INFORMATION of PERCEPTION
freedom, democracy and physics

Rastko Vuković

Working version!

Working version (2017)
Rastko Vuković:
INFORMATION of PERCEPTION – freedom, democracy and physics

Lecturer & Reviewers (2016):
Goran Dakić, Journalist & Writer
Dr. Momir Ćelić, Faculty of Electrical Engineering Banja Luka
Dr. Aleksandar Savanović, Faculty of Political Science Banja Luka

Translated from Serbian (2017)
Rastko Vuković:
INFORMACIJA PERCEPCIJE – sloboda, demokratija i fizika

Text in pdf
https://www.scribd.com/document/360939276/ – English
https://www.scribd.com/doc/315602275/ – Serbian

Foreword

This book is a retrospective of my analysis of contemporary sociological problems published over the last couple of years. These debates are sometimes triggered by a coffee issue in a tavern or by correspondence via social networks, but which still end in isolation by searching for better definitions and mathematical forms. That is why the ideas presented here will sometimes appear to be scandalous or at least exaggerated to the academician.

However, at the time of writing the first brochures (by 2014), now summarized in this booklet, the interesting stories for me were so unusual that I could hardly find a colleague in the profession who could take me seriously. In fact, anymore educated interlocutor who would think seriously that "equality leads to conflicts" was "very disappointed" by my ignorance of the essence of democracy and the lack of understanding of the fact that "just equality" guarantees peace, prosperity and freedom. Yes – if I continue – competition is just what equality is needed in order to create a "healthy conflict" – then the further discussion would stop. You have no idea – it would be an honest answer. You are mediocre – I answered back – and the spending of the state due to your diplomas has failed investment, because you will never learn to think with your head!

Of course, these "funny moments" were rare, unlike those pleasant. There are little discussions in the right, creative way, without the different interlocutors.

These are the reasons I wrote the text from light to heavy, from general to particular, that is, from less to more professional readers. I wanted the content to reach as wide a circle of readers as possible, because the parts of the story emerged from the encouragement of such ones. Regarding the quality of the idea, they were the first free associations, sometimes only cafes outbursts or attacks of originality, so that only a few would mature. What would remain are rare theses that I could not find contradiction and could find some elaborations, and the plethora of the other is best to be forgotten.

Rastko Vuković, June 2016

Contents

Introduction

At the time of seeking the principles outlined here by the relation of sociology and mathematics, two philosophical directions dominated. Society's theorists were divided over the question of the exactness of social phenomena. If there are strong laws of relations between people, can they be reached by mathematics? To a professor of mathematics in Café and Facebook discussions, it is "embedded" as a challenge, which has become more interesting over time.

Do I believe that there are "mathematical principles" of society's behaviors?

Yes, I believe they exist, but they are still unknown to us! If they exist and can be detected, then somebody has to be the first one, right? That is precisely why 'true' principles should be attractive to (future) 'true' mathematicians. So did the first talks.

I wrote in one polemic. When Blaise Pascal laid the foundations of "probability theory" in his four to five letters in the 17th century, by the 20th century the philosophers and mathematicians turned their heads full of doubt. It is not possible that something that is a consequence of coincidence, disorder and chaos to be defined and explored precisely. Such nebulous as "probability" will in particular never become math because we know that mathematics and all its parts are built up of pure absolute truths, which are completely immune to the historical period of creation, their duration, the development of other sciences or social relationships – said the skeptics.

However, at the beginning of the 20th century, a miracle happened. Not mathematics gave up of its absolute truths, but a brilliant Russian mathematician Kolmogorov discovered the axioms of probability theory. We know that these are just three axioms from which deduction could begin, an absolute deduction, which led to theorems that mostly coincided with the intuition (today we say the genius) of Pascal.

In these new mathematical areas, one more thing is strange. Using the principle of probability theory we prove various theorems of geometry, algebra, analysis, and of course and vice versa. In fact, this would have to be a new goal in seeking absolute truths within social "nebulosity".

It was not easy to persuade the interlocutors what I am saying here. Some critics were very instructive to me, and some of them did not.

"If you were not a mathematician, you might even realize that freedom is not and will never be a number," they spoke kindly to me. Or: "Only to someone with a head full of vectors can fall into the mind to talk at such a manner about intelligence and hierarchy." I do not know how you cannot understand that 'freedom' and ' restriction' have opposite meanings – one colleague convinced me, saying that these were reciprocal sizes, and not proportional to how (he though) I treated them.

These now entertaining critiques I mentioned to remind the future reader of the weight of the following topics at the time they were opened.

