since the logarithm of a product equals the sum of logarithms. It is the main property of the Hartley information that has connected it with the probability and gave her wings in use. Let's look it at the case of probability. When you flip a fair coin with two options, the probability of the outcome of one is $\frac{1}{2}$. When you flip a fair dice with six options, the probability of the outcome of one is $\frac{1}{6}$. When you flip the both, a coin and a dice, the number of equally probable pairs is twelve (2×6) with the probability $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$. The information of one of the twelve pairs is the sum of information by coin toss and information of throwing dice.

With this unexpected logarithm additivity, Hartley information from the abstract idea becomes a physical, tangible thing. Imagine that from a set of m_1 numbers we randomly draw one, and then again from the set m_2 numbers are drawn one again, and so on until the final set with m_n numbers. Information of all n draws will be equal to the sum of individual information, no matter how fast or in which order we carry out the draws. This follows generally from the properties of logarithmic functions, which gives us the idea that the base of the logarithm (number 2) of the formula can be changed. Changing the base logarithm of Hartley information simply means other units of measure. When the base of the logarithm is 2 the unit information is "bit", when the base is Euler's number $e \approx 2.71828$ the unit is "nat" (natural logarithm).

In the following we use the *natural logarithm* $\ln P = \log_e P$, so

$$H = -\ln P, \tag{1.57}$$

is Hartley information given by the probability $P \in (0,1)$ of the $N \approx 1/p$ equally random events. The first and for us the most interesting events are those with the greatest probabilities, close to one, when

$$P = 1 - x, \quad x \to 0, \tag{1.58}$$

or when $x > 0$ is a small number whose square and higher degrees can be ignored. Development of the logarithmic functions in Maclaurin series gives

$$\ln(1-x) = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 - \dots, \tag{1.59}$$

the value (1.57) then becomes

$$H \approx x. \tag{1.60}$$

As the probability is higher the information is lesser.

In addition to its logarithmic additivity, Hartley information, in general, has mentioned journalistic trait that the more certain event is less informative. The lower the number (equal) opportunities are, the more likely single outcome is, but is less its information. The principle of carrying out the most likely probability becomes the principle of stinginess in giving Hartley information of what here is called *information principle*. Also, more frequent realization of a more certain event means less frequent realization of uncertain.

This feature remains in a sense and in the set $\Omega = \{\omega_1, \omega_2, \dots, \omega_N\}$ random events with unequal probabilities $P_1, P_2, \dots, P_n$. For example, a set Ω approximate by set Ω' also with $N = 2, 3, \dots$ equally likely event, the probability

$$P_A = \frac{1}{N}(P_1 + P_2 + \dots + P_N) = \frac{1}{N}. \tag{1.61}$$

Number P_A is called *arithmetic mean*, average of numbers P_k. *Geometric mean* would be

$$P_G = \sqrt[N]{P_1 P_2 \dots P_N}. \tag{1.62}$$

Each of them, P_A and P_G, is a kind of average. These are two of the numbers between minimum and maximum in the given series, in which holds $P_A \ge P_G$, or

$$\frac{1}{N}(P_1 + P_2 + \dots + P_N) \ge \sqrt[N]{P_1 P_2 \dots P_N}, \tag{1.63}$$

where equality holds iff[23] $P_1 = P_2 = \dots = P_n$. Taking the logarithm, we get

$$-\ln P_A \le \frac{1}{N}(-\log_2 P_1 - \log_2 P_2 - \dots - \log_2 P_N). \tag{1.64}$$

In particular, when the aforementioned set of Ω makes the whole set of mutually independent outcomes (one of them has to happen), it will be

$$P_1 + P_2 + \dots + P_N = 1. \tag{1.65}$$

Then the probabilities (P_k) with $k = 1, 2, \dots, N$ is called *probability distribution*, and the arithmetic mean becomes

$$P_A = \frac{1}{N}. \tag{1.66}$$

Either way, the mean value is a replacement for Hartley set equal probability. In both cases, the average event information is not greater than the average of all information.

Whenever a physical phenomenon is approximated, we get even less information. This is intuitively understandable. Averaging lose some details. Further, due to the efforts that emit as little information and (1.64), is expected to ease nature escape into averaging to reduce the losses of its uncertainty. Namely, the Hartley formula (1.56) shows that the transformed amount of uncertainty resulting into the equal amount of the information. Realizing the uncertainty, nature produces exactly the same amount of information. Let us call this regularity *Hartley Conservation Law* of information plus the uncertainty. Similarly we have with attractors of chaos theory. Fleeing into the routine nature is trying to preserve the uncertainty and reduce the emission information.

If there is no Pauli *exclusion principle*[24] (two identical fermions cannot be in the same quantum state), who 1925 was formulated by Pauli[25], the escaping nature into ease of

[23]iff - if and only if.

[24]Pauli exclusion principle: `https://en.wikipedia.org/wiki/Pauli_exclusion_principle`

[25]Wolfgang Pauli (1900-1958), Austrian-Swiss American theoretical physicist.

averaging could go all the way. All quantum systems would be exactly the same so that it could be considered that there is only one of them. Again, in true high homogeneity and isotropy, there would be no room for a disorder, or of force and acceleration, why the uncertainty itself would become unnecessary. On the other hand, the macro-world has less need for such escape, because it has the law of large numbers, which certainly limits the emissions of information.

The random events $A, B \in \Omega$ we call the *independent events* if the probability of one of them does not affect the probability of the other. In particular, the two events are *statistically independent* iff[26] it is

$$P(A \cap B) = P(A)P(B). \tag{1.67}$$

As the logarithm of a product equals the sum of logarithms, so the Hartley information of statistically independent events is equal to the sum of information of the events.

Hartley information is generalized to the cases of *conditional probability.* For two given events, the probability of the first under the condition that the second certainly occurred is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}. \tag{1.68}$$

This number is as greater as event A more depends on B (constant B), so the conditional event information is

$$-\ln P(A|B) = -\ln P(A \cap B) + \ln P(B), \tag{1.69}$$

which is smaller when A is more dependent on B.

Accordingly, the Hartley information of conditional probability follows our expectations regarding the quantum entanglement[27]. Note here another confirmation of the principle of information that uncertainty does not realize when it is not needed. This fits well with the principle of least action[28] which we'll treat later similarly.

In physics, the "least action" is variation principle which applied to the *action* (the product of momentum and length or energy and time) gives the equation of motion of a given system. The different action is either minimized or maximized. Example: "light reflects so that it consumes the least time and reaches as quickly as possible", we consistently integrate: "to produce minimum information".

Example 1.2.1 (Independent events)**.** *If the pair A, B is statistically independent, then next pairs are also independent:*

$$A, B' \qquad A', B \qquad A', B'$$

where prime means negation, that is complement event.

Solution. For the first pair we have:

$$P(A \cap B') = P(A)P(B'|A) = P(A)[1 - P(B|A)] = P(A)[1 - P(B)] = P(A)P(B').$$

Similarly we prove the others. □

---

[26]iff - if and only if

[27]We expect that entangled quantum events are dependent and happened (to have real consequences) with reduced emissions of information.

[28]Principle of least action: `https://en.wikipedia.org/wiki/Principle_of_least_action`

1.2.2 Born information

Hartley information directly applied to the Bourn probability gives:

$$H = \ln|\psi|^2 = \ln(\psi^*\psi) = \ln\psi^* + \ln\psi, \tag{1.70}$$

which means that we can define the *complex function* $L \in \mathbb{C}$ of the wave function ψ in quantum mechanics, by

$$L = \operatorname{Ln}\psi = \ln|\psi| + i\operatorname{Arg}\psi, \tag{1.71}$$

where $|\psi| = \sqrt{\psi^*\psi}$ is *modulus* of the complex number ψ, and i is *imaginary unit*, $\operatorname{Arg}\psi$ angle in radians or argument that the complex number ψ is forming with the real axis in the complex plane. Number L we named a wavelength or *complex information* of Hartley. This logarithm is inverse of the exponential function, so

$$\psi = e^L \tag{1.72}$$

is *complex probability*. The product of complex conjugate probabilities is the real number called Born probability, as we know.

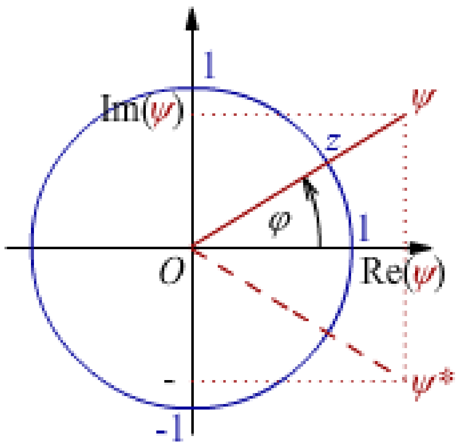


Figure 1.16: Complex plane.

On the figure 1.16 you can see complex number $\psi = x + iy$ in the complex plane, whose the real and imaginary projections on the abscissa and the ordinate respectively are $\Re(\psi)$ and $\Im(\psi)$. The unit circle represents the points $z = x+iy$ with coordinates (x, y) that satisfy equation $x^2 + y^2 = 1$, and there is always the angle φ that

$$z = \cos\varphi + i\sin\varphi, \quad |z| = 1. \tag{1.73}$$

For two such numbers z_1 and z_2 with angles φ_1 and φ_2 the product z_1z_2 is again the complex number on the unit circle, with the angle $\varphi_1 + \varphi_2$. This is easy to prove by multiplying complex number z_1 by z_2 and using addition formula for the cosine and sine of the sum. When cosine and sine functions are developed in Maclaurin series, their sum z is the development of exponential function and is

$$e^{i\varphi} = \cos\varphi + i\sin\varphi = \operatorname{cis}(\varphi). \tag{1.74}$$

This is the Euler equality[29]. Therefore, any complex number can be written as

$$\psi = |\psi|(\cos\varphi + i\sin\varphi), \tag{1.75}$$

where $|\psi|$ is mode (the length from the origin O to the number ψ), and $\varphi = \operatorname{Arg}\psi$ is the argument (the angle between the abscissa axis and the length).

For the *conjugated* complex numbers ψ^* and ψ the abscissa is the axis of symmetry, so they have the same real values but the imaginary with the opposite signs. Thus, the

$$\psi^*\psi = [\Re(\psi) - i\Im(\psi)][\Re(\psi) + i\Im(\psi)] = \Re^2(\psi) + \Im^2(\psi), \tag{1.76}$$

is the square of the length from O to the point ψ, of the hypotenuse of a right triangle with vertices O, $\mathrm{Re}(\psi)$ and ψ on picture 1.16.

In this figure is seen that the angle $\varphi = \operatorname{Arg}\psi$ remains substantially the same if we add or subtract it arbitrarily many full angles $2k\pi$ ($k = 0, \pm1, \pm2\dots$). Basic angle of these angles are usually denoted by $\arg\psi \in (-\pi, \pi]$. Thus, the number L is periodic with fundamental period 2π. However, even if you only use the basic argument and write

$$L = \ln|\psi| + i\arg\psi, \quad \arg\psi \in (-\pi, \pi], \tag{1.77}$$

complex exponential function (1.72) is the *periodic function*, due to

$$e^{L+2i\pi} = e^L, \tag{1.78}$$

so the equation (1.72) has an infinite number of solutions in the domain of complex $L \in \mathbb{C}$. It is used to be the cause of paradoxes in mathematics, such as the following.

Example 1.2.2 (Paradox of Bernoulli and Leibniz)**.** *There are a series of false deductions that supposedly proves that* $\arctan(1) = 0$*, besides* $\arctan(1) = \frac{\pi}{4}$.

Solution. The mentioned "deductions" are:

$$\arctan(x) = \int_0^x \frac{d\xi}{\xi^2 + 1} = \int_0^x \frac{1}{2i}\left(\frac{1}{\xi - i} - \frac{1}{\xi + i}\right) d\xi = \frac{1}{2i}\ln\frac{x - i}{x + i},$$

so for $x = 1$ we get:

$$\arctan(1) = \frac{1}{2i}\ln\frac{1-i}{1+i} = \frac{1}{4i}\ln\left(\frac{1-i}{1+i}\right)^2 = \frac{1}{4i}\ln(-1) = \frac{1}{8i}\ln(-1)^2 = \frac{1}{8i}\ln 1 = 0.$$

However, $\arctan(1) = \frac{\pi}{4}$.

The paradox is caused by not taking into account the ambiguity of the logarithm of a complex number, because $\operatorname{Ln} z = \ln|z| + i(\arg z + 2k\pi)$ for arbitrary $k \in \mathbb{Z}$. For example, for $k = 1$ it is $\operatorname{Ln}(1) = 2i\pi$ so we have:

$$\arctan(1) = \frac{1}{8i}\operatorname{Ln}(1) = \frac{1}{8i}2i\pi = \frac{\pi}{4},$$

and there is no paradox. □

Because the information L is a complex number, the complex probability (1.72) is periodical. We have mentioned that the physical phenomena that have no *correct real values* (as zeroes of a polynomial without real roots) we cannot represent by accurate real numbers. Now we see further, that the inverse function of such is necessarily periodic. That is why the wave function of quantum mechanics is periodic.

[29]Function $\operatorname{cis}(\varphi) = e^{i\varphi}$ is used recently.

1.2.3 Waves of matter

Waves of matter are the central part of quantum mechanics. They are derived from the properties of quantum systems that they are both waves and particles. Small pieces of matter have wave and quantum properties, which is referred to as the dualism of *wave-particle*. For example, the electrons exhibit the properties *diffraction*, grouping its energy into the pattern of weaker and stronger concentric circles, when there is an obstacle or must be passed through the opening. The waves of electrons are actually probability waves that create new and new positions of electrons (which parts as the mass, charge or spin are not still divisible) while time passes, adhering to the principle that once implemented particles at a given moment will be again the most likely in the same circumstances and in the next moment, until a force does not disrupt this probability. As attractors, waves of probability can be approximated by a sinusoid.

Picture 1.17 is shown a simple wave, represented by sinusoid $y = a \sin bx$. Wave oscillates about the abscissa (x-axis) and form the maximal deviation in the ordinate direction (y-axis) called *amplitude* (a) that are constant (a, b = const.) and periodic. The distance between two adjacent amplitude is called the *wavelength* ($\lambda = 2\pi/b$) or *base period* of the wave.

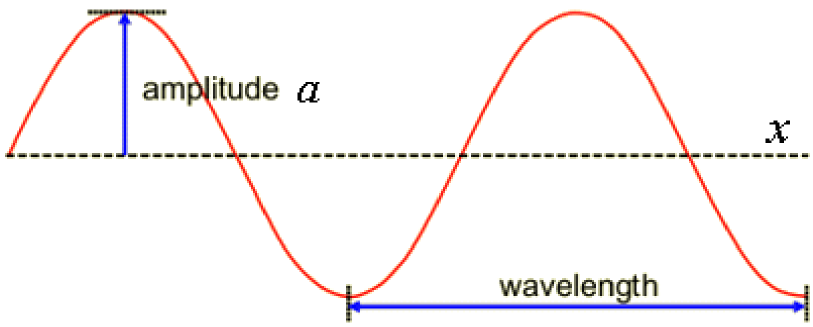


Figure 1.17: Simple wave.

Because the parameters a and b of the sinusoid are constants, the wave seems to be even as shown. If the parameters vary along the abscissa, $a = a(x)$ and $b = b(x)$, sinusoids is *stationary* but it is not necessarily uniform. In general, the stationary state we call that which does not change over time. When the parameters of the wave change over time, but the amplitude oscillate from the upper to the lower ordinate direction without changing the place on the x-axis, then we have *standing waves*. A fixed point on the axis of such waves is called *wave node*. Finally, a bunch of waves in one place that can move (not necessarily) is called the *wave package*.

A phase wave is the position of the point on the wave at a given time during its periodic movement of time. For peak amplitude of standing wave is said to be moved parallel to the ordinate (up and down), while the waves in motion move parallel to the x-axis (left-right). For example, when the sine wave, as on the given picture, move for length $-\alpha$ parallel to the x-axis, then we get a new sine wave:

$$y \to y_1 = a \sin[(x + \alpha)b] = a \sin(xb + \varphi), \tag{1.79}$$

and we say that the phase shift is $\varphi = \alpha b$. The phase shift can be varied over time and then we write $\varphi = \varphi(t)$, when the phase velocity is $\dot{\varphi} = d\varphi/dt$. This is the speed of wave propagation (along the abscissa in Figure 1.17).

In the fluid dynamics in general, the dispersion (scattering) of water waves is relating to the scattering of frequency, which means that the waves of the different wavelengths travel by different phase velocities. Water with its free surface in this sense belongs to the dispersion medium, and the water waves that spread over the water surface are driven by surface tension and gravity. The importance of gravity for moving water waves now, after preliminary examination, means that the deeper causes of the wave motion of water should be sought in the principle of probability and consequences.

In transversal (orthogonal) waves like water, the ordinate value literally means the amount of deflection wave from the direction of its propagation along the abscissa. At longitudinal waves such as sound, the value of ordinate is the amount of compression of the media along the direction of movement of the waves. Water waves deflections are called *crest* and *through*, in sounds waves so-called *compression* and *rarefaction*. The compression is the area of higher pressure; the rarefaction is areas of low. Analog to gravity, we'll show that the pressure has something with probability too.

Undercover effects of uncertainty macro-world are more pronounced in the micro-world. Embodiments of random events occurring multitude of information that formed "now" makes our present. Present then becomes our past. This is a short description of the transformation of uncertainty, which is eventually deposited in the (mostly) stable the past because of the law of conservation of information. Our past is just as much a variable as much uncertainty kept its individual events. On the other hand, what is realized in the form of information is as immutable as we can trust the information received in the corresponding experiment.

Intuitively, it is clear that the information will not be changed by moving the quantum system along the axes (including time) if you will not change the probability. The analytical form of the statements we have in the following example.

Example 1.2.3 (Conservation uncertainty)**.** *Show that for $\xi \in \{x, y, z, ct\}$ equation*

$$\frac{\partial\psi^*}{\partial\xi}\psi + \psi^*\frac{\partial\psi}{\partial\xi} = 0, \tag{1.80}$$

means conservation of uncertainty of state $\psi = \psi(\mathbf{r}, t)$ by changing coordinate ξ.

Solution. The amount of uncertainty will not change if and only if the probability along ξ will not change. A copy of the Born probability is:

$$\frac{\partial}{\partial\xi}|\psi|^2 = \frac{\partial}{\partial\xi}(\psi^*\psi) = \frac{\partial\psi^*}{\partial\xi}\psi + \psi^*\frac{\partial\psi}{\partial\xi}$$

however, due to

$$\frac{\partial}{\partial\xi}|\psi|^2 = 0$$

we obtain the required equality. □

Unchanged uncertainty means non-realization of a random event and not creating the new information of the closed quantum system. Because of the aforementioned law of conservation of total uncertainty and information, such a system should maintain unchanged the information too, whenever uncertainty kept unchanged. Indeed, from (1.70) by derivation we get:

$$\frac{\partial}{\partial\xi}H = \frac{\partial}{\partial\xi}\ln|\psi|^2 = \frac{1}{|\psi|^2}\frac{\partial}{\partial\xi}|\psi|^2 = \frac{\partial\psi^*}{\partial\xi}\psi + \psi^*\frac{\partial\psi}{\partial\xi},$$

and from $\partial_\xi H = 0$ followed by (1.80). Vice versa, from (1.80) by replacing (1.72) we get:

$$0 = \frac{\partial \psi^*}{\partial \xi}\psi + \psi^* \frac{\partial \psi}{\partial \xi} = e^{L^*} e^L \frac{\partial L^*}{\partial \xi} + e^{L^*} e^L \frac{\partial L}{\partial \xi} = e^{L^*+L} \frac{\partial}{\partial \xi}(L^* + L),$$

and hence $\partial_\xi H = 0$.

Let's show in a similar way what we intuitively announced with inertial motion and force. First of all, that the probability and the Hartley information are constants if there is no force. For example, the Dirac[30] equation reduced to the abscissa

$$\left[\beta mc^2 + c\left(\sum_{n=1}^{3} \alpha_n p_n\right)\right]\psi(x,t) = i\hbar \frac{\partial \psi(x,t)}{\partial t}, \tag{1.81}$$

is invariant to the Lorentz transformation. The following example shows that solutions of this equation satisfy the equality (1.80).

Example 1.2.4. *Show that any solution of Dirac equation conserves the information.*

Solution. From (1.81) follows, row:

$$\left[\beta mc^2 + c\left(\sum_{n=1}^{3} \alpha_n p_n\right)\right]\psi\psi^* = i\hbar \frac{\partial \psi}{\partial t}\psi^*,$$

$$-\left[\beta mc^2 + c\left(\sum_{n=1}^{3} \alpha_n p_n\right)\right]\psi^*\psi = i\hbar \frac{\partial \psi^*}{\partial t}\psi,$$

and by adding

$$0 = i\hbar\left(\frac{\partial \psi}{\partial t}\psi^* + \frac{\partial \psi^*}{\partial t}\psi\right),$$

from where the required (1.80). □

Solutions that do not include the Dirac equations are located in a Schrödinger[31] equation, which also may be written as in the abscissa and the time

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + U(x,t)\psi = i\hbar \frac{\partial \psi}{\partial t}, \tag{1.82}$$

wherein a wave function $\psi = \psi(x,t)$. The first addend is the kinetic energy, potential energy is the second, and on the right side of the equality is the total energy. The mass of particles is m, potential is $U(x,t)$ and i is the imaginary unit. Therefore, such a written Schrödinger equation expresses the law of conservation of energy.

A simpler solution to this equation we get when there is no changing the state by time $\psi = \psi(x)$ and when the potential is the function only of abscissa $U = U(x)$. In the simplest case, for the *free particle-wave*, we get the solution:

$$\psi(x) = ae^{ikx}, \quad \frac{\hbar^2}{2m}k^2 = E, \quad p = \hbar k. \tag{1.83}$$

where a is real constant, p momentum of particle, k (real) wavenumber, E energy of particle, $\hbar = h/2\pi$, $h = 6.626 \times 10^{-34}$ Js is Planck constant. This is a stationary state. That the

[30] Paul Dirac (1902-1984), English mathematician.
[31] Schrödinger (1887-1961), Austrian physicist.

information conservation law (1.80) is applied for a free particle follows from the following calculation:

$$\psi^*(x) = ae^{-ikx}, \quad \psi(x) = ae^{ikx},$$

$$\frac{\partial\psi^*}{\partial x} = -ik\psi^*, \quad \frac{\partial\psi}{\partial x} = ik\psi,$$

$$\frac{\partial\psi^*}{\partial x}\psi = -ik\psi^*\psi, \quad \psi^*\frac{\partial\psi}{\partial x} = ik\psi^*\psi,$$

$$\psi^*\frac{\partial\psi}{\partial x} + \frac{\partial\psi^*}{\partial x}\psi = 0,$$

Therefore, the equality (1.80) is true. However, information is not the same as energy, so we can expect that some of the solutions of the Schrödinger equation (1.82) do not meet the requirement of conserving information.

Indeed, for a particle in the excited state, when $a = a(x)$ is no longer constant, by similar process we found

$$\frac{d\psi^*}{dx}\psi + \psi^*\frac{d\psi}{dx} = 2a\frac{da}{dx} \ne 0, \tag{1.84}$$

when derivative $\frac{da(x)}{dx} \ne 0$. Compared with the previous, now we can see that these particles are in no inertial systems.

So when generalized Hartley definition (1.70) and the wave function is written in the form (1.72), then L is generalized information and complex number. From the conservation of probability (1.80) followed $\partial_\xi(L^* + L) = 0$, which means the preservation of the real part of this information, while the imaginary part makes a function $\psi = \ln L$ periodic. However, the force disrupts probabilities and change information, which confirms (1.84).

A free particle (1.83) is in the *stationary state*, because on it does not act the forces, and therefore its energy does not change over time. The free particles also represent the probability waves, normalized so that the total area under the graph of the square norms of the wave function and to x-axis is equal to one. Therefore, it is important from where it started (say event D_1) and how far has arrived (event D_2), on the interval "known" where it is located. And it has its strange consequences which I have already described.

Let this uniform "free" particle a photon that travels from the interaction D_1 to D_2. Due to objective uncertainty (whatever we have organized something it can always fail) photon after leaving the event D_1 cannot be quite sure when to meet the event D_2. If the event D_2 never happen, its probability amplitude of the wave function will be so stretched to the fact that the photon never existed, that the event D_1 in its history has never occurred. On the contrary, if the photon meets with the event D_2, its history must be adjusted so that the event D_1 is realistic. If these two events are real than would be respected (say) the law of conservation of momentum, otherwise, it would not.

Therefore, we consider the theory in which the *quantum entanglement* means. We believe not only in the "phantom actions on distance" which opposed Einstein, but also in changing the past by future. The Past, together with space and matter of the universe, is a dump of information and it is a permanent thing just as much as the law of conservation information applies. On the other hand, uncertainty is what didn't become information, of what our universe has significantly more than all until now created time, space and matter.

1.2.4 Shannon's definition

Working for the same company and developing the same Harley ideas, Shannon[32] in 1948 defined the information for set events of different probabilities. The basis is taken in set that define *probability distribution*. In the discrete case, when we have not more than the countable set of independent random events from Ω of which certainly would realized one, say $\omega_k \in \Omega$ with probability $P_k = \Pr(\omega_k) \ge 0$. The sum of all probability from the distribution is one. Let event ω_k orderly for $k = 1, 2, 3, \dots$ carries Hartley information $H_k = -\ln P_k$. Average value of all, *mathematical expectation*, is

$$S = -\sum_k P_k \ln P_k, \quad \sum_k P_k = 1, \tag{1.85}$$

where is added for all indices k. This is the *Shannon's information* in a discrete case. The term comes down to Hartley (1.57) when all the outcomes have equal probability. Analog defines Shannon's information in the case of the continuum, using the probability density $\rho \ge 0$, which depends on the part of $\omega \subseteq \Omega$ of a continuum Ω random events, by integral:

$$S = -\int_\Omega \rho \ln \rho \, d\omega, \quad \int_\Omega \rho \, d\omega = 1, \tag{1.86}$$

For long these definition has been considered the only and then the best generalization of Hartley information.

Shannon's information retained the journalistic trait of news that is less informative what is more likely and vice versa. This can be seen in the example of sample texts. Serbian alphabet has $n_1 = 30$ letters, England $n_2 = 26$ and we know that the various characters in the text of different languages occur with different probabilities. The average occurrence of letters is n-the part of all, although the frequency of each letter is mainly different from the average $1/n$ of the number of letters in the alphabet. However, when the $n_1 > n_2$, the probability of average occurrence of letters in the first text will be less than in the second, but the Shannon information would be higher. Using the alphabet with more letters it is possible to write more different words with the same given length.

Another example of economizing with Shannon's information we discover in the well-known *Ergodic Theorem* for Markov chains. In short, when the information transmitted by connection of channels, one used many times, due to noise (jam) that are inevitable in practice, at the end of the transfer has received a message whose code is not dependent so of sending messages as of the nature of the channel. It looks like nature hides a more complex (more informative) message by simpler, using noise. That we had and in the children's game of "deaf telephones", where every child in the row to the next on its ear quietly convey the message of the previous one. When all the children were similar, and if it was enough of them in the game, then independently of the initial message the final can be, for example, "mammy". Information is dissipated and not renewed. It does not arise from nothing.

Shannon's information also has the feature of conservation only in inertial systems. Those look at the case of discrete quantum information of Born probabilities

$$S = -\sum_k |\psi_k|^2 \ln |\psi_k|^2, \quad \sum_k |\psi_k|^2 = 1. \tag{1.87}$$

Taking a derivative of the coordinates along $\xi = x, y, z, ct$ we get

$$\partial_\xi S = -\sum_k (1 + \ln |\psi_k|^2) \partial_\xi |\psi_k|^2, \tag{1.88}$$

[32]Claude Elwood Shannon (1916-2001), American mathematician.

which means that the Shannon information is constant ($\partial_\xi S = 0$) where the distribution is constant, ie. when along the coordinates (ξ) does not alter the individual probabilities ($|\psi_k|^2$).