Chapter 1

Perception

The word *perception* (lat. *perceptio*, *percipio*) means detection, observation, appreciation, but also all those mental states and events that are directly caused by sensory stimuli. It is a word that is used in psychology only for living beings, which we also mean here. *Living* being we will consider what can made *decisions*. The decision is the result of *decision-making*, and decision-making is the process of identifying and choosing *option* based on the values and determinations of the decision-maker.

1.1 Freedom

The freedom of the individual is primarily limited by the possibilities of its overall perception, and then by the possibilities of using these perceptions. Perceptions can be understood in relation to the body of the individual as the inner and outer, according to the significance or frequency. In any case, we can also talk about the amount of perception in the way that mathematics defines information.

In the theory of probability the *information* is the "amount of indeterminacy", that is, what we get by "realization of coincidence". For now, we just notice that throwing a cube has more uncertainty than throwing coins, and that the information "dropped the number 3" when throwing a cube is greater than "dropped the tail." The greater is the news "the man has bitten the dog", than the news "The dog has bitten the man", because the first is less expected. Consequently, greater uncertainty generates a greater "amount of liberty", which we will label here with ℓ and call *freedom*.

Freedom ℓ is the number, the measure of all the options an individual can choose with respect to his own senses, or his own perceptions of power, and in relation to his environment. Therefore, in this text (total) *freedom* is the size that in the vector algebra is called *scalar*. Briefly, the scalar ℓ is the amount of opportunity. It is also (total, conscious or unconscious) information obtained through the perception that an individual can have about himself and the outside world.

Perception is a largely unconscious process that interprets data from various senses by making a whole individual. Because this process is unconscious, it is easy to apply to the wider forms of life, and even further. Hence, the need to define, i.e. to distinguish a living from a non-living being.

It has already been said that "living" is what can "decide", but we will not specify what a "decision" is. In mathematics, it is not unusual to set up "loose" axioms, formal and universal, which are easily interpreted in different ways and easily generalized. Hence, the ease of applying mathematics in different fields' comes.

At the end of the 19th century Hilbert[1] published axioms of geometry divided into five groups (see [3]). The first group has axioms or incidents, they have eight, but I only mention the first two:

I-1: For every two points A and B there exists a line a that contains them both.

I-2: For every two points there exists no more than one line that contains them both.

Hilbert's ingenious idea was to discontinue debates about the "thickness" or "length" of the point and paradox about the required number of points for one length in order to formalize Euclidean axioms and allow the generalization of geometry objects. So that the words "line" and "point" in his geometry can be freely replaced by the words "house" and "window", and that the obtained "geometry" is again correct.

Similarly, we treat the meaning of the term "decision" here. It is left to the reader at will, depending on how he understands the "ability to decide", so that he sets the seat, his level of definition of life. If only people are able to decide in order, the continuation of this theory must be true "for people only". If other recipients are considered to have the ability to decide, and that's fine, the rest of the theory will be true for primates. Similarly, if bacteria have the ability to decide, then the theory will apply to them.

For example, bacteria can decide, because they cannot be programmed. If we make some bacteria (small robots) that we can fully control then it will not be this one today, which we will continue to consider as living beings. The chess program *Deep blue*, which was defeated Kasparov in 1997, is not yet a living being, because he blindly followed the developer's code. However, we can consider as a living being "a machine" that would tell us: I would do this part of your code, and this I will not. The same criterion applies to any unknown form of life today, both on Earth and outside.

Without considering the reasons for deciding, note that it is the consequence of the existence of options, needs and choices. Here we believe that the basis of decision-making is non-determinism of physical world and the ability of living creatures to use uncertainty. Every form of life has some freedom to choose its own possibilities, according to personal ability and external circumstances. Hence, the choices of the given possibilities are what we in the probability theory call the outcomes of random events. The freedom exists because there are random events and their realizations, and then because we can perceive and use them.

Another phenomenon of perception is its diversity for different types of life. Grass perceptions are not the same as the perception of ants, even if they were on the same ground. Perception of the plant is its ability to feel and react to the environment by adjusting its morphology, physiology and phenotype. The plant reacts to chemicals, light, humidity, temperature, gravity, infections, but also to some sounds and touch.

Small flowering plant *arabidopsis thaliana* with cryptochromes (Greek $\kappa\rho\upsilon\pi\tau\acute{o}\ \xi\rho\omega\mu\alpha$

[1] David Hilbert (1862-1943), German mathematician.

– hidden color) notice the magnetic field. The poplar can recognize a change in orientation and inclination. It is known that the injured tomato produces the volatile scent of methyl jasmonate as an alarm signal that similar plants in the environment recognize and trigger the production of chemicals for the defense of insects and other predators.

Thus, different forms of life will have different total amounts of possibilities, here the freedom of ℓ, which will be slowly changed by evolution. So slowly, in the range of up to a few dozen generations, this number can be considered as a constant ($\ell =$ const.). On the other hand, the same number of freedoms changes faster by changing the environment. With the emergence of video games, mobile phones, cars, today we have some opportunities that no cavemen have had, but we also lost some because of the outbreak of wild nature.

Limitations of perception can formally be obtained by narrowing the set of all freedoms, by throwing out "opportunities" that we never choose. We can get them and give them less importance (less intensity). We do not make some choices because we do not want them, some because we are not smart enough, and some are forbidden to us.In fact, we never or very rarely choose a very large number of possibilities, and we decide on only a relatively small number of them.

Let us now notice that the meaning of the word "freedom" here defined is not equal to any previous in philosophy. On the other hand, I hope that this ℓ will be widely applicable in philosophy, in the way that mathematics is usually applied in other disciplines.

The word of freedom (*kata physin*) at Heraclitus of Ephesus (about 500 BC) is the norm: it must act according "to nature" and morally. Socrates also took the unconditional value of the moral, thus limiting his freedom. The Greek word for freedom (*eleutheria*) originally means only legal-political freedom. A free city-state (*polis*) is one that is not under the jurisdiction of aliens. Later, with Plato and Aristotle, the notion of personal freedom and ethical responsibility appears, when moral freedom becomes something completely different from the sociopolitical.

Hegel distinguished objective and subjective freedoms, maintaining that internal, subjective freedoms are an essential requirement of moral action. Freedom does not allow any kind of willingness, but it is limited by moral, responsible and human rightful action. Then the concept of liberty expanded to individual ethical and social-ethical norms, but in all historical periods it always remained only limited freedom.

Subjective freedom has been particularly negated in the early modern age. In the strict image of the world of the then mechanisms, according to which everything is necessary, uniquely determined, there was no room for free action. In this spirit is Spinoza's metaphysical determinism in the part of *Ethica*, with which Leibnitz and Kant deal each in his own way.

Only since 1700, from the time of English free thinkers and the French Revolution (when it becomes a battle password: *liberty*), it is understood what Hegel called "abstract freedom", one that is free from concrete attachments and which is lifted to unlimited spaces of freedom. Absolute freedom reaches the breadth of pure self-will.

Today's *philosophy* knows freedom as the right of choice and the ability of a man to choose and make decisions. Freedom as an autonomy of action that means the ability to act in accordance with one's own will that presupposes the absence of any coercion from outside. Then, as a free will, which means everything that does not hurt another. In ontological terms, freedom makes the essence of man, his humanity that distinguishes him from other beings determined by necessity. They call this latter also personal freedom.

Thus, we defined the freedom ℓ rather by the "space of random events" of the theory of information than as a well-known philosophical definition. Yet I am convinced that this approach will show its great efficiency in the subjects of sociology, psychology and society in general.

1.2 Intelligence

The ability of an individual to come up with choices that allow its perceptions (to the freedom ℓ) we call *intelligence* and mark with $\mathbf{i}$. Briefly, the intelligence is the ability to use options.

It is clear that the possibilities of the intelligence $\mathbf{i}$ are in some way limited by the amount of all possibilities, i.e. by freedom ℓ. On the other hand, it would be too easy to assume that $\mathbf{i}$ is a number, as ℓ. At best, if the intelligence is not a number, then it is a vector (which has direction and intensity) such as velocity, force, or the "oriented length". In the more difficult case, intelligence could be a matrix, a tensor, or an even more complicated mathematical structure, but it would be optimal to determine it as the simplest of the offered forms, yet closest to the intuitive concept of "intelligence" as it is the known coefficient IQ. This will in fact be a new meaning of the old concept.

Greater intelligence will mean a greater amount of capabilities, that is, it will take up most of the ℓ or will require a higher ℓ. Therefore, the intelligence we are talking about is in some way directly proportional to freedom. So, if we ever construct an intelligent machine, the first thing that one could ask for is more freedom. This is acceptable but a surprising conclusion to us, considering the modern understanding of robots their role among people and their obedience.

Secondly, it is clear that by learning and practicing we can acquire or increase some of our abilities. This means that the intelligence we define is not just the one IQ, but it also includes all the knowledge and skills that we have. I take concrete examples, but remind, we are searching for intelligence determinants for all living things in general, that is, for those beings that can make decisions. In this sense, the intelligence of single-celled organisms is a kind of atoms of this new concept.