In case random events of the continuum Ω, Shannon's information is

$$S = -\int_\Omega |\psi|^2 \ln |\psi|^2 \, d\omega, \quad \int_\Omega |\psi|^2 \, d\omega = 1, \tag{1.89}$$

where $\omega \subseteq \Omega$ are subsets. Wave function

$$\psi(x,t) = \phi(x) e^{-iEt/\hbar} \tag{1.90}$$

defines *stationary* quantum state. It is easy to check:

$$|\psi(x,t)|^2 = \psi^*(x,t)\psi(x,t) = \phi^*(x)\phi(x). \tag{1.91}$$

Dependence of time has disappeared. The spatial part of the wave function satisfies the *time independent* Schrödinger equation for x-axes

$$-\frac{\hbar^2}{2m}\frac{d^2\phi(x)}{dx^2} + U(x)\phi(x) = E\phi(x), \tag{1.92}$$

where $U(x)$ is potential energy, E is the energy of the system. This is a differential equation of the second order with one unknown, not simple, but pretty thoroughly explored in math.

Recall that in specific situations in quantum mechanics the solutions of the equation (1.92) are well known. General solutions are of the form

$$\phi(x) = Ce^{\pm k(x-x_0) \pm i f(x)}, \tag{1.93}$$

where C, k, x_0 are constants and $f(x)$ function, which depend on the considered case. For example, note that in the calculation of the probability the imaginary part disappears, so we can put $f(x) \equiv 0$, then choose the start point $x_0 = 0$. If the constant k is positive, in front leave minus so $\phi(x)$ converge when $x \to \infty$. The constant C determine the norms on the right half-axis. From $\int_0^\infty |\phi|^2 dx = 1$ follows $C = \sqrt{2k}$, and is

$$\phi(x) = \sqrt{2k}\, e^{-kx}, \quad x \ge 0. \tag{1.94}$$

Substitute ϕ in (1.92) and find

$$k = \frac{1}{\hbar}\sqrt{2m(U-E)} \ge 0. \tag{1.95}$$

For this solution, Shannon's information is:

$$S = -\int_0^\infty |\phi|^2 \ln |\phi|^2 dx = 1 - 2k \ge 0, \tag{1.96}$$

and hence $0 \le U - E \le \frac{\hbar^2 k^2}{2m}$. Similarly, we are working with other solutions (1.93).

The example with the scoring at the end shows one simple use of the Shannon information in the analysis of the quantum system. But despite these seemingly light possibilities, it appears that physicists do not use it enough. This is perhaps due to its unreliability, which is illustrated in the following example.

Example 1.2.5. *Indicate the example of a well-defined distribution but with Shannon's information that diverges.*

Solution. It is known that the first series diverges and the second converges:

$$A: \sum_{k=2}^{\infty} \frac{1}{n \ln n} = \infty, \quad B: \sum_{k=1}^{\infty} \frac{1}{n \ln^2 n} = b \in \mathbb{R}^+.$$

The coefficients of this second divided by b constitute a well-defined distribution of probability. However, Shannon's information such distributions diverge. Really:

$$S = -\sum_{k=1}^{\infty} \frac{1}{bn \ln^2 n} \ln \frac{1}{bn \ln^2 n} = \sum_{k=2}^{\infty} \frac{\ln b + \ln n + \ln \ln^2 n}{bn \ln^2 n} =$$

$$= \sum_{k=2}^{\infty} \frac{\ln b}{bn \ln^2 n} + \sum_{k=2}^{\infty} \frac{\ln n}{bn \ln^2 n} + \sum_{k=2}^{\infty} \frac{\ln \ln^2 n}{bn \ln^2 n} \to \infty,$$

because they middle sum diverges together with A. □

Here we'll not quite believe in *reliability* of analysis using Shannon 's information, fearing of good distribution for which it can give such poor results. Another problem with this definition of information comes from its limitation on the probability distribution. It is typical for a process that from the given set of random outcomes generates only one, or one by one like the chess game. The sum of these probabilities is one ($\sum P_k = 1$) because the outcome of a given set is a certain event. But nature does not work that way only.

1.2.5 Dot product

Nature is often *multiprocessing.* For example, various parts of the human body are triggered by different processes that control the functions of our integral organism in a way that we feel independent. They run the heart and blood vessels; liver, digestive, they move our arms and legs, vision, speech and many others. We believe that several thousand independent processes working in our body all the time.

About multiprocessing living beings I wrote the book "Information perception" (see: [2] in Serbian). Simply, this book can be considered as the check of only one formula that links freedom, intelligence, and hierarchy:

$$\ell = \mathbf{i} \cdot \mathbf{h}. \tag{1.97}$$

As in this book, there also implies the existence of different options. A "living being" is defined by its ability to make decisions. The number ℓ is *freedom* (liberty) of the individual, a number of possibilities that an individual has thanks to its senses and perceptions in general. Vector $\mathbf{i}$ is individual *intelligence* defined as the ability of using its possibilities. The vector $\mathbf{h}$ is *hierarchy*, defined as the ability of the environment to deprive individuals of their capabilities. Accordingly, the freedom of the living being is *dot product* or inner product of the vectors, individual ability with surrounding limitations.

The book is considered an individual person who lives in the social system and other circumstances. Also, it is an ant in the ant colony or a blade of grass in the lawn. Formula (1.97) is equally true for a liver cell which is individual in its environment the surrounding body of a living being. On the other hand, it also cannot be proved by a deduction from some established, and is in this respect similar to the kinematics formula that the product of speed and time is distance traveled ($vt = s$), in the time before Galileo[33].

The aforementioned book has suggested but didn't insist that freedom in expression (1.97) can be extended to inanimate things. Such expansion will not lead us formally in contradiction simply because the term is the mathematical proven and well-known scalar product of vectors. Second, it must be possible to enlargement Shannon's formula because universe ruled by randomness multiprocessing for whose description the Shannon formula is unsuitable and insufficient. Finally, formula (1.97) reduces to the Shannon's when the components of the first vector form the probability distribution, the second its logarithms.

We assume that "living beings" and "inanimate matter" are in a dualism if they are not the same so that while the first seeks as much freedom (ℓ), the second seek as little emission of information (denoted the L). That's why we need to supplement the aforementioned formula. In the book (see: [2] example 1.5.2 then theorem 1.5.4) in the example has shown a simple implication

$$(p_1 \ge p_2) \wedge (q_1 \ge q_2) \;\Rightarrow\; (p_1q_1 + p_2q_2 \ge p_1q_2 + p_2q_1), \tag{1.98}$$

which was then generalized into the theorem on arbitrary arrays. Two series of n numbers $\mathbf{p} = (p_1, p_2, \dots, p_n)$ and $\mathbf{q} = (q_1, q_2, \ dots, q_n)$ are descending if all the inequalities applied:

$$p_1 \ge p_2 \ge \dots \ge p_n, \quad q_1 \ge q_2 \ge \dots \ge q_n. \tag{1.99}$$

Then their scalar product

$$\mathbf{p} \cdot \mathbf{q} = p_1q_1 + p_2q_2 + \dots + p_nq_n \tag{1.100}$$

has the maximum. Replace any two of components in one of the vectors, without changing other members, and you get a lower value of the scalar product. This is the meaning of the mentioned theorem.

The result is that Shannon's information as a scalar product of a series of probabilities and a series of negative logarithm of the corresponding probability maximized the product. The replacement of the two probabilities (or the two logarithms) would give a lower result. Accordingly, in order to avoid divergence such as those in the example 1.2.5, we define vectors of n components $\mathbf{p}$ and $\mathbf{q}$ so to obtain a smaller dot product.

From the same implication (1.98) follows that the reverse order of the arrangement of the sequences $\mathbf{p}$ and $\mathbf{q}$ for $n = 2$ gives a smaller scalar product. Generally, if the

$$p_1 \ge p_2 \ge \dots \ge p_n, \quad q_1 \le q_2 \le \dots \le q_n \tag{1.101}$$

Then again due to (1.98) holds the inequality

$$(p_1q_2 + p_2q_1) + (p_3q_3 + \dots + p_nq_n) \ge (p_1q_1 + p_2q_2) + (p_3q_3 + \dots + p_nq_n).$$

Then easily we find that the scalar product of the reverse-kept series (1.101)

$$L = \mathbf{p} \cdot \mathbf{q} = \sum_{k=1}^{n} p_kq_k \tag{1.102}$$

is minimal.

[33] Galileo Galilei (1564-1642), Italian mathematician, physicist, astronomic, philosophy.

Example 1.2.6. *Prove the implication (1.98).*

Proof. From the assumption $p_1 \ge p_2$ and $(q_1 - q_2) \ge 0$ multiplying we get

$$p_1(q_1 - q_2) \ge p_2(q_1 - q_2),$$

and after rearrangement the conclusion. □

Theorem 1.2.7. *Dot product (1.102) is minimal iff (1.101).*

Proof. When the components of the vector $\mathbf{p}$ in the product (1.102) are not arranged in a manner (1.101), rearrange the addends. Addition is commutative. Suppose further that the components of the vector $\mathbf{p}$ are arranged in a given order and the components of the vector $\mathbf{q}$ are not. Let q_k is lowest number of all components of the $\mathbf{q}$. If this is not the first member change places q_1 and q_k in the given product. The new dot product is less than the previous one. Next, consider the all members of $\mathbf{q}$ other than the first repeating the process, so that the next smallest member of the series is found at the second position, wherein the dot product will be even lower. After n steps we will have always smaller product (1.102) and finally ordered members as (1.101). □

Because the dot products of the summands is commutative, the members of the given n-tuple can be changed in pairs, substituting members with i and j index in one n-tuple replacing also the corresponding pair of the other. If after such transpositions is achieved direct (1.99) or reverse (1,101) arrangement of the components, then we say the given vectors are *adapted*. In the case of Shannon information, or a maximum of freedom ℓ from the mentioned book, we have a positive, that is direct adaptation. In the case of a minimum of freedom from this book or herein mentioned principle of information, we have a negative or reverse adaptation. The next work continues in accordance with (1.9).

1.2.6 Lagrangian

Starting from Heisenberg's relations (1.8) follows a minimum term

$$L = \Delta p_x \Delta x + \Delta p_y \Delta y + \Delta p_z \Delta z - \Delta E \Delta t, \tag{1.103}$$

with a value of $\hbar$ in all inertial systems. Because it's invariant of inertial motion, in the example 1.1.2 we could perform the Lorentz transformation. On the other hand, from the same we get the expression (1.9) which is (generalized) dot[34] product the probability and uncertainty of the position. Similarly, we could get a dot product of the probability of position and momentum. In both cases to the higher probability corresponds the less uncertainty (in L) and, according to the theorem 1.2.7, the dot product has a minimal value. Accordingly, the term L follows the principle of least action (effect), otherwise well known in the physics of motion. Let me explain.

Because, depending on the selected components p_k and q_k, the dot product (1.102) gives the smallest value, it formally mimics *Lagrange method* in searching the *equation of motion*. Here is the explanation then proof. As is known from physics, the difference of kinetic (E_k) and potential (E_p) energy is called the *Lagrangian*

$$\mathcal{L} = E_k - E_p. \tag{1.104}$$

[34] dot, scalar and inner products are synonyms

This term is the base of the principle of least action and the *Euler-Lagrange equation* of moving, in later text E-L equation

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{q}}\right)=\frac{\partial\mathcal{L}}{\partial q}, \tag{1.105}$$

where q determined the curve which, for example, in a Cartesian right-angled system is the abscissa (x), the ordinate (y) or the applicate (z). The advantage of this method over the Newtonian is its easy generalization in different coordinates.

Example 1.2.8 (Spring). *Find E-L equation for the spring.*

Solution. Kinetic and potential energy of the weight on a spring along the x-axis provide:

$$E_k=\frac{1}{2}m\dot{x}^2,\quad E_p=\frac{1}{2}kx^2,$$

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{x}}\right)=\frac{d}{dt}\left(\frac{\partial(E_k-E_p)}{\partial\dot{x}}\right)=\frac{d}{dt}m\dot{x}=m\ddot{x},\quad \frac{\partial\mathcal{L}}{\partial x}=-kx,$$

$$m\ddot{x}=-kx,$$

and that is exactly the Newton's formula $F=ma$. □

Example 1.2.9 (Gravity). *Find E-L equations of motion for the body mass m in gravity planet's mass M.*

Solution. Kinetic and potential energy in the central symmetrical field are:

$$E_k=\frac{1}{2}m\dot{r}^2,\quad E_p=-G\frac{mM}{r},$$

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{r}}\right)=m\ddot{r},\quad \frac{\partial\mathcal{L}}{\partial r}=G\frac{mM}{r^2},$$

$$m\ddot{r}=G\frac{mM}{r^2},$$

and this is Newton's gravitational force. □

To derive the E-L equations (1.105) consider the value

$$A\equiv\int_{t_1}^{t_2}\mathcal{L}(q,\dot{q},t)\,dt, \tag{1.106}$$

which is called the *action* or effects. Physical dimensions A is (energy) × (time). Here we work with only one coordinate, but the principle is the same and the result is easy generalized. For all possible functions of the form $q=q(t)$ ($t_1\le t\le t_2$) with a fixed end-points, $q_1=q(t_1)$ and $q_2=q(t_2)$, we search one that is *stationary*, that give a local minimum, maximum or saddle of action A. This is generalized, in calculus well-known, method of seeking fixed-point using zero roots of functions.

Theorem 1.2.10. *Of all the functions $q(t)$ with fixed ends $q(t_1)=q_1$ and $q(t_2)=q_2$, it $q_0(t)$ that applies*

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{q}_0}\right)=\frac{\partial\mathcal{L}}{\partial q_0}, \tag{1.107}$$

is stationary point of actions A (local minimum, maximum or saddle).

Proof. If a function $q_0(t)$ gives stationary (unchanged) value of the action A, then every other close it functions with the same endpoints provide essentially the same A, to a first order approximation. For such a stationary position consider the function

$$q_x(t) \equiv q_0(t) + xf(t), \tag{1.108}$$

where $x \in \mathbb{R}$ and $f(t)$ holds the conditions $f(t_1) = f(t_2) = 0$. We calculate:

$$\frac{\partial}{\partial x} A[q_x(t)] \equiv \frac{\partial}{\partial x} \int_{t_1}^{t_2} \mathcal{L}\,dt = \int_{t_1}^{t_2} \frac{\partial \mathcal{L}}{\partial x}\,dt = \int_{t_1}^{t_2} \left(\frac{\partial \mathcal{L}}{\partial q_x} \frac{\partial q_x}{\partial x} + \frac{\partial \mathcal{L}}{\partial \dot{q}_x} \frac{\partial \dot{q}_x}{\partial x} \right) dt. \tag{1.109}$$

From the previous equation we have:

$$\frac{\partial q_x}{\partial x} = f(t), \quad \frac{\partial \dot{q}_x}{\partial x} = \dot{f}(t), \tag{1.110}$$

so (1.109) is

$$\frac{\partial}{\partial x} A[q_x(t)] \equiv \int_{t_1}^{t_2} \left(\frac{\partial \mathcal{L}}{\partial q_x} f + \frac{\partial \mathcal{L}}{\partial \dot{q}_x} \dot{f} \right) dt. \tag{1.111}$$

Integrate the second term partial

$$\int \frac{\partial \mathcal{L}}{\partial \dot{q}_x} \dot{f}\,dt = \frac{\partial \mathcal{L}}{\partial \dot{q}_x} f - \int \left(\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}_x} \right) f\,dt, \tag{1.112}$$

so (1.111) becomes

$$\frac{\partial}{\partial x} A[q_x(t)] \equiv \int_{t_1}^{t_2} \left(\frac{\partial \mathcal{L}}{\partial q_x} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}_x} \right) f\,dt + \frac{\partial \mathcal{L}}{\partial \dot{q}_x} f \Big|_{t_1}^{t_2}. \tag{1.113}$$

Because of the boundary conditions $f(t_1) = f(t_2) = 0$ last summand disappears. Further, use the fact that the left side must be zero for an arbitrary function $f(t)$, because we assume $q_0(t)$ is stationary. The only way that is true, if the value is in parenthesis (calculated for $x = 0$) is identically equal to zero, or if it is valid (1.107). □

Note, in the proof, the Lagrangian function $\mathcal{L}$ can be replaced by the dot product L, by replacing (1.104) to (1.102), and the theorem remains accurate. Then also function $q_0(t)$ with a given fixed ends, is a stationary state value of $\mathcal{L} = L$, if applies E-L equation (1.107). Hence the conclusion, a particle is moving inertial so the dot product, such as (1103), was minimal.

In inertial systems we distinguish two types of observations. The *proper* (own) from the observer that is stationary in a given system, and the *relative* from the observer to which the given system is moving. The proper and the relative observations generally are not equal. Because the preferred is the principle of probability (the most usual occurs what is the most likely), the relative likelihood is (mainly) lower than the corresponding own. Simply so.

The next step is the information that we define by probability in different ways, but mostly we ask: with the more likely to get less. I see no reason to now abandon these definitions, at least in terms of their own observers. However, if for the relative observers who perceive lower probabilities the realization of the information would be greater, they would have seen higher production and a faster flow of time, a smaller unrealized rest of uncertainty. As the universe has more relative than the proper observers then it would be rapidly exhausted. Because of the law of conservation of total uncertainty and information, all of the uncertainty of the universe would leak into real in a very short time. The universe would be created flashed and burned, all at one sudden.

Thus, the arbitrary particle is moving along paths of its own greatest probabilities and the smallest production of information. Relative observers their probabilities and information see even smaller. The system cannot spontaneously go from own into relative state to reduce the emission of information because it would cross into the state of the smaller probability.

Unlike non-living, *living beings* make decisions. Anyway, the freedom of (1.97) are taking only the values from the least, such as (1.103) when the smaller value of the intelligence component is multiplied by a higher value components of the hierarchy and vice versa, and to the largest, where is multiplying less with less and larger with a larger one. Minimum gives the theorem 1.2.7 and the maximum the corresponding theorem from [2]. Accordingly, the traces of life in the observed substance can be discovered by the principle of least action, that is, from the deviations from the trajectories that are solutions of the Euler-Lagrangian equation (1.107), but not too much. We have seen that this equation is the result of the principle of information, and from the same is following the "laziness" of any alive individual, which will eventually be discovered in mathematics as simple patterns in the behaviors of the living beings.

For example, the human of modest coefficients of vector *intelligence* (IQ, mastery of craft, a gift for art, etc.) it is the most optimal for success in business to make all the wayside routines down to a blind obedience. If, say, in a series of summands (1.97), that is

$$\ell = i_1 h_1 + i_2 h_2 + \dots + i_n h_n, \tag{1.114}$$

the first feature (ω_1) represents the timeliness in coming to work and the other (ω_2) planning the next working day, then it is best that the person has the maximum amount of hierarchy on these two properties, with as many $h_1 = h(\omega_1)$ and $h_2 = h(\omega_2)$, and that he (or she) as less manipulate them as could, reducing the number $i_1 = i(\omega_1)$ and $i_2 = i(\omega_2)$. With this, the larger amount of his total intellectual capacity

$$|\mathbf{i}| = \sqrt{i_1^2 + i_2^2 + \dots + i_n^2}, \tag{1.115}$$

which is substantially constant, remain free for creative work. I believe that future research will show that the organized managers, who appreciate the work habits and who strictly adhere to the timetable and work plans, are more successful. Particular confirmation of this analysis will be the discovery that such managers, who by top-quality work, order and discipline achieve top results, do not actually have to be highly intelligent people.

1.2.7 Satellite

Below we test previous conclusions. Firstly, we consider the case of *satellite circling* around a planet when the centrifugal force and gravity are balanced. A small local environment about the satellite is (approximately) the inertial system.

At the origin of this coordinate system, at the point O on figure 1.18, is located the center of the planet mass M, around which rotate two satellites A_1 and A_2 masses m_1 and m_2 at distances r_1 and r_2 respectively, in circular[35] paths k_1 and k_2. Repulsive centrifugal

[35] Instead of circles we could take any other curve of the second order (ellipse, parabola or hyperbola), but then the account would be complicated.

forces generated by the rotation of the satellites are in balance with the attractive force of gravity and satellites slide on its circle lines not feeling the effects of these forces. They are weightless and therefore in the (curved) inertial movement. The question is what with the probability and information of satellites?

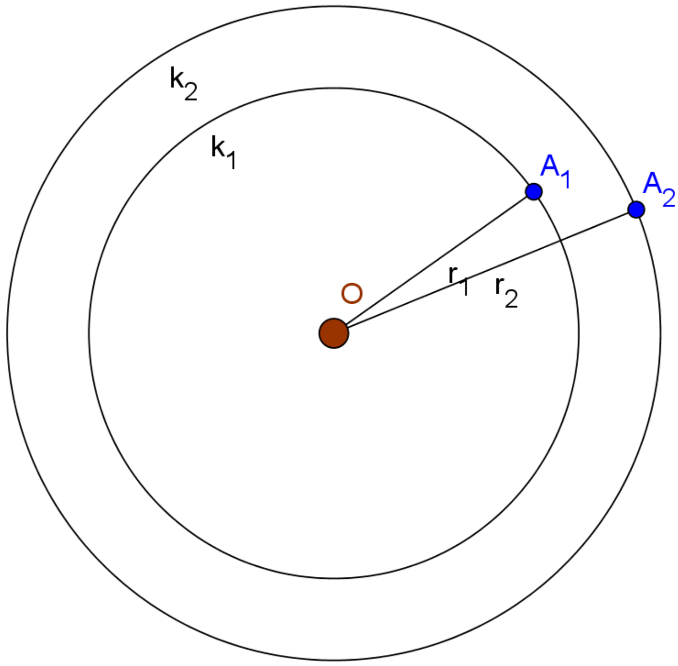


Figure 1.18: The rotation around the center of mass.

Consistently to the principle of probability, the proper probability of each of the satellites is greater than the relative on the other. Also, the proper probability of a satellite is larger than the observed at a fixed point that the satellite passes during its rotation because otherwise it could stop at the point (without the effects of other forces or collision). That's why the satellite does not exceed spontaneously from one path to another, neither stops.

Example 1.2.11. *Calculate the speed of the satellite.*

Solution. Centrifugal and *gravitational* force of the satellite of weight m at distance r of the center of mass M, are:

$$F_c = \frac{mv^2}{r}, \quad F_g = G\frac{Mm}{r^2}, \tag{1.116}$$

where $G = 6.67 \times 10^{-11}\text{N m}^2\text{ kg}^2$ is the gravitational constant. By equating we get

$$v = \sqrt{GM/r}, \tag{1.117}$$

then r change with $r_1 < r_2$, and we get two different speeds $v_1 > v_2$. As the satellite is further it moves slower, at a rate inversely proportional to $\sqrt{r}$. □

In a fixed (almost) infinitely distant point, when $r \to \infty$ and $v_\infty = 0$, it does not feel the effect of gravity. Seen from that distant place, it will pass the relative time interval

$$\Delta t_\infty = \frac{\Delta t_A}{\sqrt{1 - \frac{v^2}{c^2}}} \approx (1 + \frac{1}{2}\frac{v^2}{c^2})\Delta t_A, \tag{1.118}$$

while on the satellite A that slides with tangential speed (1.117) along the circle with the radius r pass the time interval Δt_A. At a given satellite interval Δt_A is

$$\Delta t_A = \frac{\Delta t_r}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{1.119}$$

where Δt_r is the interval of time measured by a clock at rest at a fixed point on stationary point by which the satellite passes. The composition of the last two expressions gives

$$\Delta t_\infty = \frac{\Delta t_r}{1 - \frac{v^2}{c^2}} \approx (1 + \frac{v^2}{c^2})\Delta t_r. \tag{1.120}$$

Accordingly, as seen from the position outside of gravity ($r \to \infty$), a fixed point at a height r has the *slower time flow* according to the (approximate) formula

$$\Delta t_\infty = \frac{\Delta t_r}{\sqrt{1 - \frac{2GM}{rc^2}}} \approx (1 + \frac{GM}{rc^2})\Delta t_r. \tag{1.121}$$

This will be explained in another way in the following caption, and we'll show that you get the same results from the general theory of relativity.

Time slows down in proportion to (1.121), and we conclude that by the same coefficient decreases and the relative production of information. Change of probability we'll evaluate using the relative change of frequency and wavelength (origin) of the same light in two different places. In doing so, we can rely on the known measurement and confirmation of the general theory of relativity in connection with gravitation and *red shift*.

Viewed from the position beyond the reach of gravity, with an infinite distance from the central pivot, the formula for the red-shifted frequency $\nu = c/\lambda$ (decreasing) and therefore for the energy $E = h\nu$ photons provides

$$\nu_\infty = \nu_r\sqrt{1 - \frac{2GM}{rc^2}} \approx \nu_r(1 - \frac{GM}{rc^2}), \tag{1.122}$$

where r is height fixed point of sources of photons. It is a known formula, but it is quite expected from the new (1.121) because the frequency is the reciprocal of the time. How many times relatively time flows more slowly, so times lower the frequency and wavelength increases. The wavelength is so many times higher, and

$$\lambda_\infty = \frac{\lambda_r}{\sqrt{1 - \frac{2GM}{rc^2}}} \approx \lambda_r(1 + \frac{GM}{rc^2}), \tag{1.123}$$

where λ_r is the wavelength of (the same) emitted light source which is at rest at the height of r, seen as λ_∞ from the position outside of the gravitational field. How many times was observed wavelength greater so many times is the probability less.

This last conclusion we have discussed in the framework of quantum mechanics. The greater the wavelength, the more stretched the place of a possible finding of photons become, and the probability of its finding at a given point is lessened. There it wasn't strange, but here it seems a surprising and somewhat "unacceptable". This is because, I believe, as a mechanic so runs away from the idea of a coincidence that in this sense becomes a dogma.

1.2.8 Vertical drop

In conditions of a weak gravitational field (Newton Field), when the free fall along the circular path of the satellite can still be considered as an inertial motion, the speed of the fixed point that the satellite passes, grows with the approaching to the center of gravity. In relation to the immobile observer from almost infinitely distant satellite, the relative time and emission of information slows down. So, by interpreting the emergence of time by creating information, we arrive at the conclusion that Newton's mechanics predicts a slowdown of time within gravity. Moreover, it turns out that this is in line with Einstein's general theory of relativity!

Einstein and then Schwarzschild[36] in 1916 showed that from the general theory of relativity, by approximation, is obtained the classical theory of gravity. Now we have a reverse process, the derivation of the consequences of Einstein's general theory from the classical Newtonian theory. However, the principle of equivalence is not necessary for such generalization. A special theory of relativity and classical mechanics are sufficient.

Starting from Newton's law, we keep the center of mass M still in the outcome of the coordinate system, and the body mass m is in free fall, say vertically (or otherwise) towards that center. The fall of the body mass m produces a work (the gravitational force of a body M) which converts to kinetic energy and the mass m grows. According to Galileo's principle of equivalence of the inertial and gravitational mass, the gravitational force increases with the increase in mass. Let dm is increase of mass m by moving for dr. Then we have:

$$dmc^2 = -\frac{GMm}{r^2}dr,$$

$$\frac{dm}{m} = -\frac{GM}{r^2c^2}dr,$$

$$\ln m = \frac{GM}{rc^2} + \text{const.}$$

$$m = m_0 e^{GM/rc^2}, \tag{1.124}$$

where the mass of the falling body would be m_0 in the absence of gravity.

By developing the expression in the Maclaurin series and ignoring the higher degrees of small members, we see that the coefficient of time dilation is equal to the coefficient of mass increase. With the same approximation, it turns out that the light entering the gravitational field changes the frequency just as time in the gravitational field slows down due to the decrease in entropy.