Then, we do not use all receptions equally, not always the same, nor is the same object of observation equal for all individuals. This, on the one hand, means that intelligence is not a complete perception, in terms of the absolute use of all observations, and on the other hand that it can be "redeployed", that is, the same amount of intelligence can be used for different purposes.

Animal species struggle to survive in the wild with great power, speed, with the ability to conceal, then with a good instinct rather than with significant intelligence. From the standpoint of a small-step evolution – the extreme development of a brain that is a major consumer of energy (like a human being) may have been a very risky experiment? Moreover, the perfect intelligence for evolution seems to have been an overwhelming task. To slow plants, and many other species, the small minds were enough.

It would be empirical evidence that intelligence is not (always) equal to total perception. They do not have much to do with the exact, but they can confirm that the hypothesis about the low intensity of $\mathbf{i}$ relative to the freedom ℓ is on the right path.

When we see the stains of the color, our brain interprets them as a particular object, but the number of variations we see is limited by our experience and awareness. Someone will see only a man in two folding positions on the drawing of Vitruvius man (Leonardo da Vinci around 1490), the other will see the golden ratio, and the third will remember Giuseppe Bossi who put this drawing in his a monograph on Leonardo's Secret Nights.

The many sounds and words we hear as speech will have more meaning for us if we understand the language in which we are talking. The combination of taste is recognized as a meal, and our previous tasting experiences increase the number of opinions, reactions and decisions regarding the same meal. Therefore, unlike the quantity of possibilities, intelligence connects knowledge and wishes, that is, the past and the future. It is a vector!

Recall that perception involves the organization, identification and interpretation of information that can be perceived for the purpose of customizing and interacting with the environment. It occurs when physical or chemical stimulation of the senses becomes signals that through the nervous system reach the brain or to some other organ of the individual. Perception is not the passive reception of these signals, but also what comes to them, such as attention, expectation, memory, and learning. Perception is more than a simple amount of possibility (more than information), and besides this numerical part, the intelligence also filters out some of its vector properties.

So, perception already contains vector properties (as well as properties of information). The last group of arguments speaks about the way in which intelligent performances are formed, from past experiences for future action. When we encounter danger (bomb, poison, earthquake), which might seem harmless to us, unless we have an innate or acquired fear with the occurrence, our genetic or personal experience will define our notion of experience. Thus, we arrive to the conclusion that intelligence is an experience-information-action. Our experience shapes our perceptions, so two intelligent people will understand the same situation differently. Intelligence implies memory, its meaning and application in any future action.

Note that we can formalize the words "memory" and "application", for example by putting them in the context of Newton's law of inertia. Of course, we do not have to go that far here, but only to the grass or an even simpler form of life, and to say that the data filter filters observations from their surroundings to their genetic heritage or personal experiences, and for their future benefit. Nevertheless, the importance of the past and the future for the interpretation of the observation indicates the vector's sense of intelligence. This is not the same argument as the previously mentioned "rearrangement", but leads to the same conclusion, to the vector nature of intelligence.

Intelligence **i** has direction, and intensity, such as velocity, momentum (impulse) or force in physics. In general, the intensity of the vector **v** (the vector we will write with the *bold* letter) is also called *modulus* or the norm, which is indicated by the same *italic* letter:

$$v = |\mathbf{v}| \quad \text{or} \quad v = ||\mathbf{v}||. \tag{1.1}$$

When we say "this car is faster than that", we do not usually ask the direction. When we say "this force is greater than that", we do not ask, in what direction. So we will say here, "this vector is greater than that", meaning "in relation to another vector of the same direction", unless it is emphasized differently.

Therefore, the intelligence of **i** is the ability to use options, and again it is not the same as the freedom ℓ. To the question of what is the difference between intelligence and freedom, if we exclude the foregoing (the first is the vector, the second scalar), the answer is: according to the ability and quantity of uncertainty. The smallest such difference is a series of zero uncertainties. If there could be a living being with all the zero components, it would have such a reduced perception or increased intelligence that it can receive, understand and use total, total freedom! But why there are no such ones?

Therefore, the intelligence of **i** is the ability to use options, and again it is not the same as the freedom ℓ. To the question of what is the difference between intelligence and freedom, if we exclude the foregoing (the first is the vector, the second scalar), the answer is: according to the ability and quantity of uncertainty. The smallest such difference is a series of zero uncertainties. If there could be a living being with all the zero differences, it would have such a reduced perception or increased intelligence that it can receive, understand and use complete, total freedom! But why there are no such ones?

The following group of arguments comes from Ramsey[2] theories[3], located in continue mathematics for advanced high school students[4]. Ramsey's works in the field of combinatorics, especially the so-called The Dirichlet principle, but also in the Theory of Chaos, was continued by Motzkin[5] to which the discovery was attributed that the complete disorder was impossible.

We will not go into the details of these famous views, except that we will notice that in a sufficiently large set of random words and letters (on the pages of a meaningless book), we can always find a small subgroup with a predetermined meaning. Howsoever we try to paint some surface with meaningless spots, as the picture is the bigger we'll sooner find the more recognizable parts on which we will see some meaning. This is the content of the theorem we call Ramsey.

Ramsey's theorem gives the new weight to the limits of intelligence, that is, the "residue" of the perceptions that we will now define.

---

[2]Frank P. Ramsey, 1903-1930, British philosopher and mathematician.
[3]Popular Ramsey: `https://youtu.be/JYyuPITzMgw`
[4]Ramsey Theory: `http://web.mat.bham.ac.uk/D.Kuehn/RamseyGreg.pdf`
[5]Theodore Motzkin (1908-1970), Israeli-American mathematician.

1.3 Hierarchy

Freedom ℓ is the total amount of opportunity, i.e. information about the world in and around the individual that the body of a living being receives through perception. Intelligence $\mathbf{i}$ is the ability of an individual to use the given options. *Hierarchy* $\mathbf{h}$ is the ability of an environment to deprive individuals of the given options. Hence, the hierarchy is all restrictions on the freedom of the individual, whether they come because of her incapacity, due to natural laws, or by social norms.

Note that the freedom ℓ is proportional to $\mathbf{i}$ and $\mathbf{h}$. As a glass partly filled with water, and partly with oil, which is an even greater if the volume of two of its ingredients are larger. On the other hand, as ℓ is a number, and $\mathbf{i}$ is a vector, then $\mathbf{h}$ must be a vector for their product to be a scalar, i.e. scalar product of the vector:

$$\ell = \mathbf{i} \cdot \mathbf{h}. \tag{1.2}$$

That $\mathbf{h}$ is indeed a vector we can also check indirectly, by examples of organizing. Let a group of masons build a wall, and then crush it. It resembles the action of two parallel vectors of the same intensity, and the opposite directions. The same group of masons can be organized in a third direction too.

Emphasize once again that the vector $\mathbf{i}$ is called the intelligence of an individual and it is the ability of the individual to use the given options. The vector $\mathbf{h}$ is called a hierarchy and it represents the inability of intelligence to use the given options, that is, the hierarchy is the ability of society and the environment in which the individual lives to deny her these options.

However, only at first glance, the intelligence of the individual gives freedom, and the hierarchy denies them. For example, when a monkey jumps from branch to branch, he can work with a certain goal thanks to intelligence. Whether his goal is to get food or to escape from danger, the choice of the best next branch is no longer arbitrary, random, but is forced and conditioned for efficiency. Looking such at the freedom of dialing through intelligence, the freedom becomes unfreedom, a compulsion.

On the other hand, we have something similar to the hierarchy. Only at first glance, the reducing hierarchy gains pure increase in freedom. A counter-example would be reducing of individual freedoms in constant fear that somebody can be injured, in the case (it is believed) of allowing violence and killing on the streets and society in general. Also, economists speculate[6] that by regulating market rules, it is possible to increase market competition, diversification of supply and demand, and the growth of the economy.

When we philosophize about intelligence as the ability to use options, we mean the aspect of these abilities which is positive, as an increase in the use of freedom, until we emphasize it differently. On the other hand, there is a hierarchy which we mean to represent the strict laws, natural and the social, which deny the given freedoms. These “exceptions” are enough to notice, but you do not need to worry too much about them, because formula (1.2) combines the entire necessary aspects[7].

[6] Increasing regulation can increase efficiency against options and developmental.

[7] There are no exceptions in mathematical statements, they are all always contained in the given.

Another feature of this formula is *dualism* which follows from the commutative the scalar product of the vector ($\mathbf{i} \cdot \mathbf{h} = \mathbf{h} \cdot \mathbf{i}$). A man is an individual (with intelligence $\mathbf{i}$) in the social system (the hierarchy of $\mathbf{h}$). However, we can also look at the same formula as a society of biological cells. Formally, it is not a problem that the intelligence of a hierarchy (man) is greater than the intelligence of individuals (body cells), nor is such "anomaly" hard to find in nature. Let's just remember that an organization of ants shows signs of greater intelligence than its individual.