The equation (1.224) applies to gravitational fields where the mass M is sufficiently greater than m (the acceleration of a larger body is negligible in relation to the acceleration of the smaller one), when the center of mass is at the center of a larger body and we can still have inertial systems. The mass of the photon is $\hbar\omega/c^2$, the frequency of light in infinity is ω_0. As light travels in gravity field (see [9]) the frequency is $\omega = \omega_0 e^{GM/rc^2}$. Conversely, if we put the light frequency ω' on the surface of the star of the radius of R, then it, while move from the star, change into the frequency

$$\omega = \omega' e^{-GM/Rc^2} \approx \omega'(1 - \frac{GM}{Rc^2}). \tag{1.125}$$

[36]Karl Schwarzschild (1873-1916), German physicist and astronomer

We have seen that this is a well-known formula for the gravitational *red shift*. It also indicates a lower relative frequency on the surface of the star.

Within the gravitational field, we cannot synchronize watches as in special relativity, but we can do it in the infinity of the field. The light wave oscillation arrives from infinity to a distance r from the center with an initial frequency ω_0 and the duration of one oscillation $\Delta t = 2\pi/\omega_0$. Measured by the clock in the gravitational field, the elapsed time is also Δt since the dispositions required for the two phases of the wave are the same.

However, gravity operates on the frequency of light. This is seen from the previous one, that the local frequency of light changes relative to infinity. From a distant inertial system, the local frequency becomes $\omega_0 \cdot e^{GM/rc^2}$, and the local duration of a phase wave $\Delta t \cdot e^{-GM/rc^2}$. As the time interval measured locally is Δt, so a local clock slows down in relation to the clock at infinity. In order to get time in infinity, we must multiply the local time with the factor e^{GM/rc^2}.

Similar to time dilation, using light frequencies, we can also calculate the contraction of the length in the presence of a gravitational field. Let the wavelength of light in (almost) infinite distance from gravity center is λ_0 and the distance it traverses for unit time $n\lambda_0$. As it approaches the distance r, it still makes n oscillations in the unit of time. As the local frequency increases, the local wavelength becomes $\lambda_0 e^{-GM/rc^2}$ observed from infinity, and the distance that the wave travels in the unit of time becomes $n\lambda_0 e^{-GM/rc^2}$. The length is the distance that the light travels for a given time. Compared to the length in infinity, the local length is shortened. Therefore, in order to reduce its length to infinity, the local length (in the direction of the source of the field) must be multiplied by the factor e^{-GM/rc^2}.

In other markers, if for the spectators in the gravitational field the length (in the direction of the change in the strength of the field) and the time are Δr_0 and Δt_0, the same observer outside the gravitational field will evaluate with Δr and Δt, where:

$$\Delta r = \Delta r_0 \exp\left(-\frac{GM}{rc^2}\right), \quad \Delta t = \Delta t_0 \exp\left(\frac{GM}{rc^2}\right). \tag{1.126}$$

These are expressions for the contraction of the length (in the direction of the gravitational field) and the dilation of time also known in the general theory of relativity in the case of weaker fields (small M or big r). The Maclaurin approximation reduces them to:

$$\Delta r = \left(1 - \frac{GM}{rc^2}\right)\Delta r_0, \quad \Delta t = \left(1 + \frac{GM}{rc^2}\right)\Delta t_0. \tag{1.127}$$

It can be shown that the same expressions are obtained from the Schwarzschild metric

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2 \sin^2\theta \, d\varphi^2 + r^2\, d\theta^2 - \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2, \tag{1.128}$$

Which represents the solution of Einstein's field equations (general relativity theory) in spherical coordinates of $Or\varphi\theta$ for a central symmetric gravitational field.

I hope that these calculations and explanations testify well enough about the conformity of the principle of probability and consequences here with recognized mechanics.

1.2.9 Einstein's gravity

Einstein once said: "If you cannot explain something, then you do not understand it." It is surprising because this man discovered one of the most complex and the most difficult to explain theories of exact science in general. The tensor calculus, which was at his time at the beginning, and today has become the true nightmare not only of theoretical physicists, to approach to the even better mathematicians is not at all a simple matter. Yet it is possible to be explained to laymen.

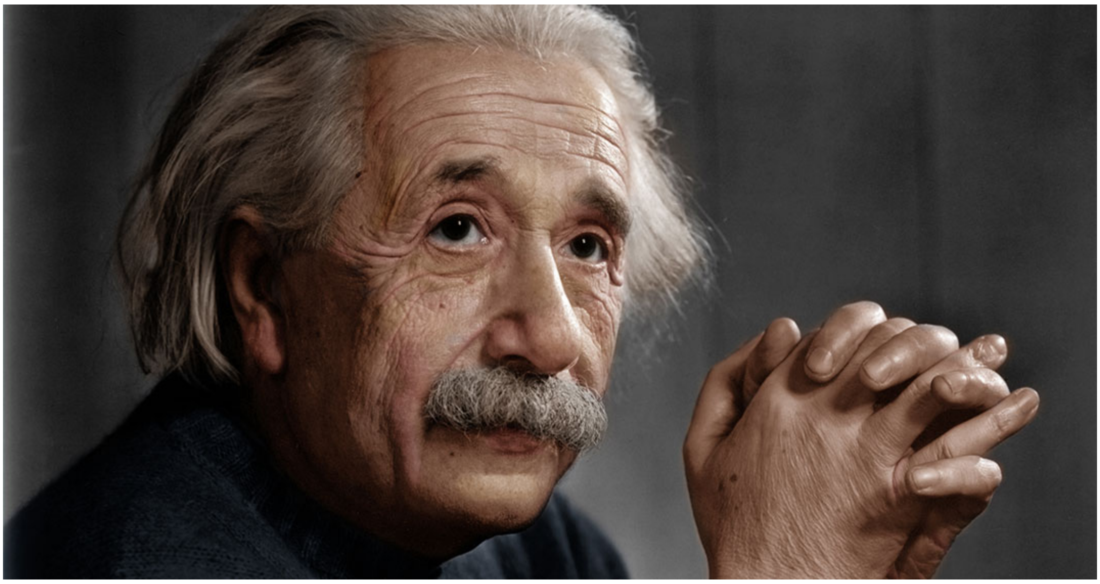


Figure 1.19: Albert Einstein.

Einstein in 1915 and then in 1916 published a system of equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{1.129}$$

Where the μ and ν are indices indicated three spatial and one time coordinates. Einstein called what arises from these equations the "field theory", but we call it the "gravitation theory" or "the theory of relativity" today. It is his General Theory. On the left side of the equation is a $G_{\mu\nu}$ tensor representing a "pure" geometry, and on the right side is $T_{\mu\nu}$ the tensor that represents matter. The coefficient between $8\pi G/c^4$ only harmonizes the physical dimensions of the two. Therefore, these *field equations* have a completely simple meaning: the geometry of space defines physical matter, and vice versa, matter defines the space.

While searching for the general theory of relativity, Einstein addressed his school friend Grossmann[37] for helping to learn the tensor calculus, these days the new field of mathematics. He was attracted by the idea of *covariance* (and dual contravariance), which is the basis of the tensors, and in fact, has a meaning similar to the principle of relativity. Tensors are such sizes around which all observers will be able to agree. Covariance is more than the mere compromise because the systems of numbers that we call the tensors after transformation of the coordinates fully preserve the form of the law they expressed. That is why Einstein asked Grossmann that the systems of numbers $G_{\mu\nu}$ and $T_{\mu\nu}$ should be the tensors.

[37] Marcel Grossmann (1878-1936), the Hungarian professor of mathematics.

The two of them quickly agreed that the simple tensor on the left side (1.129) defining the curvature of the space may be something as Ricci[38] tensor $R_{\mu\nu}$ or similar to it, not changeable on moving in space-time, because on the right side there may be some unchanging planets.

Ricci came to his tensor trying to solve, simply said, the next problem. If the ant lives on the surface of the sphere, how can it by means of measurement and calculation know that its space is curved? Ricci found that this could be (mostly) a composite term, which is a two-fold covariance tensor

$$R_{ij} = \frac{\partial \Gamma^{\ell}_{ij}}{\partial x^{\ell}} - \frac{\partial \Gamma^{\ell}_{i\ell}}{\partial x^{j}} + \Gamma^{m}_{ij}\Gamma^{\ell}_{\ell m} - \Gamma^{m}_{i\ell}\Gamma^{\ell}_{jm}, \tag{1.130}$$

which means after a possible transformation of the coordinates from one system to the other, this tensor maintains the properties of two arbitrary coordinates given by the lower indices. In the world of physics, this would mean that different relative observers have the same view of the curvature of spaces represented by numbers (1.130).

Generally, the indexes i, j, ℓ, m denote each as many coordinates as the actual geometry has dimensions. Because of the Einstein's *tensor convention* (summarized by the repeated upper and lower index) formula (1.130) represents a system of $4 \times 4 = 16$ partial differential equations, some of which are repeated. To make things even more complex in these equations are Christoffel's[39] symbols

$$\Gamma^{m}_{ij} = \frac{1}{2} g^{mk} \left(\frac{\partial g_{ki}}{\partial x^{j}} + \frac{\partial g_{kj}}{\partial x^{i}} - \frac{\partial g_{ij}}{\partial x^{k}} \right), \tag{1.131}$$

which, incidentally, are not tensors. Tensor $g_{\mu\nu}$, called *metric tensor*, are the coefficients of curly metrics that define the general term

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}, \tag{1.132}$$

which generalizes the Pythagorean Theorem. Previously $(g^{\mu\nu})$ and this $(g_{\mu\nu})$ are mutually inverse matrices.

Because the Ricci tensor derivatives $R_{\mu\nu}$ are not zero, and the tensor $T_{\mu\nu}$ on the right side of the field equation represents a stationary (unchanging) matter, Einstein for the left side of his equation came up with a tensor

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu} + \Lambda g_{\mu\nu}. \tag{1.133}$$

Simply put, according to Einstein it should have been that $G_{\mu\nu}$ is the simplest tensor that contains Ricky's tensor, but which does not change after (tensor) differentiation. The Λ scalar is the integration constant, which was soon given the name *cosmological constant.* Of course, the international community of physicists and mathematicians instantly perched on Einstein and in his way of "brutal defiance of exact science" by promoting such a "knitted" equation. Negative reviews came and from his friend Grossmann too.

In order to make matters worse, Einstein has "implanted" and the right side of his equation. He told that $T_{\mu\nu}$ represents an energy tensor, two times covariant, and before him the energy was not even really considered a vector. Tensor sizes without an index (zero order) are scalars if they have the same values for all relative observers. By the 20th

[38]Gregorio Ricci-Curbastro (1853-1925), Italian mathematician and the founder of the tensor account

[39]Elwin Bruno Christoffel (1829-1900), German mathematician and physicist.

century, it was considered, for example, that temperature and energy are surely tensors of zero order. The vector is a first-order tensor if its values are maintained by switching to another coordinate system. For example, the oriented length is a vector, but it is not a tensor because it will rotate the direction as its important qualifier. On the contrary, force vectors supporting a building are tensors, if this building looks equally stable viewed from different systems. Some second-order tensors are matrices, the third order is the blocks of the matrices arranged in depth, and so on.

Proclaiming energy two times covariant tensor, at the beginning of the 20th century, was still reckless, but not completely baseless. From the past, it was observed that energy is the fourth component of the momentum vector (mass times velocity). It only needed to be inverted and say that the three components of the momentum vector are actually the spatial components of the 4-dim "vector" of energy. It was then believed that the x component of the energy operates on the y component and in general μ at ν with the intensities $T_{\mu\nu}$. The last step, to say that the tensor $T_{\mu\nu}$ represents the matter, was lighter because of the known $E = mc^2$.

Einstein did not care much for the experiments. Of the three tests, he suggested for a general theory, first – that the clock slows down in the gravitational field – is not confirmed for his life. Moreover, the first experiments challenged his discovery. His second prediction that the brightness from distant stars should turn in the gravitational field, after measuring in 1919, made it famous, although later it turned out that such measurements were ambiguous and doubtful. The third test was the best. Because of the small anomalies of Mercury's orbit around the Sun in accordance with the calculations of general theory, Abraham Pais said, "Einstein is right" and "Nature talks to him."

Today we know his theory is true. For example, radio signals from the Cassini aircraft on the road to Saturn have confirmed the predictions of the theory of relativity with great accuracy. But Einstein would not be impressed for that. He believed his theory was correct because it was consistent, simple and beautiful. "I am convinced that the structures of pure mathematics enable the discovery of the concepts and laws that connect them, which gives us the key to understanding the phenomena of nature," he said in 1933.

1.2.10 Schwarzschild solution

Today, it is hard to see the lack of understanding for Einstein from the scientific community. At that time the people of science were hardly heard about non-Euclidean geometry, and even less of them for a tensor calculus. However, only a month after the publication of the general theory, the German physicist and astronomer Karl Schwarzschild came to the first correct equation solution (1.129), if we do not account a trivial solution for a plane space. Immediately after publishing his work in 1916, Schwarzschild died in the First World War as a German soldier. It is less known that independently and at the same time German mathematician Johannes Droste (1886-1963) (see [11]) came to a similar solution.

The Schwarzschild solution is valid for centrally symmetric gravitational fields, such as the one that creates the Sun, if we ignore the influence of the planet. Therefore, it makes sense to look for it in a metric similar to (1.128), a little more general (see [10]).

Example 1.2.12. *Derive the Schwarzschild metric from field equations, starting from*

$$ds^2 = -e^{2B(r)}c^2dt^2 + e^{2A(r)}\,dr^2 + r^2\sin^2\theta\,d\varphi^2 + r^2\,d\theta^2, \tag{1.134}$$

where $A(r)$ and $B(r)$ are unknown functions of distance r.

Solution. The metric tensor $(g_{\mu\nu})$ and its inverse $(g^{\mu\nu})$ have matrices, in the order:

$$\begin{pmatrix} -e^{2B} & 0 & 0 & 0 \\ 0 & e^{2A} & 0 & 0 \\ 0 & 0 & r^2\sin^2\theta & 0 \\ 0 & 0 & 0 & r^2 \end{pmatrix}, \quad \begin{pmatrix} -e^{-2B} & 0 & 0 & 0 \\ 0 & e^{-2A} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2\sin^2\theta} & 0 \\ 0 & 0 & 0 & \frac{1}{r^2} \end{pmatrix}$$

We use both to calculate Christoffel symbols (1.131). We put $x^0 = ct$, $x^1 = r$, $x^2 = \varphi$ and $x^3 = \theta$, so we have the following order:

$$\Gamma^1_{11} = \sum_{k=0}^{3} \frac{1}{2} g^{1k}\left(\frac{\partial g_{k1}}{\partial x^1} + \frac{\partial g_{k1}}{\partial x^1} - \frac{\partial g_{11}}{\partial x^k}\right) = \frac{1}{2} g^{11}\left(\frac{\partial g_{11}}{\partial x^1} + \frac{\partial g_{11}}{\partial x^1} - \frac{\partial g_{11}}{\partial x^1}\right) =$$

$$= \frac{1}{2} g^{11}\frac{\partial g_{11}}{\partial x^1} = \frac{1}{2} e^{-2A}\frac{\partial e^{2A}}{\partial r} = \frac{1}{2} e^{-2A} e^{2A}\frac{\partial (2A)}{\partial r} = \frac{\partial A}{\partial r} = A'(r),$$

$$\Gamma^1_{11} = A', \quad \Gamma^1_{12} = 0, \quad \Gamma^1_{13} = 0, \quad \Gamma^1_{10} = 0.$$

Due to the symmetry of the lower indexes, $\Gamma^m_{ij} = \Gamma^m_{ji}$, we get three more directly. Next:

$$\Gamma^1_{22} = \sum_{k=0}^{3} \frac{1}{2} g^{1k}\left(\frac{\partial g_{k2}}{\partial x^2} + \frac{\partial g_{k2}}{\partial x^2} - \frac{\partial g_{22}}{\partial x^k}\right) = \frac{1}{2} g^{11}\left(\frac{\partial g_{12}}{\partial x^2} + \frac{\partial g_{12}}{\partial x^2} - \frac{\partial g_{22}}{\partial x^1}\right) =$$

$$= -\frac{1}{2} g^{11}\frac{\partial g_{22}}{\partial x^1} = -\frac{1}{2} e^{-2A}\frac{\partial (r^2\sin^2\theta)}{\partial r} = -e^{-2A} r\sin^2\theta,$$

$$\Gamma^1_{33} = -e^{-2A} r, \quad \Gamma^1_{00} = e^{2(B-A)} B'(r),$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{r}, \quad \Gamma^2_{23} = \Gamma^2_{32} = \cot\theta,$$

$$\Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r}, \quad \Gamma^3_{22} = -\sin\theta\cos\theta,$$

$$\Gamma^0_{10} = \Gamma^0_{01} = B'(r),$$

and all other are zero. Then we calculate the Ricci tensor (1.130), in the order:

$$R_{11} = \sum_{\ell=0}^{3}\left[\frac{\partial \Gamma^\ell_{11}}{\partial x^\ell} - \frac{\partial \Gamma^\ell_{1\ell}}{\partial x^1} + \sum_{m=1}^{4}\left(\Gamma^m_{11}\Gamma^\ell_{\ell m} - \Gamma^m_{1\ell}\Gamma^\ell_{1m}\right)\right] =$$

$$= \sum_{\ell=0}^{3}\left[\frac{\partial \Gamma^\ell_{11}}{\partial x^\ell} - \frac{\partial \Gamma^\ell_{1\ell}}{\partial x^1} + \left(\Gamma^1_{11}\Gamma^\ell_{\ell 1} - \Gamma^1_{1\ell}\Gamma^\ell_{11}\right)_{m=1} - (\Gamma^2_{1\ell}\Gamma^\ell_{12})_2 - (\Gamma^3_{1\ell}\Gamma^\ell_{13})_3 - (\Gamma^0_{1\ell}\Gamma^\ell_{10})_0\right]$$

$$= \left[\frac{1}{r}A'\right]_{\ell=2} + \left[\frac{1}{r}A'\right]_{\ell=3} + \left[-B'' + A'B' - (B')^2\right]_{\ell=4},$$

$$R_{11} = -B'' - (B')^2 + A'B' + \frac{2}{r}A',$$

$$R_{22} = \left[1 - e^{-2A}(1 - rA' + rB')\right]\sin^2\theta,$$

$$R_{33} = 1 - e^{-2A}(1 - rA' + rB'),$$

$$R_{00} = \left[B'' + (B')^2 - A'B' + \frac{2}{r}B'\right]e^{2(B-A)}.$$

Ricci's scalar is obtained by contraction:

$$R = \sum_{\mu=0}^{3} R_{\mu\mu} = -2e^{-2A}\left[B'' + \left(B' + \frac{2}{r}\right)(B' - A') + \frac{1}{r^2}\left(1 - e^{2A}\right)\right].$$

These results enter into Einstein's tensor (1.129):

$$G_{11} = \frac{1}{r^2}(1 + 2rB' - e^{2A}),$$

$$G_{22} = r^2 e^{-2A}\left[B'' + \left(B' + \frac{1}{r}\right)(B' - A')\right]\sin^2\theta,$$

$$G_{33} = r^2 e^{-2A}\left[B'' + \left(B' + \frac{1}{r}\right)(B' - A')\right],$$

$$G_{00} = -\frac{1}{r^2}e^{2(B-A)}\left(1 - 2rA' - e^{2A}\right).$$

For the last we can see that it is

$$G_{00} = \frac{1}{r^2}e^{2B}\frac{d}{dr}\left[r\left(1 - e^{-2A}\right)\right], \tag{1.135}$$

Which will prove useful later. So much about the left side of the equation of the field.

On the right side of (1.129) is the energy tensor[40]. We assume that the energy that generates gravity within the ball (planet, star) radius r_0 with the center in the coordinate origin, with the density $\rho(r)$ is all inside the ball, and zero outside. It is known that the energy tensor for the perfect fluid in the thermodynamic equilibrium has the form

$$T_{\mu\nu} = \left(\rho + \frac{P}{c^2}\right)u_\mu u_\nu + P g_{\mu\nu}, \tag{1.136}$$

where $\rho = \rho(r)$ is density in kilograms per cubic meter, $P = P(r)$ is the hydrostatic pressure in the pascals, $u_\mu = dx_\mu/dt$ are the components of the 4-speed fluid , and $g_{\mu\nu}$ is a metric tensor. For a static fluid $u_1 = u_2 = u_3 = 0$, so the fourth component $(u_0)^2 = c^2e^{2B}$, which follows from $g^{\mu\nu}u_\mu u_\nu = -c^2$. Accordingly, the energy tensor has non-zero components:

$$T_{11} = Pe^{2A}, \quad T_{22} = Pr^2\sin^2\theta, \quad T_{33} = Pr^2, \quad T_{00} = \rho e^{2B}.$$

Putting it all in Einstein's equation (1.129), for indexes $\mu \ne \nu$ we get the trivial $0 = 0$, and for $\mu = \nu = 2$ and $\mu = \nu = 3$ we get the same equation. Interesting are only:

$$\begin{cases} \mu\nu = 11: & \frac{1}{r^2}\left(1 + 2rB' - e^{2A}\right) = \frac{8\pi G}{c^4}Pe^{2A}, \\ \mu\nu = 22: & e^{-2A}\left[B'' + \left(B' + \frac{1}{r}\right)(B' - A')\right] = \frac{8\pi G}{c^4}P, \\ \mu\nu = 00: & \frac{1}{r^2}e^{2B}\frac{d}{dr}\left[r\left(1 - e^{-2A}\right)\right] = \frac{8\pi G}{c^4}\rho e^{2B}. \end{cases}$$

From the fourth (zero) it follows:

$$A(r) = -\frac{1}{2}\ln\left(1 - \frac{2GM}{rc^2}\right), \quad M = \int 4\pi r^2\frac{\rho(r)}{c^2}\,dr.$$

We assume that $M(0) = 0$. Then, from $E = mc^2$ it follows that ρ/c^2 is the density of the mass, so $M(r)$ can be interpreted as the total mass of the ball radius r.

[40] Stress-energy tensor: `https://en.wikipedia.org/wiki/Stress%E2%80%93energy_tensor`

Finally we solve the first equation ($\mu\nu = 11$) when $r > r_0$. Outside the ball (planet or star), the mass M is constant, the pressure is zero (in the vacuum $P = 0$), so we have:

$$1 + 2rB' - e^{2A} = 0, \quad e^{2A} = \left(1 - \frac{2GM}{rc^2}\right)^{-1},$$

$$B(r) = \frac{1}{2}\int \frac{1}{1 - \frac{2GM}{rc^2}} \frac{2GM}{r^2c^2}\, dr = \frac{1}{2}\int \frac{1}{1 - \frac{2GM}{rc^2}} d\left(-\frac{2GM}{rc^2}\right),$$

$$B(r) = \frac{1}{2}\ln\left(1 - \frac{2GM}{rc^2}\right),$$

$$e^{2B} = 1 - \frac{2GM}{rc^2}.$$

Accordingly, the final solution for (1.134) is the Schwarzschild metric (1.128). □

When you enter the $r_s = 2GM/c^2$ tag called *Schwarzschild radius*, the metric (1.134) can be written

$$ds^2 = -\left(1 - \frac{r_s}{r}\right)c^2dt^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + r^2(\sin^2\theta\, d\varphi^2 + d\theta^2). \tag{1.137}$$

At a distance r_s from the center of mass[41] is *horizon events*, that is the the sphere within which is the *black hole*. This is the sphere on whose surface the relative time stops, the radial length (to the center of gravity) disappears, the energies and masses of the bodies that are found on it become infinite, and which is actually a consequence of the singularity of the metric itself.

The Schwarzschild metric was obtained by a series of simplifications, starting from the Einstein's idea that the universe was determined deterministically (that is 4-dimensional) and going to the approximate estimate of the tensor (1.136) by the perfect fluid. It is already known that it, therefore, has imprecision about the mentioned singularities and that it has a problem with the law of energy maintenance. However, it is still the best-known approximation of nature.

[41] Earth's $r_s \approx 9$ mm, the Sun $r_s \approx 3$ km.

1.3 Entropy

Carnot[42], otherwise known as the organizer of the victory in the French Revolution, discovered in 1803 that natural processes have some inner inclination to waste useful energy. Desiring to design a machine with maximum efficiency, he described the circular process of compression and propagation of gas in four cycles: 1. Isothermal (at a constant temperature T) expansion; 2. Adiabatic (at constant heat Q) expansion; 3. Isothermal compression; and 4. Adiabatic compression. These four stages, called the Carnot circular process, show the image1.20. From such works, a branch of physics is now known as *thermodynamics*.

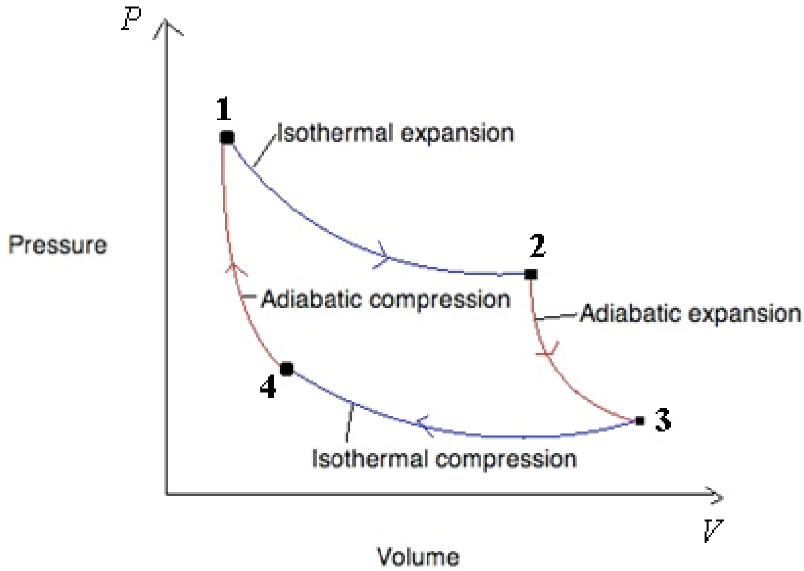


Figure 1.20: Diagram of volume V and pressure P.

Developing similar ideas, Clausius[43], since 1850, analyzed isolated systems in thermodynamic equilibrium. In 1865 he introduced the term *entropy*[44] (S) defining it as an energy that can no longer be converted into free work (W). In thermodynamics, entropy is still a magnitude representing the inaccessibility of heat (Q) for the transformation into mechanical work, which is often interpreted as the degree of disorder or coincidence of a given physical system.

Afterward, scientists such as Boltzmann[45], Gibbs[46] and Maxwell[47], gave entropy a statistical significance. The thermodynamic definition of entropy describes its use in experiments, while statistical develops this concept, giving it a deeper meaning.

[42] Lazare Carnot (1753-1823), French politician, engineer and mathematician.
[43] Rudolf Clausius (1822-1888), German physicist and mathematician.
[44] Greek $\epsilon\nu\tau\rho o\pi\eta$ - craft in.
[45] Ludwig Boltzmann (1844-1906), Austrian physicist and philosopher.
[46] Josiah Willard Gibbs (1839-1903), American engineer, physicist, and mathematician.
[47] James Clerk Maxwell (1831-1879), Scottish mathematical physicist.

1.3.1 Thermodynamics

The concept of entropy arising from the *Carnot cycle* is abstract, not experimental. In the image 1.20 the circular thermodynamic heat transfer, cycle Q, is shown from the state of the higher to the state of lower temperature T and vice versa, where $T_1 < T_2$. Heat Q_2 with temperature T_2 goes to the cold temperature tank T_1 in the heat Q_1. According to Carnot, the work of W can only be produced by the system in which temperature changes, and it should be some function of temperature difference and absorbed heat. More precisely:

$$W = \left(\frac{T_2 - T_1}{T_2}\right) Q_2 = \left(1 - \frac{T_1}{T_2}\right) Q_2 \tag{1.138}$$

is the maximum work the heat machine can produce.