I emphasize that the terms used for freedom, intelligence and hierarchy here are somewhat arbitrary, because they do not express what they expected. They are formal, because they serve to describe the relation (1.2) which is a discovery and therefore does not represent something that could be known to us earlier. In fact, here we set new definitions of concepts of freedom, intelligence and hierarchy, in accordance with new knowledge, so we must go step by step, confirming what was correct and correcting the misconceptions regarding these new old meanings.

Let's imagine the amount of ℓ from formula (1.2) again as a glass into which we pummel partly the water $\mathbf{i}$, and then to the top with the oil $\mathbf{h}$. What happens if we do not have enough oil (or water) and the glass remains partly empty? Then the formula is not correct, and we consider it unsustainable.

In reality, this unstable state, part of the "empty glasses", can become a fear or anxiety among the given individuals that lead them to new authorities. It is interesting that this unbearable condition was recognized by Erich Fromm 1941, calling it "escape from freedom". In his most famous book (v. [5]), Fromm described the irrational attempts to avoid or reduce the unbearable feeling of loneliness of a modern man, which, in fact, only increases the sense of loss. He distinguishes three main mechanisms of escape from freedom: authoritarianism, destructiveness and conformism. The first two mechanisms are characteristic for a totalitarian, and the third for a modern democratic society.

After 75 years, we see that we can understand the same problem, the fear of the surplus of liberty, by means of a simple but still very general equality (1.2). First of all, we see that this is not only a problem of psychology, but also for other forces that will direct individuals with a surplus of freedom ℓ towards development of greater intelligence $\mathbf{i}$ or a greater bondage of hierarchy $\mathbf{h}$ or both, so that the formula remains true. That what can still supplement Fromm's discovery is the universality of the "fear", which in an equivalent form must exist in all living species, whether they were created naturally, evolving or artificially, either on Earth or somewhere in the Universe.

The prisoner has the reduced capabilities $\mathbf{i}$ and the increased confine $\mathbf{h}$, filling embarrassed. However, those in loves have a narrowing of consciousness and increased attachment, with love. Therefore, fear and love can also be accompanied by changes in freedom, which is also not seen from Fromm's interpretation.

Similar problems arise with a lack of freedom. If the number ℓ on the left side of the relation (1.2) is smaller than the product to the right, then there may be produced, for example, rebellions. When the individuals do not become stupid, they reject the excess of the hierarchy that clasp them.

I believe that the same formula applies to the non-living world, but then with further new meanings of concepts of freedom, intelligence and hierarchy. We'll talk about it later (see [2]). Here, we are awaiting long list of special cases, of which I will try to distinguish only characteristic or interesting ones.

Every interest or organized group attracts individuals who at one time have the need for authority. When this attractive force over the immigrant has a culture, then we are talking about assimilation. Similar situations, with the surplus[8] of freedom ℓ and the impossibility of increasing $\mathbf{i}$, will cause sympathy for the nation, race, religion, and cause loyalty to the family. Some individuals will become tough legalists; others will seek their authorities among criminal groups.

The Taoist, Buddhist or monk going into isolation excludes some perceptions and options, reducing the total freedom of ℓ. In this way, he can experience peace and tranquility outside the regulation of society, because the loss of social norms is greater, so he remains free space for new hierarchies, which then fills with bodily exercises, imposed restrictions, or increased religiousness. By losing some knowledge and skills that might be useful to him in society, but not in loneliness, the hermit deprives his $\mathbf{i}$ leaving even more space for $\mathbf{h}$.

Hence, peace can be an indicator of the balance of freedom and regulation. For example, while working with a group of students, a teacher can monitor their mood by keeping them in the "good" and without extreme emotions. By increasing the degree of diversity (ℓ) he can instigate the need for knowledge ($\mathbf{i}$) of the group, even in the case of increasing its authority ($\mathbf{h}$).

By diversity in teaching, I mean that what could increase the perceptions and possibilities of students, and thus their ℓ. This includes more interesting materials, a way of teaching, more active participation of listeners, and increased local freedoms. This will result in greater need for both knowledge and authority. This latter becomes the attachment of students to teachers that can last.

In the event of a failure to meet the need for authority, the struggle for domination will begin. Conflicts cannot stop until the equilibrium of formula (1.2) is established, with the sharing of participants around individual leaders. The struggle does not only concern the dominant, but involves almost everyone present, so the leader who coordinates the conflict becomes even less free. He consciously or unconsciously manages his followers, they listen to him. Both the leader and the listeners become (each in their own way) unfree.