However, Carnot mistakenly assumed that $Q_2 = Q_1$ because at that time the actual calorie theory considered that in any case holds the conservation law of heat. Only after Clausius and Kelvin[48] we know today that it is actually $Q_2 > Q_1$, and hence we have the absolute Kelvin scale of temperature. The work that produces the system is the difference between the transformed heat from the warmer tank into the cooler:

$$W = Q_2 - Q_1. \tag{1.139}$$

From the previous and this equality (in case of maximum work) it follows:

$$\Delta S = \frac{Q_2}{T_2} - \frac{Q_1}{T_1} = 0. \tag{1.140}$$

Here $\Delta S = S_2 - S_1$ is the difference of an unknown function that has the physical dimension of heat divided by the temperature. This equality confirmed the *the first law of thermodynamics*: the total energy of an isolated system is constant. Energy can be transformed from one form to another, but it cannot come from nothing, or it cannot disappear.

Also, equality (1.140) implies that there is a function of the state

$$S = \frac{Q}{T}, \tag{1.141}$$

which does not change the value (is preserved) during the (optimal) Carnot cycle. Clausius called this function entropy. Further discoveries in this field were made by Boltzmann revealing that the logarithm of the number $|\omega|$ of the uniform distributions ω from the random set Ω of all ways of arranging gas in the room, are equal to Clausius entropy

$$S = k_B \ln |\omega|, \tag{1.142}$$

where $k_B = 1,38 \cdot 10^{-23}$ m^2 kg s^{-2} K^{-1} is Boltzmann constant (valid for natural logarithm, base $e \approx 2.71828$). To the number of $|\omega|$ leads such distributes ω which is the most likely, and they are evenly distributed in all possible positions of the molecules. Namely, the $n \in \mathbb{N}$ molecules have $n! = 1 \cdot 2 \dots n$ times more uniform schedules than, say, the arrangement of the same molecules in the crowd. Because Boltzmann's entropy has the maximum for scattered gases, so sometimes we say that entropy is the amount of disorder.

The picture of 1.21 shows the increase in Boltzmann's entropy $\Delta S = S_2 - S_1$ from the value S_1 to S_2 that occurs by moving the substance from the solid to the liquid and into the

[48] William Thomson, 1st Baron Kelvin (1824-1907), Scottish Irish mathematician and physicist.

gaseous state. Similarly occurs in a spontaneous evaporation process, when molecules are free to go in any direction to occupy an increasing volume. This is well known today and it seems to all of us to be quite simple. It is less known that Boltzmann committed suicide, not because he was so crazy, as because he could not handle the ridicule of his colleagues' physicists who then mocked his alleged discovery.

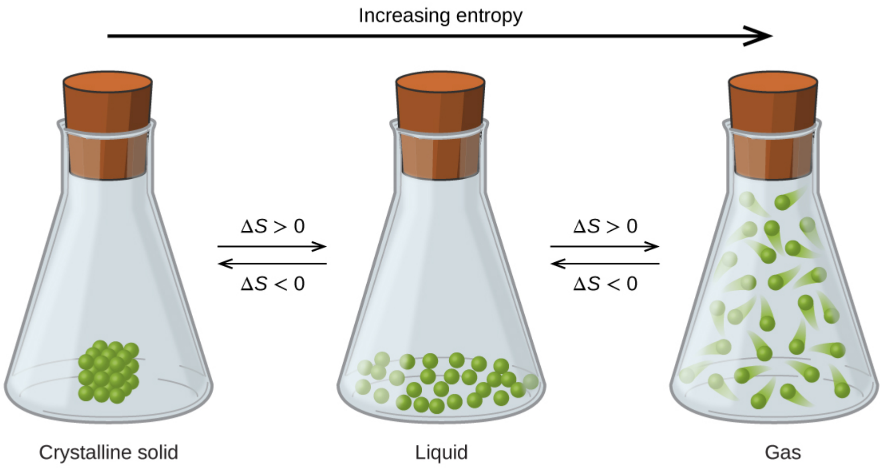


Figure 1.21: Entropy change.

In the reverse process in the same image 1.21, entropy decreases with the transition from gas to solid state. Spattered gas molecules will not just be compacted on a pile, so additional energy is needed to reduce entropy. Similarly, when the crystal vessel falls and breaks, entropy increases, but it is not possible to spontaneously return it to a lower value, or return the crystal to a previously undamaged state.

In general, when equally likely outcomes are divided into groups, more realizations will be in a group with more outcomes. For example, in the case of throwing out fair coins twice, the possible outcomes (Head and Tail) are equally likely elements of the set $\Omega = \{HH, HT, TH, TT\}$. The outcome of the double-entry Head is a subset of $\omega_1 = \{HH\}$, but the result that fall both, Head and Tail, is a subset $\omega_2 = \{HT, TH\}$. As the second subset has more than one element, $|\omega_1| = 1$ and $|\omega_2| = 2$, the second will be realized more often. This leaves the impression that nature rather realizes those random events that have more (equally probable) options as if it likes freedom, which is also a consequence of the principle of probability.

We know that nature tends to the conditions of greater entropy, as in the image 1.21, which means Boltzmann's entropy (1.142), that is, the logarithm of the number $|\omega|$ which is the number of outcomes of a larger number of the possible schedules. This is the meaning of the *second law of thermodynamics*: the total entropy of an isolated system can only grow over time. It can remain constant in an ideal case when the system is in a state of equilibrium, in a stable state, or when it is subjected to a reversible process. Natural processes of spontaneous increase in entropy are called irreversible, and because of their domination, the past is considered asymmetrical with the future. The spontaneous tendency of natural systems to preserve or increase entropy is called the *principle of entropy*.

1.3.2 Entropy information

The likelihood of "greater" schedules (some outcome of the "larger" set of outcomes) is

$$P = \frac{|\omega|}{|\Omega|}, \tag{1.143}$$

where $|\Omega|$ is the number of all possible outcomes. Therefore, the higher participation of the more likely (ω) in the set of all outcomes (Ω) gives greater probability (1.143) and greater entropy ($S = k_B \ln |\omega|$). But, unlike information, entropy is not a logarithm of probability, but rather a logarithm of its part. Changing $\omega \to \omega_1$ to a set with an even larger number of outcomes ($|\omega_1|$) of another layout (within Ω) by $\Delta|\omega_1| = |\omega_1| - |\omega|$ will result in a change in entropy and probability respectably:

$$\Delta S = k_B \ln \frac{|\omega_1|}{|\omega|}, \quad \Delta P = \frac{\Delta|\omega_1|}{|\Omega|}. \tag{1.144}$$

Because the difference in logarithms is equal to the logarithm of the quotient that is why the difference of entropy is not simply the logarithm of the difference the probabilities.

Under entropy, we mean the logarithm of the number of only the part of the layout that consists of a uniform distribution. In this sense (the example of throwing coins twice), when it can be considered as the logarithm of the number of most probable scheduling, that is, optimal but not all scheduling, it is a type of information. On the entropy related our previous conclusions about information, for example, if the relative information is smaller, then this is so the relative entropy.

With this conclusion, the story of entropy would have ended, but it's not because we still do not know exactly what information is. I recall that Hartley's definition too much simplifies, the Shannon's is not reliable (see example 1.2.5) and is generally insufficient, and generalized information (1.103) has just been discovered by adhering to the stingy principle of the preceding two. That *principle of information* that nature is economical with the expense of its uncertainty, explains entropy. Namely, assuming uniform scattering, in other words, striving for more disorder or to freedom, molecules make the gas amorphous, impersonal, and dislocated by the manifestation of any traits. Therefore, the principle of information is equivalent to the *principle of entropy*, the spontaneous transition of the system to a state of greater entropy. Both are the consequences of the *principle of probability*.

I recall that official physics is in great disagreement with all three of these principles. If (the probability principle) it is most likely that I am where I am, then for any other who is not here the probability is relative. For me, their position is relative and also less likely than mine. It is already inconsistent with the modern understanding of physical probability, but it is in line with, for example, the law of inertia: I am in my system standstill or in uniform rectilinear motion because all other states are less likely to me. Secondly, the relative information is smaller; otherwise, the universe would burn like a match. It would spend its uncertainty in a moment, as faster as there are more physical systems, particles, which are almost infinitely many. But physics has not yet come to this; it has no opinion about this because it still does not see time as the information produced. Thirdly, the principle of entropy is seen as a consequence of the principle of information on the way that the relative entropy is also smaller; despite the physics which holds that the relative entropy is equal to its proper (own).

The results of classical thermodynamics, as well as the results of the theory of relativity, are not disputed here, but we will correct the inconsistencies of the modern relativistic thermodynamics. How is it possible? Well, first of all, because relativistic thermodynamics can be treated in different ways, while classical theories, thermodynamics and the *theory of relativity* do not have to be changed. We will look at this observation and then we will introduce new settings and test their agreement with the classic ones.

During the 20th century until today, many scientific papers in the field of the relativity of thermodynamics with which we disagree here have been published. I'll only mention some. In 1907, Einstein proposed that entropy is regarded as Lorentz invariant (that it does not change the system in motion) and that the heat and temperature of the system in motion are less than their proper (seen at rest). The same was proposed by Planck[49] in 1908. Then Ott[50] in 1963 asked the opposite, that the heat and temperature in the movement were greater than the corresponding at rest, and that entropy was also the same at the same time. Landsberg somewhere at that time came to the conclusion that the body's heat in motion should be considered less and temperature and entropy immutable. Van Kampen revealed that all three were constant. In the end, Balescu proved that this is nonetheless, that relativistic thermodynamics can be treated in different ways so that classical thermodynamics and relativity theory can remain unchanged.

Due to the many contradictory findings, in physics, today *temperature* is defined only for the two bodies in rest one to another, and where the heat goes we say "there is cooler". Then we make more complex temperature measurement devices, step by step, starting from contact thermometers to non-contact thermometers. Among the most modern such devices are infrared thermometers that carry out the temperature from the amount of heat radiation (so-called black body radiation) emitted from the object of measurement. They are also called laser thermometers because they use guiding lasers. In the next section, we will see how these thermometers confirm the (hypo) theses I have here.

Contact measurement of body temperature in motion is rarely possible and, when possible, is not reliable. For example, in slower air currents, the bodies are cooled due to stronger evaporation (by blowing, we cool the soup), but at higher air speeds they are still warming up, we consider it to be the dominant conversion of kinetic energy that friction transfers to the body and passes into heat. At even higher speeds, the air cannot be folded fast enough in front of the object, it is compressed, causing, even more, warming of the body. A meteor that springs into the atmosphere at a high speed is so hot to burn.

All this indicates that the temperature of the object (heat meter) in the air stream is not the same as the temperature of the air itself. However, we note that in the aeronautics it is assumed that the approximate average difference (proper) air temperature and the heated object through which the air moves is proportional to the air velocity square, which leads us to the conclusion that the temperature increases with *kinetic energy*. Let's remind us once again what kinetic energy is.

From the special theory of relativity we know that relations of motion are valid for the body's energy:

$$E = \gamma E_0, \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{1.145}$$

where E_0 is the energy of the given body is at rest and E energy of the same body at the speed v. The speed of light in the vacuum $c = 3 \cdot 10^8$ m/s does not depend on the velocity

[49] Max Planck (1858-1947), German physicist.
[50] Heinrich Ott (1894 - 1962), German physicist.

of the light source. By developing in series of relativistic coefficient, gets:

$$\gamma = 1 + \frac{1}{2}\frac{v^2}{c^2} + \dots, \quad \frac{1}{\gamma} = 1 - \frac{1}{2}\frac{v^2}{c^2} + \dots, \tag{1.146}$$

Where the fourth and higher speeds have not been written because they are negligible for a relatively small v compared to c. When the developed coefficient γ is returned to the previous formula and put for kinetic energy $E_k = \frac{1}{2}m_0 v^2 + \dots$, we get

$$E = E_0 + E_k, \tag{1.147}$$

because $m_0 = E_0/c^2$ is the mass of the given body at rest.

The resulting expression and the procedure itself tell us several things. First, the temperature T that the thermometer will show in the flow of (ideal) air stream of the velocity v can be approximated by the expression

$$T = \gamma T_0, \tag{1.148}$$

where T_0 is the temperature that the thermometer would show outside the current. Second, the relative energy (E) of the body is a sum of its own (E_0) and kinetic (E_k). It is the total energy of the body in motion, and there is no place to increase heat energy as an oscillatory energy (molecule) or a chemical. So, the movement increases only the translational energy, and it is

$$Q = Q_0, \tag{1.149}$$

where Q and Q_0 are relative and proper heat energy. This heat Q, as *heat of entropy*, for us is even more distinctive, for example, from enthalpy, than in contemporary physics.

With this explanation, the relative entropy (1.139) becomes

$$S = S_0/\gamma, \tag{1.150}$$

where S_0 is its own (proper) entropy, and γ is a relativistic coefficient (1.145). This means that relative entropy decreases with increasing relative body speed, just as time slows down and while information is reduced.

We know that due to the entropy principle, from the room with higher pressure the air spontaneously goes into a room with a smaller one and that there is no such movement if the air pressure in the rooms is the same. Consistent with the previous one, if the closed space moves at velocity $\mathbf{v}$ = constant through the air of the same pressure at rest, if we ignore the viscosity (internal friction forces), the lateral pressure from the room to the wall will be increased proportionally to γ. Conversely, if we sit in a moving vehicle uniformly straight through the air of the same density, the lateral pressure inward will again be greater in proportion to γ, due to the principle of relativity. We will discuss this seeming paradox later, but before let's look with what we can justify a new idea of a higher relative temperature.

1.3.3 Red shift

Conclusion (1.148) supports infrared thermometer technology. Light is part of the spectrum of electromagnetic radiation (image 1.22), from the largest wavelengths (infrared, 700 nm) to the shortest (ultraviolet, 400 nm). We know that all the spectrum of the E-M waveforms in motion has greater wavelengths due to the Doppler Effect (or the Doppler shift). The

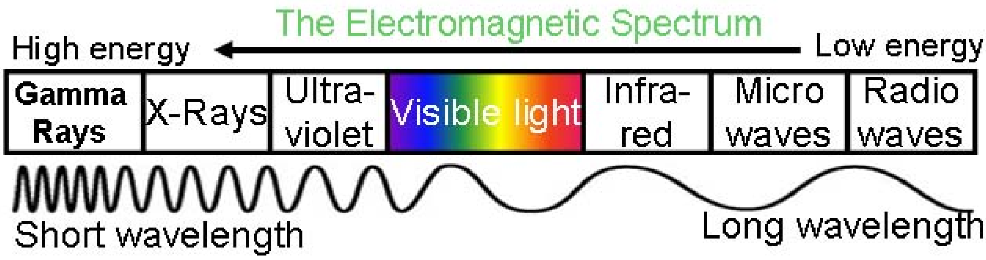


Figure 1.22: Electromagnetic waves.

wavelengths of the source are said to make the *shift to red*, which we will call here a "shift to the warmer", simply because everywhere in physics the term "Doppler Effect" can be replaced by "higher temperature", without contradiction.

The Doppler Effect is, for example, a higher tone of the siren that is approaching or lowering tone when it is moving away. The reverse to the frequency ν changes the wavelength λ, precisely their product is equal to the constant (the sound velocity c_s in the water is about 1500, in the air 340, and in the vacuum 0 meters per second). The velocity of the rest of the source of the source at rest $c_s = \lambda\nu$. It is a feature of all waves that, in the case of light, becomes especially interesting to us.

We know that the source of the wave of its proper[51] the wavelength λ_0 and the frequency ν_0, moving at the speed $\pm v$ in the direction on which the observer is located, has a wavelength and the frequency with respect to the observer, respectively:

$$\lambda = \lambda_0\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}, \quad \nu = \nu_0\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}, \tag{1.151}$$

where the sign minus is taken when the source is approaching, and the plus when it is moving away. The constant $c = 3\cdot 10^8$ m/s is the speed of light in the vacuum. If we denote λ_- and λ_+ wavelengths in the order of incoming and outgoing waves, we see that inequalities apply:

$$\lambda_- < \lambda_0 < \lambda_+, \tag{1.152}$$

when the velocity is $v > 0$. The mean value (arithmetic mean) of this *longitudinal* observed wavelength is:

$$\lambda_l = \frac{1}{2}(\lambda_- + \lambda_+) = \frac{\lambda_0}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{1.153}$$

When the observer is not in the direction of the movement of the source of the wave and looks perpendicular to that direction, it will observe the *transverse* wavelength

$$\lambda_t = \frac{\lambda_0}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{1.154}$$

Thus, the observed longitudinal and transverse wavelengths are equal. Both are larger than their own. The mentioned three: incoming λ_+, their own λ_0 and the outgoing observed λ_- wavelength, are mutually different.

[51]Proper or own - seen in moving from the source at rest.

The Relativistic Doppler effect is well known to physics, but not the interpretation I promote, that it simply means the changing of the relative probability of the medium and the temperature of the waves whose source is moving in relation to the observation of one's own observer (stationary relative to the source). Namely, the relativistic Doppler Effect points to the same changes in the space itself, the coordinate system related to the source of the EM wave.

The position of the particle-wave is smoothed along its wavelength so that the higher wavelength means the less accurate position of the particle or less the density of the probability of its finding at a given location, and vice versa, the shorter wavelength gives a more precise point of finding the particle. From the previous equations, it follows that the position of the wave of the source in motion is less likely than the same positions of the resting source.

We know that the ordinary Doppler Effect is derived from ordinary (Galilean) transforms, and that the relativistic Doppler Effect, we are dealing here, is derived from Lorentz's transformations of special relativity. However, the arithmetic mean of the relative incoming and outgoing longitudinal wavelengths (1.151), and in particular, the relative transferal wavelength (1.154), can also be understood by slowing down the time relative to the observers, which we will once again explain.

Example 1.3.1 (Time dilation)**.** *Show that for the relative Δt and the proper Δt_0 elapsed time is valid equality*

$$\Delta t = \gamma \Delta t_0, \tag{1.155}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$, and v is relative speed between two inertial systems.

Solution. From the special theory of relativity, we know that the movements are relative and that the speed of light in a vacuum is independent of the speed of the source. Hence, for two events with coordinates (r_1, t_1) and (r_2, t_2) along a direction of motion in two moments, if we put $\Delta r = r_2 - r_1$ and $\Delta t = t_2 - t_1$, the expression for the square of the interval

$$\Delta s^2 = \Delta r^2 - c^2 \Delta t^2 \tag{1.156}$$

will be the same in both inertial systems. When such two coordinate systems K and K' move along the given direction at a constant speed $\pm v$ one relative to the other, it is valid:

$$\Delta r^2 - c^2 \Delta t^2 = (\Delta r')^2 - c^2 (\Delta t')^2.$$

The observer time from K' denote with $\Delta t' = \Delta t_0$. Then $\Delta r' = 0$ (it is stationary in its system), so we get:

$$\Delta r^2 - c^2 \Delta t^2 = -c^2 \Delta t_0^2,$$

$$\left[\left(\frac{\Delta r}{\Delta t}\right)^2 - c^2\right] \Delta t^2 = -c^2 \Delta t_0^2,$$

$$(v^2 - c^2) \Delta t^2 = -c^2 \Delta t_0^2,$$

and therefore (1.155). So, the proper time observed from the other system runs slower. □

With slowing down time, the observed frequencies proportionally slow down, and because of $\lambda \nu = c$ wavelengths increase. The result is the same as the previous one. There remains only a question of different incoming and outgoing waves (1.152).

Due to the slower flow of the time of the system in motion, the particle that comes to us is in our future, and it is getting closer to our present while its distance from us decreases. While it passing near us, our "now" is the same, but leaving us it would continue to be in our past deeper and deeper. We mentioned this and with the explanation of the twin paradoxes while interpreting force. Now let's go one step further with the same observation.

We are also in the future of the particle that comes to us and in the past the one that leaves. In this way, the waves that come to us define our future, and the departing defines past. On the one hand, this means the objectivity (reality) of our history and the confirmation of the law of maintaining information. Our past is objective as far as the objective observation of the particles we perceive is concerned. On the other hand, from inequality (1.152) it follows that our future is more probable than the present and that the past is less likely than both. Therefore, we notice that time flows from our past through the present to the future. The foundation of these explanations is to confirm the basic idea of the nature of time, as well as the new hypothesis (1.148) of a higher relative temperature.

All observers in inertial motions live in the same space-time, but they do not see it as the same. But the speed of light in a vacuum ($c = 3 \cdot 10^8$ m/s) is equal to all of them, and this gives us the right to take the waves of light (EM radiation) for the definition of space-time. Therefore, the interval (1.156) is so important, since for each observer in case of light $\Delta s = 0$, on the basis of which we calculate the rest of the path ($\Delta s \ne 0$) of a slower particle complementing it by moving the light. Everything else is derived from this statement that Δs^2 is invariant, which we treat as a Pythagorean theorem.

So we came to the conclusion (1.155) that the relative time is slower than its own. Using the figure 1.23 we will understand the effect of slowing down the time to the relative contraction of the length along the direction of the movement r, otherwise derived from Lorentz's transformations in the example 1.1.3.

In relation to its own observation in the K' coordinates, perpendicular to its direction of motion, the light OA is sent, which is after a while observed again as a perpendicular but as $O'A'$. The relative observer in the K the movement of this light can see along hypotenuse OA' of the right triangle $\Delta OAA'$. The own observer sees a shorter path of light (side OA), but time passes slowly, so he sees the speed of light unchanged.

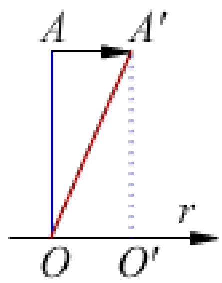


Figure 1.23: Moving.

In order for the speeds of light for them to be exactly the same, the length of the hypotenuse $\overline{OA}'$ for its own observer must be equal to the length of the shorter side $\overline{OA}$ for the relative. In other words, the lengths in the direction of the movement of their own observer must be exactly as many times shorter than the relative, how often time is slower. Hence the formula for the *length contraction*

$$\Delta r = \Delta r_0 \gamma^{-1}, \quad \gamma^{-1} = \sqrt{1 - \frac{v^2}{c^2}}, \tag{1.157}$$

where Δr_0 is its own length in the direction of motion and Δr is the corresponding relative. The lengths vertical to the direction of movement remain the same for the two observers.

By slowing down the time we can understand the increase in body mass and energy too. Firstly notice that slower time means greater body inertia, meaning sluggishness or laziness, which means an increase in its relative mass. This increase in mass is accompanied by an increase in speed, which due to the law of energy conservation means increasing the energy of the body for the kinetic energy. This is a brief explanation of known formulas:

$$m = \gamma m_0, \quad E = mc^2, \tag{1.158}$$

where m_0 is proper and m the relative mass with total energy E.

In the meantime, note that the Doppler effect and these formulas imply that we can consider that there is only one photon or one type of electromagnetic radiation with a set of relative observations. There are many systems in various relative movements for each observer within the space. From the constant product of the wavelength and photon frequency, $\lambda\nu = c$, it follows that observations of the most frequent go with the shortest wavelengths, and this means with observing the space with the highest probabilities. These are the systems of the highest energies and the highest relative speeds. In the limiting case, places without uncertainties become those with absolutely exact positions and endless energies. In general, the boundaries (beginning or edges) of the space, the horizon events, the speed of light, and the extremely small lengths, are places for which we need infinitely large energies, and these are the places of zero entropy.

So, from the theory of relativity we know that the relative masses of the molecules of the body in moving are greater, and that is exactly as many as the time passes slower. Further we evaluate that oscillator relative energy of the molecules can stay the same to the proper, because what is gaining by increase of mass, the exactly is losing by slowing the frequency, which is in harmony with the previous conclusion (1.147), that in relative increasing of energy there is no place for any other energy except kinetic. But, the conclusion that in increasing of kinetic energy of moving body has no change of its heat, that holds equation (1.149), we said, directly is opposite to the modern physics.

Let's summarize now, from the new positions, what happens when the body hits the obstacle and stops warming. The body in the inertial velocity $v > 0$ with respect to a given relative observer has a higher total energy $E = \gamma E_0$ and a higher temperature $T = \gamma T_0$, unchanged heat $Q = Q_0$, but the reduced entropy $S = S_0/\gamma$. At the moment of a stroke at an obstacle, the relative velocity of the body becomes $v = 0$, kinetic energy becomes heat, and the temperature remains unchanged – but now in relation to the (same) observer that is relatively immovable. The temperature that the body has in rest just after the collision is the relative temperature that the body had while moving.

We have explained all of this by taking only examples of special relativity, but similarly, apply in general relativity. In the presence of gravity, time slows down, which is why a gravitational *red shift* occurs, which in the sense of the previous explanation is equivalent to Doppler shift. Here we can add that this means an increase in the temperature of the position (the points that remain relative to the source of gravity) in the gravitational field. When the velocity from the γ coefficient is replaced by the gravitational potentials, the previous summary remains. The potential energy of the body in the gravitational field changes (by motion and kinetic), but the heat does not. In static points of stronger parts of the field the corresponding entropy is smaller.

Because of the principle of entropy (as well as probability), the body retains its inertial movement, since all other relative movements have less entropy. In the presence of a gravitational field, it moves with geodesics, we say freely falling, because on the path is constant entropy (and probability). Beyond its path, every other geodesic line of (free) fall is seen from the body in the conditions of lower entropy, and it is not possible for a transition in a spontaneous way. That is why satellites circulate around the Earth and, for instance, do not descend, until a collision with another body (particles of the atmosphere) or the effects of some other force. A special case of this is the body that rests in the field of a planet and can spontaneously reach its lower height only with increasing speed because it would have lower entropy at a lower altitude in a state of rest.

That the gravitational force attracts the body to a less entropy space seems absurd. However, this is logical because there is no arrival in the state of lower entropy without the action of some force, yet again, entropy disorders, probability, as well as curvature of space, are more profound causes of gravitational phenomena.

1.3.4 Pressure

In the previous two sections, the hypothesis was introduced that the relative heat energy is equal to its own (1.149). It is inconsistent with modern relativistic thermodynamics where it is assumed that the heat energy is proportional or exactly equal to the total body energy, and is greater when the body moves. That is interesting, especially because the ideas that I present here are completely in line with the classic thermodynamics and the theory of relativity. Hence there is a need for another careful consideration of the parameters that lead to disagreement. Among them, the most important pressure is.

In the classical sense, pressure creates a drum of molecules on a surface, from which physical effects come. Whether they bounce off or stick to the surface, the pressure produces the kinetic energy of the free-moving molecules, so it will not surprise us that the pressure depends on the density ρ of the pressure-generating gas or liquid, then the molecular velocity squared, v^2, and hence also from the relativistic coefficient γ from the previous equations. As the speed of the molecule can have different directions and intensities, the pressure of the same center can be different in different directions, and it is *vector* or even *tensor* before than *scalar*. For example, we know that for the lateral (transferal) pressure P_t, perpendicular to the direction of the flow (flux) of gas or liquid the velocity v and the density ρ states the Bernoulli[52] equation

$$P_t + \frac{1}{2}v^2\rho + g\rho z = \text{const}, \tag{1.159}$$

where g is the gravitational acceleration and z the height at which the flux rises. By increasing (decreasing) the squared velocity, this pressure decreases (increases), which means that P_t is not an invariant of motion. It is already an argument the pressure may not be scalar and Lorentz's invariant, which, by the way, is unusual in contemporary relativistic thermodynamics.

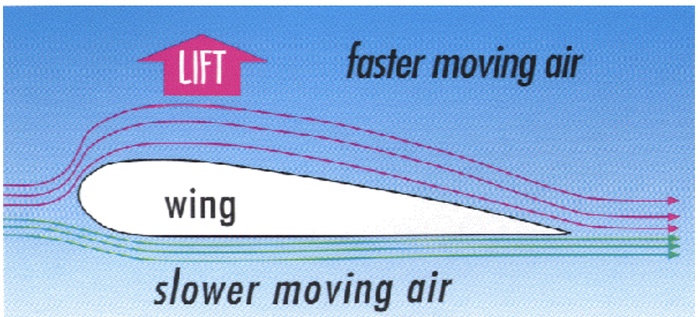


Figure 1.24: The air flows around the wing of the plane.

[52]Daniel Bernoulli (1700-1782), Swiss mathematician and physicist.

On the figure image 1.24 we see the application of Bernoulli's equation to the plane's flight. The airplane is moving at a high speed so the effect of the speed squad v^2 in formula (1.157) is as big as possible. The wing of the airplane is smooth and aerodynamic so that the air can easily slide around it, but it is such a form that the air path from the upper side is longer, and from that side, the air speed is higher. It creates a higher lateral pressure (lift) on the air stream than on the lower side of the wing. That is why the thrust from the bottom up is higher, which in the case of the horizontal flight plane is exactly in balance with the weight of the aircraft. In the more accurate description, it is also necessary to add a swirling air around the wing that helps the buoy.

The Bernoulli equation must also work in the case of the passage of two rooms A and B, which are shown below in the image 1.25. Between the rooms are openings for free passage of air. If in relation to the external observer both rooms move at the same speed, the room A right and the room B left, due to symmetry there is no movement of air between. However, what will see the observer who sits in the room A and who, according to a special theory of relativity, can consider that only the B room moves? Or vice versa, what will see the observer who is stationary in the room B and is equally right when he considers that only other room is moving?

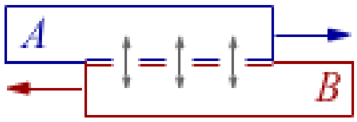


Figure 1.25: Rooms A and B in mutual movement.

Of course, an external observer who thinks that both rooms are moving equally must be right, so we conclude that there is no movement of air between the rooms. However, Bernoulli's lateral thrust would for the monitors from A have to cause the flow of air from his room to the room B, unless in the room B due to this point, the effects of relativity, no relative air suppression appears in the room B itself. This overpressure would have to be precisely so that it could prevent the movement of air between A and B.

Let's get this problem on the other side. In the same image 1.25, we continue to consider that the own observer is in the moving room B and that he in his room observes the pressure P_0 (in all directions), and this room is another observer looking from A who considers himself immobile. The pressure P is the action of the force vector $\mathbf{F}$ on the surface of area μ, that is the ratio of force perpendicular to the surface on unit of that surface.

However, the relative "parallel surface" to the direction of movement decreases proportional to γ^{-1}, while the "vertical surface" to the direction of movement does not change. If for the "own" force we take a product of mass and acceleration ($\mathbf{F}_0 = m\mathbf{a}$) and change the momentum in the unit of time ($\mathbf{F} = d\mathbf{p}/dt$) for "relative", as in equations (1.39), we will have the "relative" parallel and vertical component:

$$\mathbf{F}_{||} = \gamma^2 m\mathbf{a}_{||}, \quad \mathbf{F}_{\perp} = m\mathbf{a}_{\perp}, \tag{1.160}$$

So the appropriate pressure components are:

$$P_{||} = \gamma^2 P_0, \quad P_{\perp} = \gamma P_0. \tag{1.161}$$

So we see that in the room B there is a relative overpressure that could stop the suction effects of the Bernoulli equation. Bernoulli (transversal) vacuuming pressure P_t should, instead of (1.159), be exactly aligned with

$$P_t = P_0\gamma^{-1}, \quad \gamma^{-1} = \sqrt{1-\frac{v^2}{c^2}}, \tag{1.162}$$

to cancel the effect of the relative lateral overflow $P_\perp$ from (1.159).

The deviations due to the viscosity (contraction of the moving room and transfer the higher parallel pressure into the lateral) must be in balance with (1.159). Then there are no paradoxes in the image 1.25. This lateral overpressure is possible by closing the room than by the second (parallel) component of even greater relative overpressure. Due to the parallel pressure, the air in the room tends to get out, which would happen if there is no back and front walls of the room. In a seemingly similar situation, when we measure the temperature of the gas in motion, putting the thermometer directly in the power, the eventual closing of the room is of no such significance.

Because of the relative contraction of the length, only in the direction of the velocity $\mathbf{v}$, the volume of the moving room is

$$V = V_0\gamma^{-1}, \tag{1.163}$$

where V_0 is its own volume. The relative volume of the room decreases until the pressure rises. With new formulas (1.159) we will need a new interpretation of Boyle's law. Here it is still Ok for your own system to say: the product of pressure and volume of the gas is constant, but not for a relative. The product of relative volume and air pressure has two "components", parallel and perpendicular to the vector $\mathbf{v}$, in the following order:

$$VP_{||} = \gamma V_0 P_0, \quad VP_\perp = V_0 P_0, \tag{1.164}$$

where V_0 and P_0 are the volume and air pressure in the immobile room. When we look a little more closely, this new interpretation of pressure will prove to be better aligned with the tensor character of energy (1.136) than the usual one, but about that some other time.

Here too, we will not be much concerned about possible disagreements with ideal gas equations, for which in more realistic conditions one needs to look for better terms. The perfect or *ideal gas* is a theoretical gas consisting of point particles in random motion whose only interactions are perfect elastic collisions. One *mole* of an ideal gas has a volume of 22.71 liters under normal conditions, at a temperature of 273.15 Kelvin (zero Celsius) and under an absolute pressure of 10^5 Pascal (about one atmosphere). Under normal conditions, many gases (nitrogen, oxygen, hydrogen, and noble gases) behave like an ideal one. Other gases are more similar to ideal at higher temperatures and lower pressures, when potential energy among molecules becomes less important in relation to kinetic energy and when the distance between molecules is higher.

It should be known that the ideal gas model fails at lower temperatures or higher pressures when the intermolecular forces and distances become important. Also, this model is not applicable to many heavy gases. At lower temperatures, the pressure of the real gas is often significantly lower than in the ideal gas. For example, the ideal gas model does not foresee the transition from gaseous to liquid aggregate, common to real gases with a decrease in temperature. For the same reason, the relative ideal gas by increasing the speed becomes less and less ideal.

In an ideal gas, the relationship between pressure, volume, temperature and amount of gas is expressed by the ideal gas equation

$$PV = nR_uT, \tag{1.165}$$

where P is the absolute pressure (in Newton per square meter), V container volume (in meters cubic), n is the number of gas moles present, $R_u \approx 8.31$ J$\cdot$mol$^{-1}\cdot$K^{-1} is a universal gas constant, and T temperature (in degrees Kelvin). This formula is the strongest argument that the temperature of the gas in motion does not change. However, with a significant increase in relative velocity, the gas loses the properties of the ideal (it is compressed by the length, the mass of the molecule grows), so we cannot rely on the formula of ideal gases at that time.

If we ever accept the entropy and consequence treatment performed here, much of the relativistic thermodynamics will be more logical or simpler. Boltzmann's constant $k_B = R/N_A$ and further defines the quotient of the gas constant R_u and Avogadro[53] constant $N_A = 6.02 \times 10^{23}$ mol^{-1}. It remains consistent with the well-known classical thermodynamic formula, $PV = nRT = k_B NT$, where n is the number of the moles in substance, and N the number of gas molecules. For $n = 1$ mole, the number N is equal to the number of particles in one mole, i.e. Avogadro number. On the other hand, the kinetic theory gives an average pressure of the ideal gas $P = Nm\bar{v^2}/3V$, from where the average translational kinetic energy is $\frac{1}{2}m\bar{v^2} = \frac{3}{2}k_B T$, and it has three degrees of freedom (one $\frac{1}{2}k_B T$ for each dimension).

It is known in physics, *enthalpy* (H) is the sum of the internal energy (E) and the product of pressure (P) with the volume (V) of the thermodynamic system. Enthalpy is a feature of a thermodynamic system, independent of its history:

$$H = E + PV, \tag{1.166}$$

where E is the internal energy of the system, P pressure, and V volume. If energy changes according to the form (1.145), and for a product of pressure and volume we take the parallel component from (1.164), then the relative enthalpy

$$H = \gamma H_0, \tag{1.167}$$

where H_0 is its own. Then the relative enthalpy is greater than its own, in proportion to γ, which already differs it from the heat (1.149). However, because of the promoted vector nature of the pressure of this (1.164), and especially because of the known tensor nature of energy (1.136), things are even more complicated, actually harmonized, which we will later be discussed more.

Note that only the differences of the relative pressure components (1.161) give a new meaning to the realizations of coincidence. The same own pressure, for example, gas at rest, creates various relative realizations of random events and, accordingly, different realities to observers who move in relation to him in different inertial systems. This is in line with the new old principle of probability, but not with the determinism of mechanics.

1.3.5 Relative realities

Without the presence of force, nature should not have an objective uncertainty, nor should we need probability. We concluded this by considering the number of dimensions of the universe. However, one should not go so far to notice that without the physical forces there would be no different speeds of movement and that without force there would be no relative observers at all. Turning off forces means turning off chances, which means turning off the universe. Therefore, the principle of probability is much more general than it seems at first glance.

[53] Amadeo Avogadro (1776-1856), the Italian scientist.

Hence, the seemingly excessive assertion, the arbitrary body is where it is at a given moment because that position in that time is for the given body the most likely. Realizing the most probable random events, nature seems to want to preserve its uncertainties by allowing only the most certain coincidences to become information. Therefore, the consequence of the probability principle is the principle of information: spending the most probable saves uncertainty. This also leads us to conclude that the law of conservation of the amount of uncertainty stays for those events that are not realized into the information.

Imagine a man sitting on a bench in a park watching a nearby tree. He is in his own physical system in which, due to objective coincidence, a constant transformation of a piece of uncertainty into information is carried out. All of the new information which concerns him makes his present, the "now". The objectivity of randomness requires from nature to renounce of its uncertainty little by little, creating a layer by the layer of the present, continually creating matter, space and time, again and again. The bench is almost always the same, and the tree swings slightly due to the effect of air forces, and it altogether proves that nature always creates the same in the same given circumstances, due to the same given probabilities. The past of the aforementioned man arises by the accumulation of the layers of his present. So the time arises.

Because of their complexity and because of the laws of large numbers, macro-bodies seem to us as they can be free from the law of probability. Because we live in the macro-world and because of the presence of strict abstract laws of mathematics, we almost believe in the deterministic nature of the world. But even the largest bodies must be in a given condition at a given location, precisely because, from their own point of view, any other situation is less likely. When the observed probability would not be relative, all observers would be in the same place! They would be in a spatial conflict.

From the conclusion that time does not exist without the emergence of information, regardless of the above, follows the same law of conservation of the amount of uncertainty. We have seen that classic definitions of information are also in harmony with this law. Moreover, it follows from these definitions that the amount of information generated is exactly equal to the amount of uncertainty consumed. All the uncertainties involved converted into the material world of the given observer, his (proper) universe: time, space and substance.

If the principle of probability does not apply, let's say that less probable events are equally realized, for the observer from the bench would be usual to observe the tree in the park and in the next moment something quite different, then again the third thing very different from both previous. The principle of achieving the most probable events guarantees the maximum (possible) stability, the continuity of nature over time. The change of faith is possible by force, but this process is gradual. The force gives mass an acceleration, changing its speed. A relative observer of these changes will perceive at each level of the speed the corresponding information of the system (in which the data is stationary), lesser the information with higher the speed. The force removes the body from the observer's system, reducing the amount of loss of uncertainty of the proper universe, as if it goes onto the other side of the hill, reducing the damage in uncertainty from the viewer's viewpoint. In this sense, force is even more supportive of the principle of probability.

Uncertainty prior to the realization of a random event becomes information after. The total amount of "uncertainty plus information" of a particular closed system of any random event is constant, it does not change during the production of the given information. It is the constant of the system, so it is the constant of the universe. However, nor the information produced neither quantity are not the same for the each, the proper or the relative observer because otherwise, the universe would quickly waste its uncertainty. There are much more relative observers than proper, but the relative perceives the smaller production of information than the proper, for each random realization, so the (proper) universe lasts.

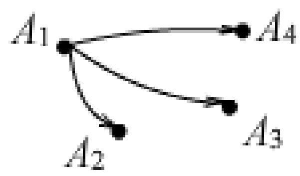


Figure 1.26: Proper and three relative observations from A_1.

In the figure 1.26 we see one of our proper observers A_1 which is relative to systems A_2, A_3 and A_4. These four different systems represent $n = 4$ proper observers, each of which is still $n - 1 = 3$ times relative, so we have the total

$$V_2^n = n(n-1) = 4 \cdot 3 = 12 \tag{1.168}$$

relative. The number of variations V_2^n grows with the square of the number n. In the case of equal production information among all of them, large numbers of relative systems would reduce the duration of the space relative to each individual observer. The time, which is actually generated information, would be quickly wasted. It might happen that the universe burns in a blinking, as in the next case.

Example 1.3.2 (Vacuum)**.** *Creation and annihilation of particle and antiparticle.*

Explanation. According to Heisenberg's uncertainty about energy and time, $\Delta E \Delta t \ge \hbar$, in the vacuum can always be some particles, matter and antimatter, of arbitrarily large energies, up to ΔE, only if their duration is no longer than Δt. All the events of such a particle are their own (proper) for one observer from the vacuum, so if the greater energy of such a particle means a greater number of events, then it means a shorter duration. If such a creation created a whole space like ours universe, it would, due to its enormous energy, more precisely because of the huge number of systems it contained, have to disappear in an extremely short interval of time. For the inhabitants of such space, if they are relative one to another, our flicker would be its eternity. □

Creation with the annihilation of a pair of particle-antiparticle in the case of a vacuum is in accordance with previous considerations for some more details. All "galaxies" and all other "bodies" formed as particles of the new "spaces" are from an observer in our vacuum – equal. They all are proper, so the duration (Δt), therefore the spending such a "universe" for an (external) observer is inversely proportional to the total content (ΔE). That is the result also expected in Heisenberg's principle of uncertainty. Furthermore, this "universe" would be, say, either matter or antimatter, and it may not have any other symmetry, which is unexpected. For example, the "universe" mustn't be electrically neutral. In such a consideration, we do not assume anything about the (inner) "inhabitants" there.

From the above example, we see how important relative observers are for space, for which time has slowed down relative to one's own. They observe a slower production of information and thus contribute to reducing the consumption of uncertainty and reducing the spending of the universe itself. We have already seen that parsimony is the essence of the principle of information, and as entropy is part of the information, the same applies to it. Why the principle of entropy (the physical system spontaneously tends to increase entropy) and the principle of information (information stinginess) are the consequences of the same principle of probability (realization of the most probable) can be understood in the following ways.

Spontaneous growth of entropy made arrangement disorderly; the physical system strives to a faceless, amorphous state with little to say. The goal of entropy growth is "not giving information". We have already mentioned this, and now let's see that it is also proved by the *Maxwell's demon*[54], in the image 1.27. The partition divides the room into two sections, A and B, in which at the beginning the air pressure is equal, it is equal the number of fast (red) and slow (blue) molecules. We imagine that there is a demon that controls the information so that it passes only the fast molecules from A to B, and only slow from B to A, when the molecule reaches the partition. Over time, all the fastest molecules will be found in the B section, and all of the slow in the A section. However, then, from the state of greater entropy (disorder), we get into the state of less entropy (order). We get the paradox that the author and generations of physicists were unable to explain afterward.

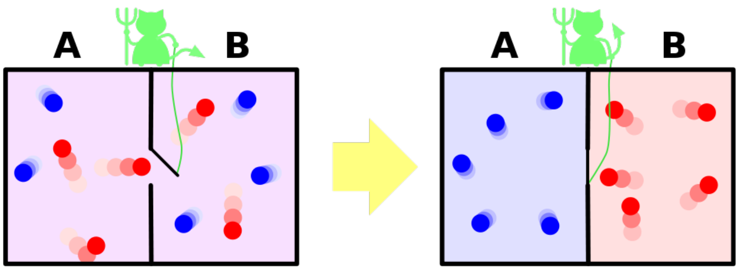


Figure 1.27: Maxwell's demon.

Now, on the way we promote space, time, and matter as forms of information, the Maxwell demon case becomes clearer. By giving to the demon the ability to handle disorder, irrespective of whether this consumption requires spent of energy, we will give the demon the ability to consume information. However, nature will not allow it, precisely because of its stinginess (the principle of information). It does not give information except when is really needed. Therefore, the principle of entropy is a consequence of the principle of information, both of which are the consequences of the principle of probability.

Let's go back to the unusual properties of time by explaining them using the example *Schrödinger's cat*[55], in the image 1.28. If the cat was a small quantum system for which the law of large numbers is not important, then any information about the cat would significantly change its material state. Let's say that we have a cat in the box for which we do not know exactly whether it is alive or dead, and then we open the box to get such information. For a small quantum system, any such information is a huge thing. It changes a non-substantial state of the cat's uncertainty into a substance. Taking out the uncertainty of the previous state of the cat, due to the law of conserving the quantity (uncertainty plus information), the conditions of the cat become more certain, and even at the time of opening the box it was all the same whether we would find a living or dead cat inside, after opening it is no longer. Establishing that the cat is, for example, "alive", its previous condition of

[54]Maxwell's demon: `https://en.wikipedia.org/wiki/Maxwell%27s_demon`
[55]Schrödinger's cat: `https://en.wikipedia.org/wiki/Schr%C3%B6dinger%27s_cat`

"living" is also defined. If by opening the box was found that the cat was "dead", then its previous condition would become "dead" too. At the time of the opening, it was objectively all the same, because until the opening of the box, the cat objectively was neither "alive" nor "dead".

Figure 1.28: Schrödinger's cat.

Only after the discovery of quantum entanglement, we have become able to understand the deeper nature of the relativity of time that Schrödinger's cat override. The same is the basis of Heisenberg's famous sentence on measurement: "only after measurement, the path of a particle gets the real meaning." So, not only do we know something about the particle by measurement, but the particles reality in the past emerged by the act of measuring in the present. By giving information about itself, it reveals itself to the material world, weakens its vagueness, and thus defines its material past. We have the same thing in the here called creation of the reality of the free particle by two consecutive interactions.

With the theory of "material information" outlined here, the mentioned Heisenberg's explanation of measurement gets a deeper meaning. By realizing a random event a part of the uncertainty turns into information, which makes the quantum system more certain. In this way, every macro-system becomes more certain too, but due to the small share of individual information in the new reality, it remains seemingly unchanged.

The free particle after the interaction A travels as very uncertain towards interaction B. This indeterminacy is permitted by Heisenberg's uncertainty principle in earlier more specified, now we can say more realistic, interactions. Within the limits of these uncertainties are respected the physical laws, the conservations of energy, impulse, spin, and for the same reason the variations of its future are possible. Therefore, the further defining of the A interaction is possible even after the interaction B. It looks weird if we think that the time flow is some kind of a precise deduction, determinism, which is not. The past is a deposited present which, due to both, the conservation of information and objective coincidence, is partly kept and partly changed. So we arrive at the conclusion that *real* is just what has *communicated* with something we have communicated with.

The principle of information is also austerity in the consumption of uncertainty, but also a denial of communication. Restriction the ability to exchange matter, interaction and information means that there is no communication with everyone. Photon communicates with a charged particle, electron through a photon with the same or another charge, but always in the manner described by the pattern of information disclosure and loss of uncertainty. Let's check it out on the Feynman diagrams.

In the figure 1.29 we have, for example, on the left, first electron e^- that emits a *virtual photon* γ, which encounters another electron e^- on the right. We accept Feynman's description that there is a constant field of virtual photons around each charged particle and on the figure is one that subtracts the momentum (or mass, energy, spin) from the first electron and hands it to the other. Conservation laws are respected and electrons are rejected. This leads to the refusal of the same charges. When the right electron has opposite course of the time, the left is still the same first electron, but on the right is positron e^+, and the same exchange of momentum becomes the reason for the mutual attraction of the now contrary charges.

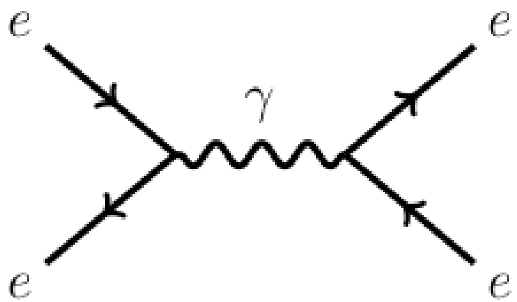


Figure 1.29: Feynman's diagram.

We further clarify Feynman's description, first by means of information. Sooner or later, some virtual photon γ communicates (exchange information) between two charged particles, as in the given figure 1.29. This increases the certainty of the space between them and the particles turn into uncertainty. For two homonymous particles (electron-electron) the uncertainty is outside, and for different (electron-positron) it is inside.

We mentioned "two passers-by" (now an electron and a positron) whose run of the time goes in *opposite directions*, and therefore could not communicate in the usual (say, typical) way. When the first asked question he expects an answer later in his time, which in the time of the second means giving the answer then listening to the question. Due to the absence of a typical exchange of information, both come to the conclusion that there is greater uncertainty in the space between them and they are attracted. Note that this explanation of the rejection (for the nuance) is different from Feynman's. In this dominates the "desire" for the greater uncertainty, that is, the principle of information.

The same story has more implications, which complement the wider picture. For example, how is it possible to consider the real entities without communicating? The answer is in atypical communications. Even in the macro-world, we meet people who are so clever or know each other so well that one will give an (exact) answer to a question that the other has not yet come to ask. We will also get the same considering the interlocutors as very stupid but in the appropriate simple situations.

Such are elementary particles. When they are completely "stupid", the smallest particles cannot imagine anything, organize or plan, but have to respond to "the first ball". It does not matter to them at that time what time the opponents will be able to "communicate". The third example is communication behind the hill. In the case of increasing relative speeds, the emissions of information appear to be less and their whole reality as if they were going to an inaccessible side. In each of these cases, we have an interaction with something that interacts with given and we are on the same side of reality.

Modern cosmology holds that the universe was created by a big bang and has been expanding ever since 14 billion years ago. By the expansion the universe is diluted and at some time it became transparent. The light that would then set off toward us had to go beyond the larger and larger distance, so due to the rapid expansion of the cosmos, the light could remain on the same side of the range. Simply put, so much space is created between the long gone light path so it can never reach us. Also, the light that moves from us in depth due to the expansion of the universe can never cross the boundaries of visible space. But there are also matter out there, which communicated with something with which we commutated, and so it is real for us.

No matter how strange it looks to us, this indirect reality is found with the explanation of the twin paradoxes, in the part about slowing the time of an inertial system that leads to lagging behind in the past or in the future of the observers. A particle that goes from us inertially has relatively slowed down the flow of time and is constantly in our past as more as it is further from us. As long as we see that particle we communicate with it, and equally, it can communicate with some other parts of our system. Therefore, our past is (indirectly) real with us.

The similarly we have with a particle that is inertial approaching to us, but then we have the conclusion that our future is also realistic, that it is the only possible one, which is in line with our conclusions on the dimensions of space-time in the case when there is no effect of force. When a particle approaches us non-inertial, because the force acts on it, it then changes the speeds and interacts with different variants of our future. Accordingly, different versions of our future are equally real to us, in the form of atypical (indirect) communications.

All this indicates that the force changes the both, the probability and the perception of the reality of the observer. Force is able to change the primary reality with the second one and replace the secondary one. Because of the universality of these phenomena, it is equally true to say vice versa, that probability distortion means the presence of force. The accumulation of air molecules closer to the floor of the room in the gravitational field is a sign of the presence of force, as is the reverse; the (uniform) probabilities are disturbed by the presence of force.

From the point of entropy, the interpretation of the Feynman diagram 1.29 is similar to the previous one. The existence of photons between the electrons interrupts the uniformity of the space, reduces the mess and thus the entropy, so the electrons spontaneously move into the space with larger entropy. It is *rebounded.* In the absence of typical communication, the electron and the positron work the same, but that now means they are *attracted.* In the case of the expansion of the universe, the principle of entropy (the physical system spontaneously goes into a state of greater entropy) is obvious. The new formulas (1.148) and (1.149) only improve the official explanation. The first ($T = \gamma T_0$) means that more galaxies (due to higher velocity and higher γ) are warmer. This is also logical because they are deeper in the past when the universe was denser. The other ($Q = Q_0$) means that the heat energy is unchanged. Now the law of energy conservation remains important because the force (1.42) that drives this expansion of the universe is ubiquitous and constant from its onset[56]. In simple terms, it is like a gravitational force of a planet that does not need fuel, adding the new energy. It is not necessary to "burn" the gravity of the earth to keep it appealing.

This "foreign" reality is also found in cosmic proportions. Let's imagine three galaxies A, B and C distant millions of light years moving away from one another. The force (1.42) which removes C from A, observed from A, grows in time and grows with γ^2. It has the direction of the vector $\overrightarrow{AC}$, and its perpendicular component does not change. Similarly happens with the C galaxy viewed from B, except that it runs under the force of $\overrightarrow{BC}$ that is not equal to the first. Because there are different forces on C from A and B, *different realities* are observed. Both are some realities because A sees C, but also see B that sees C. Already in inertial motion, we had different relative views of the same occurrence (information), but now we have much greater differences, similar to those of the particle that approaches us with speed changes. In particular, even the spontaneous development of entropy on C is not the same from the point of view of A and B.

This brings us back to those absurd explanations from the beginning of this book. According to the principle of probability, events flow from the past to the future so that the physical system always occupies mostly the most probable states. Accordingly, the real-time flow would only be the one in which this system would go backward again, in the given conditions, to the most probable states. It's the same with entropy. If each step of one flow of time was into the state of maximum entropy, then the one who walked backward over time would have to go through, under given conditions, again into the conditions of the greatest entropy.

[56]The supposed force $F_0 \neq 0$ that rejects the galaxies does not have to be constant.

1.4 Symmetry

Here are the mathematical supplements that are implied in the previous section of the book. These are mostly known topics supplemented by my ideas. The first is the derivation of Lorentz transformations from Einstein's principles, the other leads to this transformation under geometric symmetry, and the third is the formal basis for the expansion of space-time to several time dimensions. They are also the announcement of further work.

1.4.1 Lorentz transformation

In the Cartesian's Rectangular Coordinate System $Oxyz$, on the figure 1.30, all three coordinates (abscissa x, ordinate y, and applicant z) are equally calibrated and mutually orthogonal straight lines with a common outcome at the point O. In the plane Oxy is a point A with coordinates $(x, y, 0)$, and on the high z above A is the point B with coordinates (x, y, z).

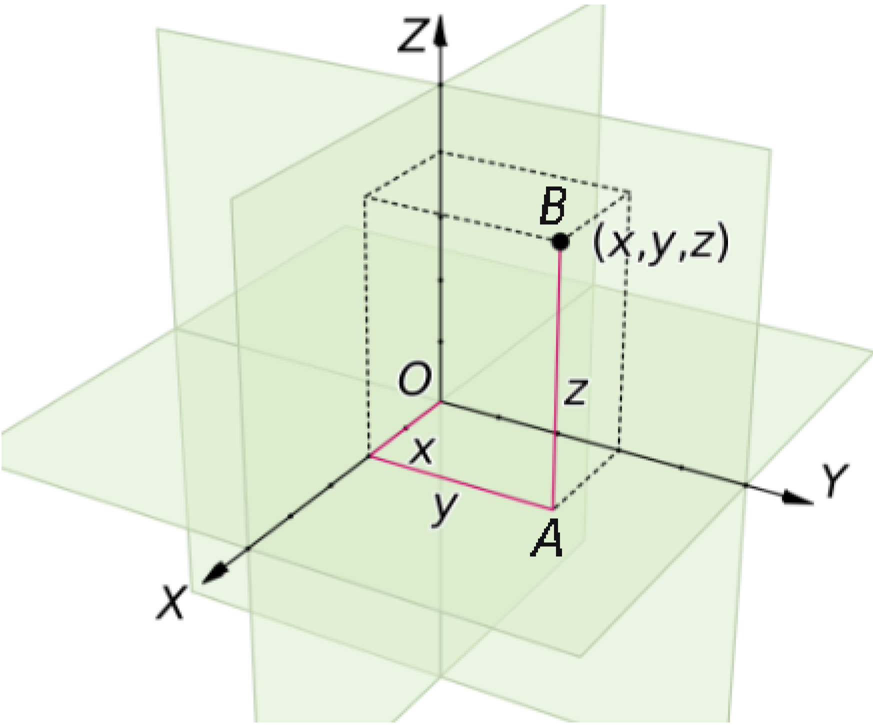


Figure 1.30: Cartesian coordinate system.