This is a mechanism of conflict that today's psychology mostly does not understand, holding that the essence of treating peer violence in schools is in conversation and kindness, not emphasizing that the very appearance of the authority of the psychologist or the institution is that which heals. To confirm this thesis, I state the behavior of gangs in the streets. So ever the "insane" bandits were they can be controlled by their leader. In his presence, without his permission, there are no internal conflicts. It's a similar thing in the army. A good officer, in the sense of "just enough authority", can share to his soldiers the weapons without fear of abuse.

[8] It is thought onto the right side of the equality.

Especially, the question is what still leads us to solving the "authority problem", *bit-by-bit*, how does practical psychology work? Only one of the reasons is the need to treat special groups, over which the very person of a physician-psychologist would be insufficient authority, and then a growing percentage of such doctors generated by the education system. Democracy "feminizes" a society in the sense that it lessens the characteristics that we would sometimes call "men's". But, about it later.

Example 1.3.1. *Use formula (1.2) to explain production relations.*

Solution. Slave work has been replaced by serfs when slavery has become economically less efficient than feudalism. In harmony with the equation $\ell = \mathbf{i} \cdot \mathbf{h}$, serfs were given more freedom (ℓ) than slaves, while simultaneously increasing their intellectual engagement ($\mathbf{i}$), but with a slight drop in control ($\mathbf{h}$) by landlords.

The workers are even more efficient in today's democrat "enslavement" because their working ability ($\mathbf{i}$) and its discipline ($\mathbf{h}$) have increased, which is possible because of greater freedom (ℓ). This freedom has grown not only because of some illusion of greater freedom in democracy, but also because of the real increase in the number of choices (options) due to work on society's tolerances, as well as the development of new technologies. □

From these examples, we see that "we do not have to know all about everything to know some about something". Intuitively or in some other ways we had the parts of the knowledge we are talking about here, but with uncertainties or mistakes, without considering a wider sense. As Einstein once said that they "do not see the wood from the tree" and fail to "catch" the whole. What I would like to demonstrate here is the huge number of examples I have listed in the previous brochures, in so-called Cafes or Facebook discussions, which confirm the relation (1.2) and, on the other hand, I hope, with the absence of at least one single contradiction.

Example 1.3.2. *Who are the leaders of local hierarchies?*

Solution. Both cases are possible – when the leader is capable and when is not.

The first case is maybe a mafia boss, a good military leader, the best surgeon appointed to be the head of surgery. Because of its larger[9] $\mathbf{i}$, it requires less $\mathbf{h}$ for itself. Because of his greater curiosity, acuteness and general abilities, he can become a charismatic leader of a quality group, and for this reason, but because of the tendency to be in the lesser clasp of a hierarchy too, he easily comes into conflict with the leaders of the wider community.

The second case is the leader appointed by the wider community, for example, the ruling political group. With a low $\mathbf{i}$ such it is easier to load a larger $\mathbf{h}$, when it becomes a real obedient. Higher wages, and sometimes even higher workloads, are justified by the increased "responsibility" that really is. There are no brilliant successes, but there is no charisma, nor disobedience, and in a stable society, his alienation of work can last. □

[9]Larger or "bigger" vector has higher intensity.

All types of perceptions, including emotions, are proportional to the freedoms, as well as intelligence[10] or hierarchy and accordingly must be contained in formula (1.2). We have already seen this by discussing Fromm's "fear of freedom", or finding that "fear" can be both anxiety and even love. Danger and safety fall into this category as well.

1.4 Scalar product

The scalar product of the two vectors is a product of their intensities and cosine of the angle between them. On the figure 1.1 we see the scalar product of the vector **i** and **h**:

$$\ell = \mathbf{i} \cdot \mathbf{h} = ih \cos\varphi, \tag{1.3}$$

where $i = |\mathbf{i}|$ and $h = |\mathbf{h}|$ are intensities of given vectors, and $\varphi = \angle(\mathbf{i}, \mathbf{h})$ angle between them. The orthogonal projection of **i** to **h** has the length $i_{pr} = i \cos\varphi$.

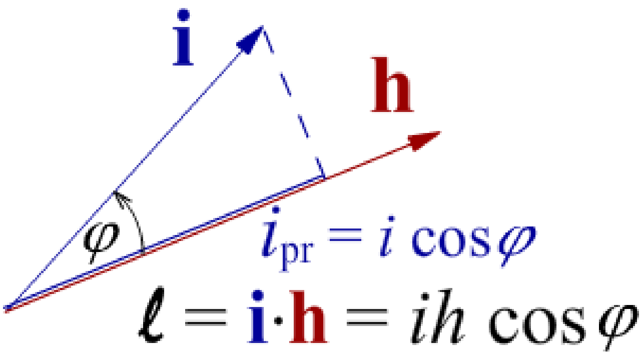

Figure 1.1: Scalar product

As we see from the given image (1.1) – when the lengths (intensities of i and h) of the vector grow, without changing directions – and without changing the angle φ, their scalar product (1.3) grows proportionally, since $\cos\varphi$ is a constant number. I hope this clarifies some of the concerns in the previous consideration. The next figure (1.2) represents the additivity of the hierarchies.