According to Pythagorean Theorem, we find the following:

$$\overline{OA}^2 = x^2 + y^2, \quad \overline{OB}^2 = \overline{OA}^2 + \overline{AB}^2,$$

$$l^2 = x^2 + y^2 + z^2, \quad l = \overline{OB}. \tag{1.169}$$

When the cuboid of the given image 1.30 is translated for the vector $\overrightarrow{OO'}$, then the point O of the cuboid cross into the point O', and the point B in the point B', the previous equation becomes

$$(\Delta l)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2, \tag{1.170}$$

where $\Delta\xi$ are the lengths of the *cuboid* moved parallel with the coordinates, in the order $\xi = x, y, z$, and Δl is the diagonal, the length of the displacement $\overrightarrow{OO'}$.

After this timeless, mathematical translation, we use new tags for the lengths of the square pages, but all the appropriate lengths remain the same. When we already say that the system $Oxyz$ has been moved to the $O'x'y'z'$ by translation, it is easy to add that this shift depends on the time elapsed t, so that it happens at a constant speed defined by the vector $\mathbf{v} = (v_x, v_y, v_z)$. This means, along the coordinate $\xi \in \{x, y, z\}$ for the time period Δt the system O' is moved for the length $v_\xi \Delta t$ in relation to the system O. If at the starting point ($t = 0$) the two systems coincide ($O \equiv O'$), then at an arbitrary time t the coordinate transformation is valid:

$$x' = x - v_x t, \quad y' = y - v_y t, \quad z' = z - v_z t, \quad t' = t. \tag{1.171}$$

These are the Galilean transformations, where we write the fourth equation only to emphasize that in both systems, a fixed O and a moving O', time runs at the same speed. There is no significant loss of generality when we put the motion only along abscise. Then the total speed is $v = v_x \neq 0$, while the other two components are zero ($v_y = v_z = 0$).

Galilean transformations have been enhanced by Lorentz's, due to the Michelson-Morley experiment from 1887, who found that *light moves at the same velocity in a vacuum* approximately by speed $c = 3 \times 10^8$ m/s, regardless of the source speed. Einstein took the knowledge of M-M experiments in his work in 1905, which is now known as the special theory of relativity (bibliography [3]). His first principle was the relativity of motion, which says the physical laws of the system in which the observer is stationary, are independent on uniform rectilinear movement.

In short, Einstein's principles of relativity are: 1. all uniform rectangular movements are equal; 2. the speed of light in a vacuum does not depend on the speed of the source.

Example 1.4.1. *Derive the Lorentz's transformation from Einstein's principles.*

Solution. Einstein's relativity for the fourth coordinate uses ct and this is the path that the light travels during time t. For the O' system moving at speed v relative to the system O along the abscise, in the most general case we can put:

$$x' = \gamma(x - \beta\, ct), \quad y' = y, \quad z' = z, \quad ct' = A\, ct - Bx, \tag{1.172}$$

where γ, β, A, B are unknown numbers yet to be determined. The first and fourth coordinates are functions dependent on the mutual speed v of the movement of the two systems, and possibly only, the speed of light c. These transformations would become Galileo's (1.171) if there were $\gamma = 1$, $\beta = v/c$, $A = 1$ and $B = 0$.

When an object rests in O' at the position $x' = 0$ it moves at constant speed v by abscissa of the system O, so $x = vt$, $\beta = v/c$ and $x' = \gamma(x - \frac{v}{c}\, ct)$. According to the principle of relativity, the inverse transformation between O' and O must have the same form, but with the opposite sign of velocity, so we find $x = \gamma(x' + \frac{v}{c}\, ct')$ for the same coefficient γ. In this case, $t = x/c$ whenever $t' = x'/c$, then with the change in the previous equations, and by multiplying we get $xx' = \gamma^2(1 - v^2/c^2)xx'$. Hence, for the first equation:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \beta = \frac{v}{c}. \tag{1.173}$$

The first coefficient is also called Lorentz's factor and we have already mentioned it. The equation in (1.172) that defines the transformation of time can be obtained from the conditions $x' = ct'$ and $x = ct$ by substitute to the previous spatial coordinates from where $ct' = \gamma(ct - \beta x)$. Accordingly, Lorentz's transformations are:

$$x' = \gamma(x - \beta\, ct), \quad y' = y, \quad z' = z, \quad ct' = \gamma(ct - \beta x), \tag{1.174}$$

where γ and β are given with (1.173). □

Because of the second Einstein principle, in both systems measured, the light travels some distance for given time at the same speed, which means $\Delta l/\Delta t = \Delta l'/\Delta t' = c$, and hence we find that the term

$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2(\Delta t)^2 \tag{1.175}$$

is invariant of the Lorentz transformations. This is a generalized Pythagorean Theorem for the 4-dim spacetime of the special theory of relativity.

Example 1.4.2. *Prove that the expression (1.175) is the invariant of Lorentz transformations.*

Solution. From the (1.172) we get:

$$\Delta x' = \gamma(\Delta x - \beta\, c\Delta t), \quad c\Delta t' = \gamma(c\Delta t - \beta\Delta x),$$

$$(\Delta x')^2 = \gamma^2(\Delta x - \beta\, c\Delta t)^2, \quad c^2(\Delta t')^2 = \gamma^2(c\Delta t - \beta\Delta x)^2,$$

$$\begin{cases} (\Delta x')^2 = \gamma^2[(\Delta x)^2 - 2\beta\,\Delta x\, c\Delta t + \beta^2\, c^2(\Delta t)^2] \\ c^2(\Delta t')^2 = \gamma^2[c^2(\Delta t)^2 - 2\beta\, c\Delta t\,\Delta x + \beta^2\,(\Delta x)^2]. \end{cases}$$

Hence, by subtraction:

$$\begin{aligned}(\Delta x')^2 - c^2(\Delta t')^2 &= \gamma^2[(\Delta x)^2 - c^2(\Delta t)^2] - \gamma^2\beta^2[(\Delta x)^2 - c^2(\Delta t)^2] \\ &= \gamma^2(1-\beta^2)[(\Delta x)^2 - c^2(\Delta t)^2] \\ &= (\Delta x)^2 - c^2(\Delta t)^2.\end{aligned}$$

By adding the interval squares to the other two coordinates, we get $(\Delta s')^2 = (\Delta s)^2$, and this is what we needed to prove. □

The *wave equation* that follows from Maxwell's papers on electromagnetism is the

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = 0, \tag{1.176}$$

where $\mu_0 = 4\pi \times 10^{-7}$ N/A^2 is magnetic, and $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m the electric constant of *vacuum permeability*.

Example 1.4.3. *Prove that the expression (1.176) is Lorenz invariant.*

Solution. Using (1.172) we get:

$$\frac{\partial\Psi}{\partial x} = \frac{\partial\Psi}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial\Psi}{\partial y'}\frac{\partial y'}{\partial x} + \frac{\partial\Psi}{\partial z'}\frac{\partial z'}{\partial x} + \frac{\partial\Psi}{\partial ct'}\frac{\partial ct'}{\partial x} = \gamma\frac{\partial\Psi}{\partial x'} - \beta\gamma\frac{\partial\Psi}{\partial ct'},$$

because $\frac{\partial y'}{\partial x} = \frac{\partial z'}{\partial x} = 0$, and $\frac{\partial x'}{\partial x} = \gamma$ so $\frac{\partial ct'}{\partial x} = -\beta\gamma$. The second derivative is the derivative of the first derivative, so similarly we get in a row:

$$\frac{\partial^2\Psi}{\partial x^2} = \frac{\partial}{\partial x'}\left(\frac{\partial\Psi}{\partial x}\right)\frac{\partial x'}{\partial x} + \frac{\partial}{\partial y'}\left(\frac{\partial\Psi}{\partial x}\right)\frac{\partial y'}{\partial x} + \frac{\partial}{\partial z'}\left(\frac{\partial\Psi}{\partial x}\right)\frac{\partial z'}{\partial x} + \frac{\partial}{\partial ct'}\left(\frac{\partial\Psi}{\partial x}\right)\frac{\partial ct'}{\partial x} =$$

$$= \frac{\partial}{\partial x'}\left(\gamma\frac{\partial\Psi}{\partial x'} - \beta\gamma\frac{\partial\Psi}{\partial ct'}\right)\frac{\partial x'}{\partial x} + \frac{\partial}{\partial ct'}\left(\gamma\frac{\partial\Psi}{\partial x'} - \beta\gamma\frac{\partial\Psi}{\partial ct'}\right)\frac{\partial ct'}{\partial x},$$

$$\frac{\partial^2\Psi}{\partial x^2} = \gamma^2\frac{\partial^2\Psi}{\partial(x')^2} - 2\beta\gamma^2\frac{\partial^2\Psi}{\partial x'\,\partial ct'} + \beta^2\gamma^2\frac{\partial^2\Psi}{\partial(ct')^2}. \tag{1.177}$$

For the second and the third coordinate we find:

$$\begin{cases} \frac{\partial\Psi}{\partial y} = \frac{\partial\Psi}{\partial x'}\frac{\partial x'}{\partial y} + \frac{\partial\Psi}{\partial y'}\frac{\partial y'}{\partial y} + \frac{\partial\Psi}{\partial z'}\frac{\partial z'}{\partial y} + \frac{\partial\Psi}{\partial ct'}\frac{\partial ct'}{\partial y} = \frac{\partial\Psi}{\partial y'} \\ \frac{\partial\Psi}{\partial z} = \frac{\partial\Psi}{\partial x'}\frac{\partial x'}{\partial z} + \frac{\partial\Psi}{\partial y'}\frac{\partial y'}{\partial z} + \frac{\partial\Psi}{\partial z'}\frac{\partial z'}{\partial z} + \frac{\partial\Psi}{\partial ct'}\frac{\partial ct'}{\partial z} = \frac{\partial\Psi}{\partial z'}, \end{cases}$$

then similarly for another derivations:

$$\frac{\partial^2\Psi}{\partial y^2} = \frac{\partial^2\Psi}{\partial(y')^2}, \quad \frac{\partial^2\Psi}{\partial z^2} = \frac{\partial^2\Psi}{\partial(z')^2}. \tag{1.178}$$

By the same procedure we get the first partial derivative for the fourth coordinate:

$$\frac{\partial\Psi}{\partial ct} = \frac{\partial\Psi}{\partial x'}\frac{\partial x'}{\partial ct} + \frac{\partial\Psi}{\partial y'}\frac{\partial y'}{\partial ct} + \frac{\partial\Psi}{\partial z'}\frac{\partial z'}{\partial ct} + \frac{\partial\Psi}{\partial ct'}\frac{\partial ct'}{\partial ct} = -\beta\gamma\frac{\partial\Psi}{\partial x'} + \gamma\frac{\partial\Psi}{\partial ct'},$$

because $\frac{\partial y'}{\partial ct} = \frac{\partial z'}{\partial ct} = 0$, and $\frac{\partial x'}{\partial ct} = -\beta\gamma$ so $\frac{\partial ct'}{\partial ct} = \gamma$. Hence:

$$\frac{\partial^2\Psi}{\partial(ct)^2} = \frac{\partial}{\partial x'}\left(-\beta\gamma\frac{\partial\Psi}{\partial x'} + \gamma\frac{\partial\Psi}{\partial ct'}\right)\frac{\partial x'}{\partial ct} + \frac{\partial}{\partial ct'}\left(-\beta\gamma\frac{\partial\Psi}{\partial x'} + \gamma\frac{\partial\Psi}{\partial ct'}\right)\frac{\partial ct'}{\partial ct}$$

$$\frac{\partial^2\Psi}{\partial(ct)^2} = \beta^2\gamma^2\frac{\partial^2\Psi}{\partial(x')^2} - 2\beta\gamma^2\frac{\partial^2\Psi}{\partial x'\,\partial ct'} + \gamma^2\frac{\partial^2\Psi}{\partial(ct')^2}. \tag{1.179}$$

Substituting and after the arrangement we have

$$\frac{\partial^2\Psi}{\partial(x')^2} + \frac{\partial^2\Psi}{\partial(y')^2} + \frac{\partial^2\Psi}{\partial(z')^2} - \frac{1}{c^2}\frac{\partial^2\Psi}{\partial(t')^2} = 0, \tag{1.180}$$

and this was to be proven. □

Finding this invariant (1.180), Lorentz[57] before Einstein came to the transformations that are called after him. However, he did not know how to interpret them. Lorentz made the correct conclusion that the M-M experiment shows a contraction of the length in the direction of movement, but it did not see it in any broader context. Nevertheless,

[57] Hendrik Lorentz (1853-1902), Dutch physicist.

he developed the electromagnetic theory of light, studied the diffraction of all crystals in binary gases and was the first that calculated the splitting of the singlet's of the spectral lines into three components in the magnetic field. In 1902, Lorentz received the Nobel Prize in Physics for Works of Electromagnetic Theory of Light.

From Lorentz's transformation, a special theory of relativity emerged, and the way Einstein explained them and how he further interpreted evenly straightforward movements left further traces in physics.

1.4.2 Rotations

What is the same in different structures in geometry is called *symmetry* and in physics *law of nature*. Examples of geometric symmetry are mirror reflections, axial symmetries, central symmetries, translation, each of which can be obtained by rotation. From the significance of rotations in geometry follows its importance for physics.

Figure 1.31: Reflections of Nature on the Water.

The axial symmetry of the points in the plane is the mapping we construct when we draw normal (vertical) straight lines to the axis of symmetry through the given points, and then transfer them to the same distance on the other side of this axis. For example, any three points that make up the triangle of one orientation will be mapped by the axial symmetry in three points that made the congruent triangle of the opposite orientation. Therefore, their matching cannot be obtained by moving a given triangle at the plane of symmetry, but this is possible by a 3-dim rotation of the given plane around its axis of symmetry. Similarly, the mirror symmetry that reflected the left orientation to the right and vice versa could have a 4-dimensional rotation of 3-dimensional space around the plane of the mirror.

The central symmetry in the plane has one common point of that plane that is the center of any line segment beginning with the original point and ends with its copy. It symmetry can be obtained by rotating this plane around the central point for the straight angle. Translation, moving points for a given vector, can be obtained by using two central symmetries, that is, with two rotations. All this is learned in high schools, so we will not be more specific about this. Just note that due to unusual geometric connections with physics, it's not a surprise that Lorentz's transformations can be represented by rotation.

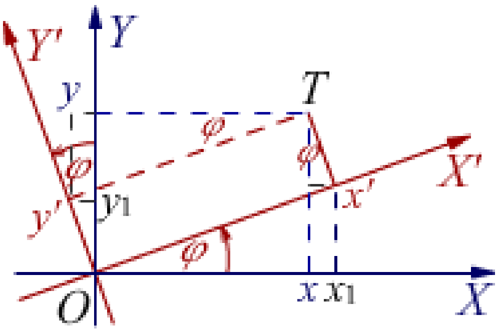


Figure 1.32: Rotation of coordinates for the angle φ.

On the figure 1.32, the orthogonal Cartesians coordinate system OXY is rotated in the same plane to the system $OX'Y'$ for the orientated angle φ around the origin. The same point T of the plane in the two systems has the coordinates $T(x,y)$ and $T(x',y')$. From the picture we read:

$$\overline{Ox_1} = \overline{Ox} + \overline{xx_1}, \quad \overline{Oy_1} = \overline{Oy} - \overline{yy_1},$$

$$x'\cos\varphi = x + y'\sin\varphi, \quad y'\cos\varphi = y - x'\sin\varphi.$$

Hence direct and inverse transformations:

$$\begin{cases} x = x'\cos\varphi - y'\sin\varphi \\ y = x'\sin\varphi + y'\cos\varphi, \end{cases} \quad \begin{cases} x' = x\cos\varphi + y\sin\varphi \\ y' = -x\sin\varphi + y\cos\varphi. \end{cases} \tag{1.181}$$

These transformations can also be written matrix, say direct:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}, \tag{1.182}$$

Or shorter $\mathbf{r} = \hat{R}\mathbf{r}'$, where the components of the vectors $\mathbf{r}$ and $\mathbf{r}'$ or the rotation matrix $\hat{R}$ are obvious.

If we take the components of these vectors to Lorentz's abscissa $x_1 = x$ and the time axis $x_4 = ict$, we obtain Lorentz's direct and inverse rotation:

$$\begin{cases} x_1 = x_1'\cos\varphi - x_4'\sin\varphi \\ x_4 = x_1'\sin\varphi + x_4'\cos\varphi, \end{cases} \quad \begin{cases} x_1' = x_1\cos\varphi + x_4\sin\varphi \\ x_4' = -x_1\sin\varphi + x_4\cos\varphi. \end{cases} \tag{1.183}$$

After substitution $\varphi = i\phi$, $i = \sqrt{-1}$ and changing to sine and cosine hyperbolic, these common becomes hyperbolic Lorentz rotations:

$$\begin{cases} x = x'\cosh\phi - ct'\sinh\phi \\ ct = -x'\sinh\phi + ct'\cosh\phi, \end{cases} \quad \begin{cases} x' = x\cosh\phi + ict\sinh\phi \\ ct' = x\sinh\phi + ct\cosh\phi. \end{cases} \tag{1.184}$$

Relations between them are known relationships between ordinary and hyperbolic (hyperbolic functions have introduced Italian mathematician Vincenzo Riccati, 1707-1775) trigonometric functions: $\sin\varphi = -i\sinh(i\varphi)$ and $\cos\varphi = \cosh i\varphi$. Putting $x' = x_1' = 0$ we find the speed of the prime system in the unprimed:

$$\beta = \frac{v}{c} = \frac{x}{ct} = \frac{ict'\sinh\phi}{ct'\cosh\phi} = i\tanh\phi = \tan\varphi. \tag{1.185}$$

The relation between the hyperbolic tangent (tanh) and the ordinary tangent (tan) is $\tanh\phi = -i\tan(i\phi) = -i\tan\varphi$, and hence mentioned $\beta = \tan\varphi$.

The basic hyperbolic trigonometric functions are:

$$\begin{cases} \sinh\phi = \frac{e^{\phi}-e^{-\phi}}{2} & \cosh\phi = \frac{e^{\phi}+e^{-\phi}}{2} \\ \tan\phi = \frac{e^{\phi}-e^{-\phi}}{e^{\phi}+e^{-\phi}} & \coth\phi = \frac{e^{\phi}+e^{-\phi}}{e^{\phi}-e^{-\phi}}. \end{cases} \tag{1.186}$$

These are in row sine, cosine, tangent, and cotangent hyperbolic. That they really lead to relations (1.184) we see from the development of ordinary and hyperbolic trigonometric functions in the Maclaurin series:

$$\begin{cases} \sinh\phi = \phi + \frac{\phi^3}{3!} + \frac{\phi^5}{5!} + \frac{\phi^7}{7!} + \dots, & \cosh\phi = 1 + \frac{\phi^2}{2!} + \frac{\phi^4}{4!} + \frac{\phi^6}{6!} + \dots, \\ \sin\varphi = \varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \frac{\varphi^7}{7!} + \dots, & \cos\varphi = 1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} - \frac{\varphi^6}{6!} + \dots, \end{cases} \tag{1.187}$$

and putting the mentioned substitution $\varphi = i\phi$, $i = \sqrt{-1}$. From the definitions (1.186) we easily find $\cosh^2\phi - \sinh^2\phi = 1$, and then the other basic identities:

$$\begin{cases} \tan\phi = \sinh\phi/\cosh\phi & \coth\phi = \cosh\phi/\sinh\phi \\ \sinh 2\phi = 2\sinh\phi\cosh\phi & \cosh 2\phi = \cosh^2\phi + \sinh^2\phi, \end{cases} \tag{1.188}$$

or say $\tanh 2\phi = 2\tanh\phi/(1+\tanh^2\phi)$, when we already mention double angles. All other identities of ordinary trigonometry are also similar to hyperbolic. For example, the addition formula for the sum and difference of angles of hyperbolic functions are:

$$\begin{cases} \sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y \\ \cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y \\ \tanh(x \pm y) = (\tanh x \pm \tanh y)/(1 \pm \tanh x \tanh y). \end{cases} \tag{1.189}$$

However, there are also differences.

Lorentz transformations (1.182) aggregate x and ct axis, opposed to the rotation (1.183) where the angle between abscise and ordinate remains the same after rotation – right angle. In the image 1.33, we see this effect.

When the speed of the received system is 60 percent of the light velocity, it will be $v = 1.8 \times 10^8$ m/s, or $\beta = v/c = 0.6$. Then the angle of rotation is $\varphi = \arctan 0.6 \approx 31°$. Both axes, x and ct, by Lorentz's "rotation" are inclined to the angle φ toward the symmetric line of the quadrant (dashed blue line) to coincide in the case that $v = c$. Events at the same place of the moving system $O'x'ct'$ are on the same (dashed) parallels with ct'-axis, while concurrent events are located on some dashed parallels with x'-axis. In the immovable system $Oxct$, events at the same place would be on the parallels with ct-axes, while concurrent events would be on parallel to x-axes. From the image 1.33, we see that in two systems O and O' neither events "in the same place" nor "simultaneous" are not the same.

As the O' system goes off, events in it go further into the past of the O system, and vice versa, the fixed system is simultaneous with further and further events from the past of the mobile system. Conversely, when an observer from the O' system approaches to the observer in O, they are each other in the nearer and nearer future. This primarily means that the relative time flows slower than the proper, and then leads to an appearance that we call the "twins paradox". We have already discussed these effects, and we also mentioned the following properties of complex numbers.

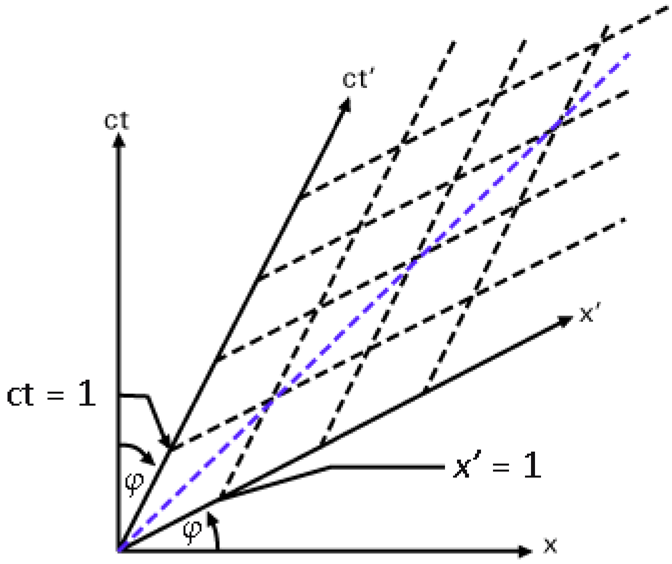


Figure 1.33: Lorentz's transformations tighten the axes.

Rotations (1.181) can also be obtained using complex numbers. The complex number $z = x+iy \in \mathbb{C}$ with its real $\Re(z) = x$ and the imaginary part $\Im(z) = y$, which are real numbers ($x, y \in \mathbb{R}$), in the complex plane is the point z with abscise and ordinate, respectively:

$$x = r\cos\alpha, \quad y = r\sin\alpha. \tag{1.190}$$

The modulus or intensity of a complex number is a real positive number $r = |z| = \sqrt{x^2+y^2}$. The complex number argument is the angle $\alpha = \angle xOz = \arctan(y/x)$. When given a unit complex number $z_0 = x_0 + iy_0$, such that $|z_0| = 1$, where $x_0 = \cos\varphi$ and $y_0 = \sin\varphi$, then the product is:

$$zz_0 = (x+iy)(z_0+iy_0) = r(\cos\alpha + i\sin\alpha)(\cos\alpha_0 + i\sin\alpha_0) =$$

$$= r[(\cos\alpha\cos\alpha_0 - \sin\alpha\sin\alpha_0) + i(\sin\alpha\cos\alpha_0 + \cos\alpha\sin\alpha_0)],$$

$$zz_0 = r[\cos(\alpha+\alpha_0) + i\sin(\alpha+\alpha_0)], \tag{1.191}$$

Where the addition formulas for cosine and sine are applied. Therefore, the multiplication by the unit complex number $z_0 = x_0 + iy_0$ of the complex number $z = x + iy$ represents the rotation of the number z around the origin of the complex plane for the argument, the angle $\alpha_0 = \arctan(y_0/x_0)$, of number z_0. By multiplying the same z_0, the entire complex plane rotates for the same angle α_0.

Because of this additivity of the arguments of complex numbers and the known additivity of the exponents in multiplying the degrees of functions of the same bases, it turns out that

$$z = x + iy = re^{i\alpha}, \tag{1.192}$$

where $r = \sqrt{x^2+y^2}$ and $\tan\alpha = y/x$. The proof can be made by the development of the exponential function (here $e^{i\alpha}$) in the Maclaurin Series. This, otherwise known complex number record using the exponential function, with the Euler number $e = 2.71828\ldots$, throws a new light on here previously derived gravitational field equations (1.137). It opens the possibility of observing the gravitational field as a continual space of a set of small, *infinitesimal inertial systems* of coordinates analogous to Lorentz's.

Instead of the entire coordinate axis x and ct, we will use their small (infinitesimal) parts dx and cdt, whereby Lorentz's "abscise", the analogy of the direction of motion of special relativity, will always be radial in direction of the gravitational field. That is why we will mark that direction differently, say with dr instead of dx. Other coordinates, such as $d\varphi$ and $d\theta$, which in the case of the spherical system $Or\varphi\theta$ are orthogonal to the direction of expansion of the gravitational field, gravity will not deform. Namely, if all the coordinates were altered equally, the result of the action at the plane would be again the Euclidean plane space in which there is no gravitational force. On the other hand, this asymmetrical, radial change will also be the cause of the known gravitational effect of lateral compression of the body, perpendicular to radial field strengths.

Here we mean spherical coordinates as in the image 1.34. If we designate them with *covariant* (lower) indices, it will be in the order $x_1 = r$, $x_2 = \varphi$ and $x_3 = \theta$, with associated time coordinate $x_4 = ict$, where i is imaginary unit, and c light speed. It is the same order in the case of labeling with the contravariant (upper) indices x^μ, $\mu = 1, 2, 3, 4$.

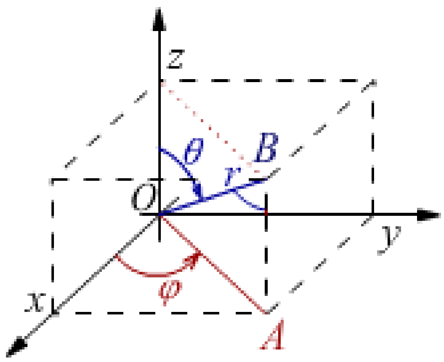


Figure 1.34: Spherical coordinates $Or\varphi\theta$.

From the image, see $r = \overline{OB}$, and:

$$\begin{cases} x = \overline{OA}\cos\varphi, & \overline{OA} = r\sin\theta, \\ y = \overline{OA}\sin\varphi, & z = r\cos\theta, \end{cases} \tag{1.193}$$

then we find the transformations:

$$\begin{cases} x = r\cos\varphi\sin\theta \\ y = r\sin\varphi\sin\theta \\ z = r\cos\theta, \end{cases} \quad \begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \varphi = \arctan\frac{y}{x} \\ \theta = \arccos\frac{z}{r}. \end{cases} \tag{1.194}$$

Derivations of these coordinates are:

$$\begin{cases} dx = \cos\varphi\sin\theta\,dr - r\sin\varphi\sin\theta\,d\varphi + r\cos\varphi\cos\theta\,d\theta \\ dy = \sin\varphi\sin\theta\,dr + r\cos\varphi\sin\theta\,d\varphi + r\sin\varphi\cos\theta\,d\theta \\ dz = \cos\theta\,dr - r\sin\theta\,d\theta, \end{cases} \tag{1.195}$$

then by squaring and substituting (by addition) to the corresponding relativistic expression (1.175), after the arranging we get

$$ds^2 = dr^2 + r^2\sin^2\theta\,d\varphi^2 + r^2\,d\theta^2 - c^2dt^2. \tag{1.196}$$

This is the *relativistic interval* for infinitesimals in spherical coordinates $Or\varphi\theta$ for space-time.

Recall that a metric can be obtained by analyzing the vertical drop (1.128) as

$$ds^2 = e^{2GM/rc^2}\,dr^2 + r^2\sin^2\theta\,d\varphi^2 + r^2\,d\theta^2 - e^{-2GM/rc^2}\,c^2dt^2, \tag{1.197}$$

and then by approximating the term

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2\sin^2\theta\,d\varphi^2 + r^2\,d\theta^2 - \left(1 - \frac{2GM}{rc^2}\right) c^2dt^2, \tag{1.198}$$

which is equal to the Schwarzschild metric (1.137). In general, the gravitational field at a given point at a given moment is a force that is a vector and forces the body to "free fall". If at this point we set the infinitesimal coordinate system so that r-axis has the direction of the force effect, then we can observe the (variables) plane $O\,r\,ct$ within which the transformations are analogous to Lorentz's are valid. This follows from the symmetry that we expect to get from geometry into physics. This is a new idea confirmed by the following theorem.

We suppose the spherical coordinate system $Or\varphi\theta$ with the center of gravity in the source, and we observe the central symmetric field, so that any two such systems have the same angles φ and θ.

Theorem 1.4.4. *Differential transformations are given*

$$\begin{cases} dr = \chi\,dr' + i\gamma^{-1}\sqrt{1-\gamma^2\chi^2}\,cdt' \\ cdt = i\gamma\sqrt{1-\gamma^2\chi^2}\,dr' + \gamma^2\chi\,cdt', \end{cases} \tag{1.199}$$

where i is imaginary unit, $\gamma = (1 - 2GM/rc^2)^{-1/2}$, and $\chi \in \mathbb{C}$ arbitrary parameter. Show that these are general transformations that transform the metric of gravity (1.198) into an inertial metric (1.196).

Proof. Ignore the coordinates that do not affect the result ($d\varphi = d\varphi'$ and $d\theta = d\theta'$) and start from the system:

$$\begin{cases} dr = \alpha_{rr}\,dr' + \alpha_{rt}\,cdt' \\ cdt = \alpha_{tr}\,dr' + \alpha_{tt}\,cdt', \end{cases} \tag{1.200}$$

where α_{mn} depends only on r. Substituting in the abbreviated term (1.198), we get:

$$ds^2 = \gamma^2\,dr^2 - \gamma^{-2}\,c^2dt^2 =$$

$$= \gamma^2(\alpha_{rr}\,dr' + \alpha_{rt}\,cdt')^2 - \gamma^{-2}(\alpha_{tr}\,dr' + \alpha_{tt}\,cdt')^2$$

$$= (\gamma^2\alpha_{rr}^2 - \gamma^{-2}\alpha_{tr}^2)\,dr'^2 + 2(\gamma^2\alpha_{rr}\alpha_{rt} - \gamma^{-2}\alpha_{tr}\alpha_{tt})\,dr'\,cdt' + (\gamma^2\alpha_{rt}^2 - \gamma^{-2}\alpha_{tt}^2)\,c^2dt'^2.$$

Equating this interval with $dr'^2 - c^2dt'^2$, we obtain a system of equations:

$$\begin{cases} \gamma^2\alpha_{rr}^2 - \gamma^{-2}\alpha_{tr}^2 = 1, \\ \gamma^2\alpha_{rr}\alpha_{rt} - \gamma^{-2}\alpha_{tr}\alpha_{tt} = 0, \\ \gamma^2\alpha_{rt}^2 - \gamma^{-2}\alpha_{tt}^2 = -1. \end{cases}$$

These are three equations with four unknown alpha which means that we have an arbitrary parameter, let it be $\alpha_{rr} = \chi \in \mathbb{C}$. The first equation gives $\alpha_{tr} = \pm i\gamma\sqrt{1-\gamma^2\chi^2}$. From the third equation, we get $\alpha_{rt} = \pm i\gamma^{-1}\sqrt{1-\gamma^{-2}\alpha_{tt}^2}$, which is included in the middle:

$$\gamma^2\chi(\pm i\gamma^{-1}\sqrt{1-\gamma^{-2}\alpha_{tt}^2}) - \gamma^{-2}(\pm i\gamma\sqrt{1-\gamma^2\chi^2})\alpha_{tt} = 0,$$

$$\gamma\chi\sqrt{1-\gamma^{-2}\alpha_{tt}^2} = \gamma^{-1}\alpha_{tt}\sqrt{1-\gamma^2\chi^2},$$
$$\chi^2(\gamma^2-\alpha_{tt}^2) = \alpha_{tt}^2(\gamma^{-2}-\chi^2),$$
$$\chi^2\gamma^2 = \alpha_{tt}^2\gamma^{-2},$$

From where $\alpha_{tt} = \pm\gamma^2\chi$ and $\alpha_{rt} = \pm i\gamma^{-1}\sqrt{1-\gamma^2\chi^2}$. When we take only the above signs, we get the required system. □

Some my colleges were skeptical about this result, that a flat (Euclidean) metric of Minkowski spacetime can be transformed into a metric of curved gravity space, so I support it with the additional explanations.

When we look better at transformations (1,200), we see that by multiplying the first by γ and the other with $i\gamma^{-1}$ we get:

$$\begin{cases} \gamma\,dr = \gamma\chi\,dr' + \sqrt{1-\gamma^2\chi^2}\,icdt' \\ \gamma^{-1}\,icdt = -\sqrt{1-\gamma^2\chi^2}\,dr' + \gamma\chi\,icdt'. \end{cases} \tag{1.201}$$

Substituting $dy_1 = \gamma dr$, $dy_4 = \gamma^{-1}icdt$ and $dx_1 = dr'$, $dx_4 = icdt$, then:

$$\cos\varphi = \gamma\chi, \quad \sin\varphi = \sqrt{1-\gamma^2\chi^2}. \tag{1.202}$$

they become ordinary rotations, because

$$\begin{cases} dy_1 = \cos\varphi\,dx_1 + \sin\varphi\,dx_4, \\ dy_4 = -\sin\varphi\,dx_1 + \cos\varphi\,dx_4. \end{cases} \tag{1.203}$$

By changing $dy_0 = \gamma^{-1}cdt$, $dx_0 = cdt$ and $\varphi = i\phi$ we get

$$\begin{cases} dy_1 = \cosh\phi\,dx_1 + \sinh\phi\,dx_0, \\ dy_0 = \sinh\phi\,dx_1 + \cosh\phi\,dx_0, \end{cases} \tag{1.204}$$

which are hyperbolic rotation, or Lorentz transformation. Certainly

$$ds^2 = (\gamma dr)^2 - (\gamma^{-1}cdt)^2 = (dr')^2 - (cdt')^2, \tag{1.205}$$

where $\gamma = 1/\sqrt{1-2GM/rc^2}$ is taken from (1.198), but it could also be taken from (1.197) or some other similar metric.

In order for the hyperbolic transformations (1.202) to be "real" (equivalent to Lorentz's), it is enough $|\gamma\chi| \le 1$. Then from (1.204) and $\beta = i\tanh\phi$ we obtain

$$\gamma\chi = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \frac{v}{c}, \tag{1.206}$$

which reveals the nature of the second coefficient, the parameter χ. It is a number without dimensions that determines the initial speed of the body in free fall in the gravitational field. In other words, the number χ defines the height from which the body began to fall. For example, in the case of a zero initial velocity in infinity ($v \to 0$ when $r \to \infty$), we calculate $v^2 = 2GM/r$ from $ma = F_g$, and hence $\gamma\chi = 1$. Transformations (1.199) become $\gamma\,dr = dr'$ and $\gamma^{-1}cdt = cdt'$ which means that the proper values $dr_0 = dr'$ and $dt_0 = dt'$ of stationary point give:

$$dr = dr_0\sqrt{1-\frac{2GM}{rc^2}}, \quad dt = \frac{dt_0}{\sqrt{1-\frac{2GM}{rc^2}}} \tag{1.207}$$

relative to the falling body. And this result is known.

The relativistic interval symmetry (1.205) indicates a possible formal replacement of r with ict. No matter how strange it was it would be superficial, for example, to look at it with a mockery and not explore the possibility of replacing space coordinates with time. The transformations (1.201) or (1.199) also are indicating to such inversion. But it is not the subject of this, but an another story.

1.4.3 Quaternions

We know that a set of real numbers is a subset of a set of complex numbers ($\mathbb{R} \subset \mathbb{C}$) and that a set of complex numbers is sufficient to solve each polynomial equation (the basic algebraic theorem). In this way, due to the impossibility of accurate solutions in real terms, we understood the need of quantum mechanics for complex numbers. For those who believe in deeper links between mathematics and physics, this could have been the cause of discovering the mathematics of the quantum world, as it is.

With this book, I am at a similar turning point. An analysis of the dimensions of space-time proves that there are more than four, that is, the time dimensions have as many as spatial we know, and the question arises is there a mathematical apparatus that could describe them all. The theme of this subtitle is to remind us that such an apparatus really exists.

One of the simpler upgrades of a set of complex numbers is a set of 2×2 matrices. The unit matrix is the

$$\hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{1.208}$$

As we know, it is the only neutral in multiplying other matrices so that for each matrix $\hat{A}$ (of the same type), $\hat{I}\hat{A} = \hat{A}\hat{I} = \hat{A}$. Other basic matrices will be divided into two groups, according to the sign on the right side of the square equality:

$$\hat{\sigma}^2 = \pm\hat{I}. \tag{1.209}$$

By solving this equation for the plus sign (+) we obtain Pauli's[58] matrices:

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \hat{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{1.210}$$

By solving the above equation for the sign minus (−) we get *quaternions*:

$$\hat{\sigma}_4 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad \hat{\sigma}_5 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \hat{\sigma}_6 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}. \tag{1.211}$$

Matrices are numbered such that:

$$\begin{cases} \hat{\sigma}_1\hat{\sigma}_2 = i\hat{\sigma}_3 & \hat{\sigma}_2\hat{\sigma}_3 = i\hat{\sigma}_1 & \hat{\sigma}_3\hat{\sigma}_1 = i\hat{\sigma}_2 \\ \hat{\sigma}_4\hat{\sigma}_5 = \hat{\sigma}_6 & \hat{\sigma}_5\hat{\sigma}_6 = \hat{\sigma}_4 & \hat{\sigma}_6\hat{\sigma}_4 = \hat{\sigma}_5, \end{cases} \tag{1.212}$$

where i is imaginary unit. In addition, there is

$$\hat{\sigma}_3\hat{\sigma}_4 = \hat{\sigma}_6\hat{\sigma}_1 = i\hat{I}. \tag{1.213}$$

[58]Wolfgang Pauli (1900-1958), Austrian-Swiss-American theoretical physicist.

Otherwise, the matrix multiplication is not commutative.

Besides the usual use of these matrices, Pauli's are associated with the spin of fermions, and all six for the representation of rotations, the quaternion matrix can also be used to distinguish different *imaginary dimensions* of time.

The unit matrix together with the three Pauli (or quaternion) makes a complete set of matrices of type 2×2. This means that for an arbitrary matrix $\hat{A}$ there are unique four numbers x, y, u, v such that

$$x\hat{\sigma}_m + y\hat{\sigma}_n + u\hat{\sigma}_p + v\hat{I} = \hat{A}, \tag{1.214}$$

when m, n, p are indexed row 1, 2 and 3 or indexes are 4, 5 and 6. Such four matrices can form a 4-dimensional *vector space*. On the other hand, this means that with the three matrices of Pauli (or quaternion), a single matrix cannot be obtained by linear operations (by multiple additions).

Example 1.4.5. *Show that Pauli's matrices with a unit make a complete set.*

Solution. Start from (1.213):

$$x\hat{\sigma}_1 + y\hat{\sigma}_2 + u\hat{\sigma}_3 + v\hat{I} = \hat{A},$$

$$x\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + y\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + u\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + v\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

$$\begin{pmatrix} u+v & x-iy \\ x+iy & -u+v \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

$$\begin{cases} u+v=a \\ x-iy=b \\ x+iy=c \\ -u+v=d, \end{cases} \Rightarrow \begin{cases} x = \frac{b+c}{2} \\ y = i\frac{b-c}{2} \\ u = \frac{a-d}{2} \\ v = \frac{a+d}{2}. \end{cases}$$

Therefore, for arbitrary given numbers a, b, c, d there are always unique numbers x, y, u, v, and this is what is to be shown. □

Example 1.4.6. *Show that quaternions with a unit matrix make a complete set.*

Solution. Using the above, we get in the order:

$$x\hat{\sigma}_4 + y\hat{\sigma}_5 + u\hat{\sigma}_6 + v\hat{I} = \hat{A},$$

$$x\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} + y\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + u\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + v\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

$$\begin{pmatrix} ix+v & y+iu \\ -y+iu & -ix+v \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

$$\begin{pmatrix} x & y \\ u & v \end{pmatrix} = \begin{pmatrix} -i\frac{a-d}{2} & \frac{b-c}{2} \\ -i\frac{b+c}{2} & \frac{a+d}{2} \end{pmatrix}.$$

For arbitrary given numbers a, b, c, d there are unique x, y, u, v, which should have been shown. □

Formal 4-dimensional space-time can also be obtained from three Pauli matrices plus $\hat{\sigma}_4$, that is, with three Pauli and linear combinations of quaternions that must contain $\hat{\sigma}_4$. Such a quaternion combination mark with $\hat{\sigma}_0$ and call it a time dimension. Due to the compulsory presence of $\hat{\sigma}_4$, and optional $\hat{\sigma}_5$ and $\hat{\sigma}_6$, after choosing Pauli's matrices for the space base, we see that there is some asymmetry in the choosing of quaternion matrix as time dimensions. Moreover, in the opposite case, when we chose the quaternion matrices for the base of the space, for the fourth, time dimension, we could take only the unit matrix.

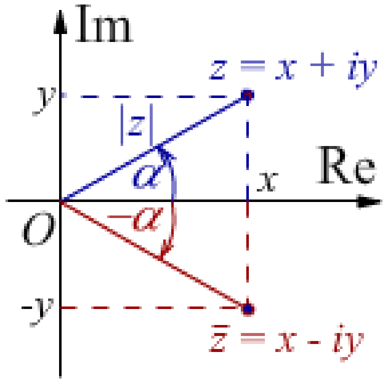


Figure 1.35: Complex plane $\mathbb{C}$.

Using this picture, we understood ordinary rotations, and now we continue this story in a little more detail. The distance from the source O to the complex number z, to the point z, is the modulo of the complex number z which is the real number $|z|$. The angle between the positive direction of the real axis (abscissa) and the oriented line segment Oz is the argument of the complex number z which is the real number $\arg(z)$. From the picture 1.35 we read easily:

$$|z| = \sqrt{x^2 + y^2} = \rho, \quad \arg(z) = \arctan\frac{y}{x} = \alpha. \tag{1.215}$$

Reverse transformations are:

$$x = \rho\cos\alpha, \quad y = \rho\sin\alpha. \tag{1.216}$$

Therefore, the complex number $z = x + iy$ can be written:

$$z = \rho(\cos\alpha + i\sin\alpha). \tag{1.217}$$

This is the representation of the number z in polar coordinates.

Theorem 1.4.7. *For all complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ is valid:*

1. $z_1z_2 = (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$;
2. $|z_1z_2| = |z_1||z_2|$;
3. $\arg(z_1z_2) = \arg(z_1) + \arg(z_2)$.

Proof. 1. By direct multiplication, due to distribution laws, we get:

$$z_1z_2 = (x_1 + iy_1)(x_2 + iy_2) = x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2,$$

whereby due to the law of commutation, after grouping followed the required equality.

2. From the previous expression we find:

$$z_1 z_2 = (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2) = \rho\left(\frac{x_1x_2 - y_1y_2}{\rho} + i\frac{x_1y_2 + y_1x_2}{\rho}\right), \tag{1.218}$$

where $\rho = \sqrt{(x_1x_2 - y_1y_2)^2 + (x_1y_2 + y_1x_2)^2}$. Note that it is

$$\left(\frac{x_1x_2 - y_1y_2}{\rho}\right)^2 + \left(\frac{x_1y_2 + y_1x_2}{\rho}\right)^2 = 1,$$

which means that $\rho = |z_1 z_2|$, and the fractions are $(x_1x_2 - y_1y_2)/\rho = \cos\alpha$ and $(x_1y_2 + y_1x_2)/\rho = \sin\alpha$, where $\alpha = \arg(z_1z_2)$.

3. Let us show that the angle α is the sum of the angles $\alpha_1 = \arg(z_1)$ and $\alpha_2 = \arg(z_2)$. From:

$$z_1 = x_1 + iy_1 = \sqrt{x_1^2 + y_1^2}\left(\frac{x_1}{\sqrt{x_1^2 + y_1^2}} + i\frac{y_1}{\sqrt{x_1^2 + y_1^2}}\right), \tag{1.219}$$

it follows that there is an angle α_1 such that:

$$\cos\alpha_1 = \frac{x_1}{\rho_1}, \quad \sin\alpha_2 = \frac{y_1}{\rho_1}, \quad \rho_1 = \sqrt{x_1^2 + y_1^2}, \tag{1.220}$$

because the $\cos^2\alpha_1 + \sin^2\alpha_1 = 1$. Similarly we find for z_2. After that, we use otherwise known trigonometric formulas:

$$\begin{aligned}
\cos(\alpha_1 + \alpha_2) &= \cos\alpha_1\cos\alpha_2 - \sin\alpha_1\sin\alpha_2 \\
&= \frac{x_1x_2}{\rho_1\rho_2} - \frac{y_1y_2}{\rho_1\rho_2} = \frac{x_1x_2 - y_1y_2}{\sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2}} \\
&= \frac{x_1x_2 - y_1y_2}{\sqrt{x_1^2x_2^2 + x_1^2y_2^2 + y_1^2x_2^2 + y_1^2y_2^2}} \\
&= \frac{x_1x_2 - y_1y_2}{\sqrt{(x_1^2x_2^2 - 2x_1x_2y_1y_2 + y_1^2y_2^2) + (x_1^2y_2^2 + 2x_1x_2y_1y_2 + y_1^2x_2^2)}} \\
&= \frac{x_1x_2 - y_1y_2}{\sqrt{(x_1x_2 - y_1y_2)^2 + (x_1y_2 + x_2y_1)^2}} = \frac{x_1x_2 - y_1y_2}{\rho}, \\
\cos(\alpha_1 + \alpha_2) &= \cos\alpha.
\end{aligned}$$

We also find $\sin(\alpha_1 + \alpha_2) = \sin\alpha_1\cos\alpha_2 + \cos\alpha_1\sin\alpha_2 = \sin\alpha$. □

These were somewhat unusual evidence of generally known attitudes. It is also unusual the prove of the following theorem.

By replacing $i \to -i$, the complex number $z = x + iy$ is changed into *conjugate* $\bar{z} = x - iy$. In a complex plane, the conjugate complex numbers z and $\bar{z}$ are axially symmetric with respect to the real axis, as seen in the picture 1.35.

Theorem 1.4.8. *For all complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ is valid:*

1. $\bar{z}_1 z_2 = (x_1x_2 + y_1y_2) + i(x_1y_2 - y_1x_2)$;

2. $\Re(\bar{z}_1 z_2) = |z_1||z_2|\cos\beta$;

3. $\Im(\bar{z}_1 z_2) = |z_1||z_2|\sin\beta$;

where $\beta = \angle z_1 O z_2$ is angle between oriented lengths Oz_1 and Oz_2.

Proof. 1. It follows with the change of $y_1 \to -y_1$ from the previous, 1 of the theorem 1.4.7.

2. Apply the cosine rule to the triangle Oz_1z_2:

$$|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos\beta.$$

Hence:

$$|z_1||z_2|\cos\beta = \frac{1}{2}(|z_1|^2 + |z_2|^2 - |z_1 - z_2|^2) =$$

$$= \frac{1}{2}[(x_1^2 + y_1^2) + (x_2^2 + y_2^2) - (x_1 - x_2)^2 - (y_1 - y_2)^2]$$

$$= \frac{1}{2}[2x_1x_2 + 2y_1y_2] = \Re(\bar{z}_1z_2),$$

$$|z_1||z_2|\cos\beta = \Re(\bar{z}_1z_2),$$

which was to be proved.

3. Use the trigonometric identity $\cos^2\beta + \sin^2\beta = 1$, that is:

$$|z_1|^2|z_2|^2\sin^2\beta = |z_1|^2|z_2|^2 - |z_1|^2|z_2|^2\cos^2\beta =$$

$$= (x_1^2 + y_1^2)(x_2^2 + y_2^2) - (x_1x_2 + y_1y_2)^2$$

$$= x_1^2y_2^2 - 2x_1x_2y_1y_2 + y_1^2x_2^2$$

$$= (x_1y_2 - y_1x_2)^2 = \Im^2(\bar{z}_1z_2),$$

$$|z_1||z_2|\sin\beta = \Im(\bar{z}_1z_2),$$

what is required. □

In analogy to the vectors, we can define the *scalar product* and the intensity of the *vector* product of given complex numbers, with:

$$z_1 \cdot z_2 = \Re(\bar{z}_1z_2), \quad |z_1 \times z_2| = \Im(\bar{z}_1z_2). \tag{1.221}$$

By using these definitions, we join the definitions [59] *composite sum* and *composite difference* of complex numbers, respectively:

$$z_1 \oplus z_2 = \frac{1}{2}(\bar{z}_1z_2 + z_1\bar{z}_2), \quad z_1 \ominus z_2 = \frac{1}{2}(\bar{z}_1z_2 - z_1\bar{z}_2), \tag{1.222}$$

For which it is easy to verify that equality applies:

$$z_1 \oplus z_2 = \Re(\bar{z}_1z_2), \quad z_1 \ominus z_2 = i\Im(\bar{z}_1z_2). \tag{1.223}$$

The first term (sum) resembles "real product" of complex numbers (the scalar product) in the mathematical literature, but in the book [1] it can be seen that in the general case (which is normally implied) this is not true.

[59] Names that I have temporarily used.

1.5 Opinions

The raw version of the text of the book, on March 2017, has greatly improved by the lecturers and reviewers (in Serbian) listed at the beginning. This assistance was significant and much better than expected, so I thank them again. Their statements and opinions follow in the same order as they came.

Dragana Galić

This book links math, physics, chemistry, and philosophy. Some of the ideas I hear for the first time, but they are accurately mathematically proved.

What does that tell us?

That the author is probably on a good path to something new, unexplored ...

The book can be used by students of natural and technical sciences, professors and all those who want to become acquainted with the application of mathematics in other sciences. I think the readers will enjoy it, happily!

Dragana Galić, prof. Mathematics,
Gimnazija Banja Luka, June 6, 2017.

Aleksandra Radić

While many basic concepts and phenomena are imply in physics, and many of them have not yet been clarified, this book gives us a new way of explaining some of them, which largely fits into already solid foundations of areas such as thermodynamics and the theory of relativity, but gives them a new dimension and unexpected innovations.

What is the information and how it arises, what is its connection with space-time, as well as the uncertainty of events around us and entropy – are just some of the answers that will be revealed in an interesting and instructive way in this book.

Interestingly, the phenomenon of "stinging" of nature on information is noted, which, together with the principle of entropy and probability, logically clarifies puzzling phenomena such as Maxwell's demon and Schrödinger's cats, as well as the interaction of particles and the expansion of the universe.

Uncertainty – unknown, and information – known, first goes to another, and while nature prefers the first, the probability points to the other, but there are rules that regulate it, and they are that their totality is a constant in the universe. However, like so many, it's also relative. Everyone else perceives the same information differently from its own position.

Only some of the interesting features in thermodynamics, but also the mechanics, gravity, cosmology that originate from these interpretations of nature are both the nature of the pressure (but also the components of the force) and the heat that is placed under the magnifier in well-known examples of its demonstration and throws new light on Relativity of these quantities and phenomena. The entire story is eventually linked to a coherent whole, which unites many areas of physics, in many respects in a new, yet elegant and completely logical way.

Aleksandra Radić, prof. Physics,
Gimnazija Banja Luka, June 13, 2017.

Zoran Rajilić

The author, who quite well understands physics, sees the unity of natural phenomena through probability, information, and entropy. Yet I perceive this text more as an art than as a science.

Viewed strictly from the angle of physics, it is very difficult to follow frequent crossings from the area to the area. For example, on pages where Maxwell's demon, Schrödinger's cat, and Feynman's diagram was explained, it should be more careful to build links to these very different topics.

Dr. Zoran Rajilić, Physics
Faculty of Natural Science and Mathematics
Banja Luka, June 23, 2017.

* * *

The next observation from my fellow a computer engineer is as extensive as it is current, that I had to state it completely. It examines the basic idea of this book, an objective randomness, explaining it as an axiom as it is, something that can neither be proved nor disputed, and that is why the elaboration of the same hypothesis in the exposed physics is at the border of (mathematical) fantasy. However, it as well shows that it and also another, already recognized physics that starts from not accepting objective coincidence, is also a fantastic one.

About randomness

The whole concept of physics promoted by the author in his work is based on what he calls the principle of probability as well as the concept of objective randomness. The logic of the author seems to me to be flawless, although I admit that I am not completely clear with the notion of objective randomness. Therefore, I would try to present here some ideas about the notion of coincidence, of course not mine, but of authors whom many, today consider the highest authority in this field of research. This is Gregory Chaitin, an American mathematician, and more details about him interested can be found on Wikipedia.

From the numerous Chaitin works, I decided to present here the work "Randomness and Mathematical Proof" published in 1975. It should be noted that what Chaitin actually deals with in almost all his works is to explore what he calls "the limits of mathematics".

The paper begins with the following words singled out as the motto: "Although randomness can be precisely defined and can even be measured, a given number cannot be proved to be random. This enigma establishes a limit to what is possible in mathematics."

Almost everyone has an intuitive notion of what a random number is. For example, consider these two series of binary digits:

$$01010101010101010101$$
$$01101100110111100010$$

The first is obviously constructed according to a simple rule; it consists of the number 01 repeated ten times. If one were asked to speculate on how the series might continue, one could predict with considerable confidence that the next two digits would be 0 and 1. Inspection of the second series of digits yields no such comprehensive pattern. There is no obvious rule governing the formation of the number, and there is no rational way to guess the succeeding digits. The arrangement seems haphazard; in other words, the sequence appears to be a random assortment of 0's and 1's.

The second set of binary digits is generated by throwing a coin 20 times and writing 1 if the result is "head" and 0 if it is a "letter". Throwing coins is a classic procedure for producing a random number, and it might be thought that exactly such an origin of a series confirms that it is coincidental. But that's not the case. Throwing a coin 20 times can produce any of 2^{20} (just over a million) binary sequences, and each of them has exactly the same probability. Therefore, there should be no greater surprise if a string is obtained with an obvious pattern than if a series appears that looks random; Each represents an event with a probability of 2^{-20}. If the origin of the probabilistic event is the only coincidence criterion, then both sets must be considered random, and of course all others, since the same mechanism can generate all possible sequences. This conclusion is not helpful in distinguishing a coincidental from the orderly one.

The second set of binary digits is generated by throwing a coin 20 times and writing 1 if the result is "head" and 0 if it is "tail". Throwing coins is a classic procedure for producing a random number, and it might be thought that exactly such an origin of a series confirms that it is coincidental. But that's not the case. Throwing a coin 20 times can produce any of 2^{20} (just over a million) binary sequences, and each of them has exactly the same probability. Therefore, there should be no greater surprise if a string is obtained with an obvious pattern than if a series appears that looks random; Each represents an event with a probability of 2^{-20}. If the origin of the probabilistic event is the only coincidence criterion, then both sets must be considered random, and of course all others, since the same mechanism can generate all possible sequences. This conclusion is not helpful in distinguishing a coincidental from the orderly one.

It is clear that a more sensible definition of coincidence is needed, which will not contradict the intuitive concept of a number without a pattern. Such a definition was developed relatively recently, in the 1960s. It does not take into account the origin of the number, but it depends exclusively on the characteristic of the sequence of digits. The new definition allows us to describe the properties of a random number more precisely than previously possible and to establish a randomness hierarchy. The possibilities of this definition may be of even greater concern to its limitations. In particular, the definition cannot help determining, except in very special cases, whether or not a set of digits, such as our second set, is or is not given, is really random or just seems random. This limitation is not a failure in definition; This is a consequence of a subtle but fundamental anomaly in the foundations of mathematics. It is closely related to the famous theorem which was put forward and proved by Kurt Gödel in 1931, which became known as Gödel's theorem on incompleteness. And this theorem and recent discoveries concerning the nature of coincidence help to define the boundaries that limit certain mathematical methods.

Algorithmic Definition

The new definition of coincidence has its origins in information theory, a science that developed largely after the Second World War, which studies the transmission of messages. Let's assume that we want to send someone a message containing the tables of trigonometric functions values. We can simply translate numbers into the appropriate code (for example, binary numbers) and send them directly, but even the most modest tables of these functions have several thousand digits. A much simpler way to forward the same information would be to pass instructions for calculating these tables from the corresponding trigonometric formulas, such as Euler's equation

$$e^{ix} = \cos x + i \sin x.$$

Such a message may be relatively short, but it will contain all the information that would be even in the largest tables.

Suppose, on the other hand, that we would like to transfer information about football. Someone wants to know the results of all matches of the first league in the last few decades. In this case, it is extremely unlikely that there is some formula that could compress the information into a short message; In such a series of numbers, each digit is essentially an independent unit of information, and cannot be predicted by means of adjacent digits or by any rule. There is no alternative to transmitting a full list of results.

In these two strange messages, there is a call to the new definition of coincidence. It is based on the observation that information contained in a random number of numbers cannot be "compressed" or reduced to a more compact form. When formulating the actual definition it is desirable to observe communication not with a person but with a digital computer. A person can have the ability to draw conclusions about numbers or construct sequences of partial information or based on foggy instructions. The computer does not have this capability, and for our purposes this deficiency is an advantage. The instructions given to the computer must be complete and explicit, and must enable it to do step-by-step without the need to understand the results of any part of the operations it performs. Such an instruction set is an algorithm. It may be required to carry out any final number of mechanical manipulations with numbers, but cannot be asked to judge their meaning.

The definition also requires that it be possible to measure the information content of the message in some precise way. The basic unit of information is a "bit", which we can define as the smallest unit of information that is able to indicate the choice between two equally probable things. In binary notation, one bit is equivalent to one digit, 0 or 1.

Now we are able to more precisely describe the differences between the two sets of digits from the beginning of this comment:

$$\begin{gathered}01010101010101010101\\01101100110111100010\end{gathered}$$

The first one can be specified for a computer using a very simple algorithm, for example, "Print 01 ten times". If the string was expanded by the same rule, the algorithm would only be slightly higher; For example, "Print 01 Million Times". The number of bits in such an algorithm is a small part of the number of bits in a string that specifies, and as the sequence increases the size of the algorithm (program) grows much slower.

There is no corresponding shortcut for the second string of digits. The most cost-effective way to express that string is to print it out as a whole, and the shortest algorithm for entering a string into a computer would be "Print 01101100110111100010". If the string was much longer (but still obviously without a pattern), the algorithm would have to be extended to the correct length. This "incomprehensibility" is a feature of all random numbers. Now we can continue to define the coincidence in terms of incompatibility: A series of numbers is random if the smallest algorithm capable of that string specifies a computer has the same (or almost the same) number of information bits as the string itself.

This definition was independently proposed around 1965 by A.N. Kolmogorov from the Academy of Sciences of the Soviet Union and Gregory Chaitin, who was then a student at City College in New York. Neither of them then knew about a similar proposal given by Ray

J. Solomonoff in 1960 to Zator Company in an attempt to measure the simplicity of scientific theories. In the following years, research into the meaning of coincidence continued. The original formulations have been improved and the correctness of this approach has been strongly endorsed.

Inductive Method Model

The algorithmic definition of randomness sets new foundations of probability theory. In no case does it suppress the classic probability theory, which is based on the ensemble of possibilities where each possibility is assigned a probability. In fact, the algorithmic approach is complementary to the ensemble method by giving a precise meaning to concepts that were intuitively attractive but could not be formally accepted.

The classic theory of probability, which dates back to the 17th century, still remains of great practical importance. It forms the basis of statistics, and is applied in many other areas of science and technology. Algorithmic theory also has important implications, but they are primarily theoretical. Here, the area of greatest interest is the extension of Gödel's theorem on incompleteness. The second application (which actually preceded the formulation of the theory itself) is in Solomon's model of scientific induction.

Solomon presented scientific observations as a series of binary digits. The scientist is trying to explain these observations through a theory, which can be seen as an algorithm capable of generating that array and expanding it, i.e. to anticipate future observations. For each given series of observations, there are always several theories that compete with each other, and the scientist must choose one of them. This model requires that the smallest algorithm, one of the smallest bits, be selected. Said otherwise, this rule is similar to the Occam's razor formula: If we have more theories of obviously equal value, we prefer the simplest.

Therefore, in Solomon's model, a theory that allows us to understand a series of observations is seen as a small computer program that reproduces observations and provides predictions about possible future observations. The less the program is, the more comprehensive the theory is and the higher the level of understanding. Observations that are random cannot be reproduced using a small program and therefore cannot be explained using theory. In addition, the future behavior of the random system cannot be predicted. For random data, the most compact way for a scientist to report his observations is to publish them in their entirety.

Choosing a particular computer can be considered an irrelevant question, so we can choose the ideal computer for our calculations. For him, we assume that he has unlimited memory capacity and an unlimited time to complete his calculations. The input and output from the computer are in the form of binary digits. The machine starts to run as soon as it can be programmed and continues to work until it completes the output of the binary string that is the result. The machine then stops. If there is no programming error, the computer will produce exactly one output for each given program.

Minimum Programs and Complexity

Any sequence of numbers can be generated using an infinite number of algorithms. However, of the utmost interest are the smallest that will produce a given numerical array. The smallest programs are called minimal programs; For a given string there may be one minimal program or may be more.

Any minimal program is necessarily random, regardless of whether the array is generated by random or not. This conclusion is the direct result of the way in which we defined coincidence. Let's look at the program P, which is the minimum program for the S string. If we assume that P is not random then, by definition, there must be a second program, P', significantly less than P that will generate it. Then we can generate S using the following algorithm: "From P' calculate P, then from P calculate S". This program is only a few bits longer than P', and therefore must be significantly shorter than P. P is therefore not a minimal program.

The minimum program is closely related to another fundamental concept of algorithmic randomness theory: the concept of complexity. The complexity of a set of digits is the number of bits that must be inserted into the computer to get the original string as the output. The complexity is therefore equal to the size of the bits of the minimum program series. Since we have introduced this concept, we can now reiterate our definition of coincidence in a more rigorous way: A random sequence of digits is one whose complexity is approximately equal to its size in bits.

The notion of complexity serves not only to define coincidence but also to measure it. For the given series of numbers each of which has n digits, it is theoretically possible to identify all those with the complexity of $n-1$, $n-10$, $n-100$, etc. And thus rank the arrays at a declining degree of coincidence. The exact value of the complexity below which the string is no longer considered to be random is somewhat arbitrary. This value should be set low enough that the numbers with obviously random features would not be excluded and high enough so that the numbers with a clearly expressed form were disqualified, but to set a certain numerical value means to estimate what degree of coincidence makes a real coincidence. It is precisely this uncertainty that is reflected in the qualified claim that the complexity of the random sequence *approximately* is equal to the size of the string.

Random Number Properties

Methods of algorithmic theory of probability can clarify many properties and random and incidental numbers. For example, it can be shown that the distribution of the frequency of the digits in a series has an important effect on the randomness of a sequence. A simple insight suggests that a string consisting of only 0 or 1 is far from random, and the algorithmic approach confirms this conclusion. If such a string has a length of n digits, its complexity is approximately equal to the logarithm of base 2 of n. (The exact value depends on the machine language used.) A string can be generated by a simple algorithm such as "Print 0 n times", where in fact all the necessary information is contained in the binary number for n. The size of this number is about $\log_2 n$ bits. Since even for moderately long strings the n logarithm is much smaller than just n, such numbers have low complexity; Their intuitively perceived pattern is mathematically confirmed.

Another binary string that can be successfully analyzed in this way is the one where 0 and 1 are present with relative frequencies of 3/4 and 1/4. If the string is n length, it can be shown that its complexity is not greater than 4/5 of n, that is, a program that produces a string can be written with $4n/5$ bits. This maximum applies regardless of the sequence of digits, so there is no sequence with such a distribution of frequency that could be considered very random. In fact, it can be shown that in any long binary sequence that is random relative frequencies 0 and 1 must be close to 1/2. (In random decimal sequences, the relative frequency of each digit is, of course, 1/10.)

Numbers that have an inconsistent distribution of frequency are an exception. For example, of all possible n-digit binary numbers, there is only one that all consists of 0, and only one that consists of 1. All others are less organized, and the vast majority of the sea, according to any reasonable standard, is considered to be random. In order to choose an arbitrary limit, we can calculate which part of all n-digit binary numbers has a complexity of less than $n-10$. There are 2^1 programs length of one digit that can generate a n-digit string; There are 2^2 programs of two-digit lengths that can give such a set, 2^3 programs of lengths of three digits, etc., until the longest programs allowed within the allowed complexity; It has 2^{n-11} of them. The sum of this string $(2^1 + 2^2 + \cdots + 2^{n-11})$ is equal to $2^{n-10} - 2$. So there are less than 2^{n-10} programs of size less than n^{-10}, and since each of these programs can specify no more than one set of digits, less than 2^{n-10} of 2^n numbers has a complexity less than $n-$ 10. Since $2^{n-10}/2^n = 1/1024$, it follows that of all n-digit binary numbers only about one of 1000 has a complexity less than $n-10$. In other words, only about one string of 1000 can be compressed into a computer program that is more than 10 digits smaller than it itself.

The necessary consequence of this calculation is that more than 999 of every 1000 n-digit binary numbers have a complexity equal to or greater than $n - 10$. If this degree of complexity can be taken as a suitable random test, then almost all n-digit numbers are actually random. If a fair coin is dropped n times, the probability is greater than 0.999 that the result will be incidental to that extent. It therefore seems like it's easy to show a copy of a long series of random digits; Actually, this is impossible to do.

Formal Systems

It can easily be shown that a certain number of digits are not random; It is enough to find a program that will generate that sequence and that it is significantly smaller than the string itself. The program does not necessarily have to be a minimum program for that set; It's just enough to be small. On the other hand, in order to show that a certain number of digits are random, we have to prove that there is no small program that calculates it.

In the world of mathematical proof, Gödel's theorem of incompleteness is an exceptional landmark. The Chaitin version of this theorem states that *the required proof of coincidence cannot be found*. The consequences of this fact are equally interesting because of what they discover about Gödel's theorems, as well as what they indicate about the nature of random numbers.

Gödel's theorem is the resolution of controversy prevalent by mathematicians during the early years of the 20th century. The question was: "What makes valid proof in mathematics and how can such a proof be recognized?" David Hilbert tried to solve this controversy by developing an artificial language in which valid evidence could be mechanically found, without any need for human insight or estimation. Gödel showed that such a perfect language does not exist.

Hilbert has established the final alphabet of the symbol, an unambiguous grammar that specifies how a meaningful statement can be formed, a final list of axioms, or initial assumptions, and a final list of the conclusion rules for the deduction of theorems from axioms or other theorems. Such a language, with its rules, is called a formal system.

The formal system is defined so precisely that the proof can be made using a recursive procedure that includes only simple logical and arithmetic manipulations. In other words, in the formal system, there is an algorithm for testing the validity of proof. Today, which was not possible in Hilbert's time, the algorithm could be executed on a digital computer and the machine could be required to "evaluate" the validity of the evidence.

Because of Hilbert's requirement that a formal system has an algorithm for verifying evidence, it is theoretically possible to list one by one all the theorems that can be proved in a particular system. First, we list alphabetically all the sequences of symbols that are of one character length and apply the test algorithm to each of them, which we find all theorems (if any) whose evidence consists of one character. Then, we test all the two-character sequences of the symbol, etc. In this way, all possible evidence can be checked, and in the end, all theorems can be discovered in the order of their size. (This method is, of course, only theoretical, the procedure is too long to be practical.)

Unprovable Claims

Gödel showed in his evidence from 1931 that Hilbert's plan for complete systematic mathematics cannot be fulfilled. He did this by constructing a claim on positive integers in the language of a formal system that is true but cannot be proven in the system. The formal system, no matter how large or carefully constructed, cannot include all the exact theorems and therefore is incomplete. Gödel's technique can be applied to practically any formal system, which leads to a surprising and, for many, an unfortunate conclusion that there can be no definitive answer to the question "What is valid proof?"

Gödel's proof of the incompleteness theorem is based on the paradox of Epimenides Crete, who allegedly said, "All Crete are liars". This paradox can be pronounced in a more general way than "This claim is incorrect", which is a statement that is true if and only if it is inaccurate, and therefore is neither accurate nor inaccurate. Gödel replaced the concept of truth with the concept of evidence and thus constructed the sentence "This claim is inconclusive", which is a statement that, in a certain formal system, is provable if and only if it is inaccurate. Therefore, if the inaccuracy is provable, what is prohibited, or the exact claim is inconclusive, then the formal system is incomplete. Gödel then applied a technique that uniquely numbered all the arguments and evidence in the formal system, thus converting the sentence "This claim is incalculable" in the statement of the characteristics of positive integers. Because such a transformation is possible, the incompleteness theorem is equally convincingly applied to all formal systems in which it is possible to work with positive integers.

A close connection between Gödel's proof and random number theory can be explained through another paradox, in the form of a similar Epimenides paradox. It is a variant of Berry's paradox, first published by Bertrand Russell in 1908. It reads: "Find the smallest positive integer that to be specified requires more characters than there are in this sentence." This sentence has 35 characters (counting the spaces between the word and the point at the end, but not the quotation marks), and yet allegedly determines the whole number that, by definition, requires more than 35 characters to be specified.

As before, in order to apply the paradox to the theorem on incompleteness, it is necessary to move it from the world of truth to the world of proof. The phrase "requiring" must be replaced with "for which it can be proved to be required", which means that all claims will be expressed in a particular formal system. Also, the foggy notion of "the number of characters needed to determine the whole number" can be replaced by a precisely defined concept of complexity, which is measured in bits, not in characters.

The result of these transformations is the following computer program: "Find a series of binary digits that can be proven to be more complex than the number of bits in this program." The program tests all possible evidence in a formal system in order of magnitude until it hits the first which proves that a particular binary sequence has a complexity greater than the number of bits in the program. He then prints the string he has found and stops. Of course, the paradox in the statement from which the program was executed is not eliminated. The program allegedly calculates a number that no program of its size should be able to calculate. In fact, the program finds the first number that can be proven that it cannot be found.

The absurdity of this conclusion only shows that the program will never find the number for which it was made to search for it. In a formal system, it cannot be shown that a certain set of digits has a complexity greater than the number of program bits used to specify that string.

It is possible to make further generalizations about this paradox. It's not the number of bits in the program itself that is the limiting factor, but the number of bits in the formal system as a whole. Hidden in the program are axioms and rules of locking that determine the behavior of the system and provide an algorithm for testing evidence. The information content of these axioms and rules can be measured and can denote the complexity of the formal system. The size of the entire program, therefore, exceeds the complexity of the formal system for the fixed number of bits c. (The real value of c depends on the machine language used.) The theorem proved using the paradox can be expressed as follows: In the formal complexity system n it is impossible to prove that a certain set of binary digits has a complexity greater than $n + c$, where c is a constant that is independent of which system is used.

Formal System Boundaries

Since complexity is defined as a coincidence measure, this theorem implies that in a formal system no number can be proved to be random unless the complexity of that number is less than that of the system itself. Since all the minimal programs of a random theorem also imply that a system of greater complexity is needed to prove that the program is minimal for a certain set of digits.

The complexity of the formal system has a very heavy weight in proving the case because it is a measure of the amount of information the system contains and therefore the amount of information that can be derived from it. The formal system rests on axioms: fundamental claims that are non-conductive in the same sense as the minimum program. (If an axiom could be expressed in a more compact way, then this shorter assertion would become a new axiom, and the old claim would become the theorem.) The information embodied in the axioms is, therefore, itself incidental, and can be used to test the coincidence of other data. The coincidence of some numbers can thus be proven, but only if they are smaller than the formal system. In addition, each formal system is necessarily finite, while any number of digits can be arbitrarily large. Therefore, there will always be numbers whose coincidence cannot be proved.

The undertaking of defining and measuring coincidences has greatly clarified the importance and implications of Gödel's theorem on incompleteness. This theorem is now seen not as an isolated paradox but as a natural consequence of the constraint imposed by the theory of information. In 1946, Hermann Weyl said that the suspect posed by such discoveries as Gödel's theorem was "a constant blow to the enthusiasm and determination of which I myself went into my research work." From the point of view of the theory of information, however, Gödel's theorem gives no reason for depression. Instead, it seems that it simply suggests that mathematicians to advance, like researchers in other sciences, have to search for new axioms.

Duško Milinčić, El. Engineer, prof. IT,
Gimnazija Banja Luka, June 25, 2017.

* * *

The next opinion was given by a lecturer Goran Dakić, a professor of Serbian language and literature, writer, and a journalist. He is the author of the novel "Dalj", which was recently proclaimed the best novel of the year published in the Republic of Srpska, and after a few years, Dakic was named as the best journalist in Bosnia and Herzegovina in the year. He is known for his sharp attacks on current politics. In the commentary here, he remained consistent with himself and again found a policy, but surprisingly, in the way even I mentioned somewhere.

State of Higher probability

In the preface of the book "Space-time", Rastko Vuković mentions, among others, Da Vinci, Pascal, and Tesla, who in every way defied to academies and scientific intelligence of their time. This defiance will, for centuries, lead Serbian poet Stanislav Vinaver into a cynical aphorism: "It must be blithely despised" and thus show that the spirit of the genius is free as much as the departments are and will never be. This type of contempt and defiance is peculiar to those who see further in time; they do not care about the titles, they do not care about their career, they can easily give up their titles. Genius or the one who searches for it burns in the fire of idea; everything else for him is only morning, a January grill where icebergs cannot warm up.

It is almost certain that I do not know anything about Lorenz's transformations and the theory of sets, but I do know about some of our academies and academics. I have noticed in my mind many times that the originality of scientific work in our country is measured by quotations and footnotes; as many citations as possible, making more references to other authors and other researchers – then the scientific work is more original! The paradox is, of course, obvious, but nobody sees it or it's actually part of a larger game. For years I mocked with scientific papers or even dissertations that explored and analyzed the accumulation of initial capital in Defoe's novels, but now I see that they have made itself enviable academic careers based on pseudo-scientific jumbo-jumping. In this order, things do not exist for the original. After all, remember that Einstein passed with some discoveries that only later, after his death, was celebrated as revolutionary.

Our higher education is as it is: in order for a young scientist to become an assistant professor, he needs a certain number of scientific papers, one master, participation in international meetings; Five years later, the scientist is no longer so young, but his career continues and his career as a full-time professor is ahead of him, and it takes twice as many works, twice as many sets and one or two textbooks, or monographs; But our scientist, who now looks backward and wondering where he was and what he did last year, has another regular goal in front of him, and it is, of course, and the most difficult ... In such an order of things, it is clear places there is no originality. The idea was killed by dogma and bureaucracy. After all, there is an anecdote according to which Einstein remained without a certain scientific title because he was not original enough. Nobody mentions those who decided on this, not even in the footnote.

The originality of Vukovic's book is undisputed; as a writer and journalist I can follow her philosophical side; about the mathematical and physical one, others will have to judge and judge. I believe his views are correct, but not because I understand them in the end, but because his potential reviewers refused to recommend the book to the publisher with a loose argument: It's all true, but you're wrong! Certifications are worthy of fanatics. Vukovic is a heretic; he does not believe in anything, because he simply does not have to believe in anything. If something is wrong, then it is not true and there is no department that can prove it to be different. This, I believe, is Vuković's motto. My colleagues are my dear, but the truth is dear to me more, says Vukovic, and the story ends. For the young, as once said the prof, it remains to be seen whether they will quote him in coming decades, as is quoted Leibnitz today.

Academics do not believe in the idea; the idea is everything in what Vukovic believes. Namely, that is the difference.

Goran Dakić, Journalist,
Banja Luka, July 19, 2017.

Dimitrije Čvokić

In his manuscript, the author presented a kind of construction of a view of the physical world that takes on the basis of the so-called objective coincidence. I would like to mention that the "curious" term given in this text, in a way used Heisenberg in 1958. In essence, according to Heisenberg, objects do not have explicitly defined and unchanged properties, i.e. there are only their probabilistic characterizations. The wave function of quantum mechanics can be seen as a unique cognitive record of a physical system and at the same time as a complete (but probabilistic) description of all outcomes in some future views. It is clear that this objective coincidence is directly derived from this objective, and that it is apparent in a close relationship with the concept invoked by the author. That is observed from the angle of modern quantum mechanics as if a small castling was made. It begins from coincidence (the objective) and ends with the description of the diversity of the partially known world. It's like there's something metaphysical. A man, caught between an infinitely big and infinitely small one, from which he can not understand, is only certain of his uncertainty.

Otherwise, the quest for the primordial principles through which all the confusion/orderliness and complexity of the universe in which we live (and which we study) will be considered, is not the end of a more recent date. It is interesting that even the Tales of Miletus once claimed that there must be a general principle that connects all natural phenomena and on the basis of which they can be reasoned at all. Moreover, he was convinced that there was a given principle or the first element from which all matter emerged and that the search for the root should be the ultimate goal of all natural sciences. Therefore, we could say that the author takes coincidence (i.e. probability) as that principle, and introduces two more: the principle of information and the principle of entropy.

By reading the interpretations and evidence of some well-known claims in physics, the impression is that it is a probability packet of already known deterministic approaches in understanding the world around us and the laws that govern it. However, in addition to showing how the three principles can be used to explain and describe various physical phenomena, quite simple alternative interpretations of some tricky places in the prevailing theories of modern physics are also offered. In a way, as if on the work itself we supplement the existing theory, guiding Montaigne's understanding that there are two main branches of philosophy: dogmatism and skepticism. Dogmatism, in a sense, would fit in here with a deterministic view of the world, and skepticism would be probable.

Among the various problems in the field of physical sciences are considering quantum entanglement, deterministic chaos, dimensions in physics, nature of gravity, some claims from thermodynamics, redshift, and a slightly different path to basic claims in the theory of special relativity. Some of the statements and interpretations I have found particularly interesting, such as the impact of the future on the past, although it is not clear from the text whether the author puts emphasis on anti-causality, a-causality, a mixture of everything with causality, or is something about the fourth.

We can say that due to the complexity and the plant in its basic settings, this book poses a challenge in terms of terminology, style, and language. Initially, it moves from some commonly known concepts and some interesting interpretations, which gives the impression that a book can be read by a layman. However, things soon become much more complicated and much more perplexed. Do some of the concepts in theoretical physics need to be described in greater detail, or not? Should one really look at some physical phenomena or not? Is it really necessary to mention Gödel's theorems, which, unfortunately, have recently become synonymous with pomp? Of course, it all depends on the reader itself, but it's also one of the problems in that: who belongs to the target group?

I believe that some of the claims could really jolt the imagination of many physicists, especially since the matter looks a bit like the Pondicherry interpretation of quantum mechanics, but also that many of them will be confused by the role of (abstract) information in all of this, as well as some mathematical games (so call it). On the other hand, the mathematician should not have difficulties with the mathematical apparatus used; the role of information, the understanding of the outlay relating to Lorentz transformations, but the background to the problem (dimensions in physics, strings, gravity, Carnot cycle, particle annihilation and antecedents, etc.) represented quite certainly a psychological blockade. Again, the philosopher would, for example, skip numerous examples and mathematical proofs of various claims, and instead he was bothered by their interpretation by linking them with Peirce's pragmatism[60], accidentalism, libertarianism, supremacy, but at least, in my opinion, there is a stumbling block. The interpretations should be very cautious, and if the mathematical background of the problem is not properly known, it can become quite fanciful fantasies.

Of course, I do not think that this text is bad for all this. Moreover, I can say that the author managed to fight so many times with all of this and to be honest, probably better than me if I was on the same occasion working on the same issue. In some ways, Pascal's thought is applicable here: "The last thing we can do when composing a piece is what to put in the first place". From experience, I know that this "last station" is not so easy to get. On the other hand, it is not easy to get caught up with the "world", because we know it is not easy to please the world.

Finally, I think that the text is very interesting, because it forces a person to some kind of thinking about nature, about society, but also about some ontological issues. For example, one of the problems that could break the imagination would be the information "skepticism" (or, sufficiency?). Is there a measure of this scantiness? What does the "ultimate uncertainty" mean to the extent of the cosmos (Greek $\kappa\acute{o}\sigma\mu o\zeta$ – ordered world)? Where is the origin and role of human personality? How is it that the ocean of uncertainty (and fuzziness) is subjected in this case to a cruel and icy, i.e. extremely certain and distinct mathematics? What does it really mean when it says that nature "must do"? Is there time of time? What could be done if we added the probability principle, now quite elaborated, the notion of negative probability? In other words, the value of the book should not be seen exclusively by the accuracy, consistency, nor the originality of the exhibited, but during this process, it should also take into consideration the questions that it might have in the readers of various educational profiles. It is not doubtful that the book of Rastko Vuković, in addition to providing in some cases simple and interesting answers to many tricky questions of contemporary physics, allows the reader to think about new connections, new leaps, new doors that should lead to scientific sources and buckles.

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Banja Luka, August 23, 2017.

[60]Charles Sanders Peirce (1839-1914), American philosopher, logician, mathematician, and scientist who is sometimes known as "the father of pragmatism" – author's remark.

Duško Jojić

In the text "SPACE-TIME", the author on a little over a hundred pages speaks of interesting questions that link the theory of probability, theory of information and modern physics.

At first, the probability axioms, Gödel's theorems and Heisenberg's uncertainty are discussed. Later, the author observes the nature of space, time and matter, skillfully interpreting familiar examples (lift paradox, butterfly paradox, etc.). From the plenty of interesting examples, I emphasize (probably from personal affinities) the connection between Chaos theory and Ramsey's[61] theory. The first chapter ends with the introduction of the concept of dimensions, examples of symmetry and the discussion of Einstein's special theory of relativity.

In the second chapter, the author begins with the notion of information and associates it with axioms of probability. Afterwards, the Euler-Lagrange equations are described and for some specific problems solutions of these equations are given. As an interesting detail in this chapter, I highlight the story of the role of tensors in Einstein's theory of gravity.

The third chapter looks at entropy and connects the basic concepts of thermodynamics with probability and a special theory of relativity. Using the principles of the theory of probability from the example of thermodynamics, the author concludes that time does not exist without information!

In the last chapter, Lorenz's transformations are derived from Einstein's principles, and then these transformations are interpreted as symmetries. According to the author's words, this chapter is the announcement of his future work.

Observing the weight and variety of the examples, the sophisticated mathematical apparatus needed to describe the topics it deals with, the author should be acknowledged in the skill to approach these questions to the ordinary reader. This does not mean that the text is easy to read. Unfortunately, mathematics is difficult (and the physics is even more difficult!) And the understanding of concepts important for understanding the world around us requires the efforts and strains of the interested reader. Those interested in the book of Rastko Vuković will find attractive and challenging text, elegant presentation of different theories and an original interpretation of known examples.

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Banja Luka, September 4, 2017.

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