In the preceding text we talked about a glass partially filled with water (**i** in the image 1.2), and partly with oil (the two layers $\mathbf{h}'$ and $\mathbf{h}''$), which creates problems if it is not filled exactly on the volume of the glass. Now we are considering a different aspect of this charge.

That it is possible to substitute one hierarchy with another we prove by using vectors algebra. Namely, when we look at the hierarchies of $\mathbf{h}'$ and $\mathbf{h}''$ within the same society boarders (constant intelligence and the same perceptions) we have equations:

$$\ell = \mathbf{i} \cdot \mathbf{h} = \mathbf{i} \cdot (\mathbf{h}' + \mathbf{h}'') = \mathbf{i} \cdot \mathbf{h}' + \mathbf{i} \cdot \mathbf{h}'' = \ell' + \ell'', \tag{1.4}$$

[10] By the definitions introduced here.

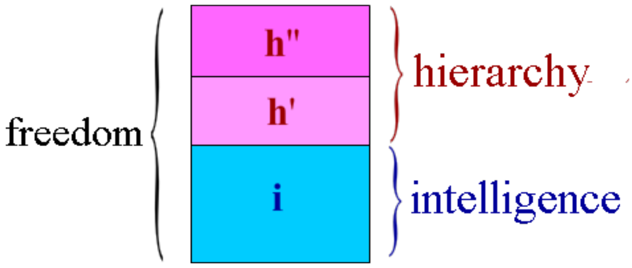

Figure 1.2: Additivity of the hierarchy

the consequences of distribution in scalar multiplication. This distribution generates the trait of a hierarchy we call *additivity*. On the previous figure, this would mean that increasing the hierarchy $\mathbf{h}'$, with constant intelligence $\mathbf{i}$, must lead to suppression, reducing the hierarchy $\mathbf{h}''$.

For example, the hierarchies in our society are family, religion[11], the legal system. According to (1.3), they are additive, which means that the strengthening of one hierarchy, say, the legal system, will suppress and choke the other two, the family and the religion. We both see it at work today. The countries in Europe in which democracy works for a long time, because of "protection", are more and more taking the right for them to judge in interfamily conflicts and even to take away children from their parents, presuming social laws before the natural. In similar European countries, with increasing legal systems, the influence of religion is decreasing. In the same formula (1.4) we believe if we want to "suppress the rule of crime" by "the rule of law".

Example 1.4.1. *Explain the Stockholm syndrome.*

Solution. Stockholm syndrome is the name of a psychological state that arises in situations where a rapprochement between hijackers and hostages appears. Incorporating this into the formula $\ell = \mathbf{i} \cdot \mathbf{h}$ looks like an impossible mission. However, it is not.

In the company of renegades, seemingly successful, which are not too bad for the victims, it weakens the perception of external authorities and overthrows it by the authority of the kidnappers. This attachment is enhanced by adaptation (1.11). □

The mechanism of the Stockholm syndrome has various experiences, as well as deliberate manipulations that attack our perceptions. It is possible to deal with untruths and through the subjective get on an objective plan. Successful manipulation confuses the perception, changing not only the "desired", but also the "real" freedom, i.e. the amount of opportunity $\ell = \mathbf{i}\cdot\mathbf{h}$. Here are some examples that are obvious, I hope, because the formula is correct.

1. When we use less intelligent employees to enforce the law (in the case of a stable freedom society, $\ell = \text{const.}$), we can expect that they will better implement them, as they (as well as the authorities in general) will appreciate them better.

[11] With the assumption that religion reflects a tendency towards authority.

2. Increasing the diversity of material and teaching can increase the perception of student freedom, so with the same intelligence their interest in laws may be increased as well as teacher's authority. After increasing the student's knowledge (**i**), by the time, can be degraded (**h**), and thus the interest in learning and the power of the school to learn.

3. In company, as well as in the army, members must have exactly the reduced intelligence as they need to feel its hierarchy in the each case of freedom. Namely, few of them remain in the company (or the army) when discover that his personal abilities are too great for the power of the institution.

4. When a leader promise to people more freedom by conquest (with the same intelligence of the people), he actually tries to increase their desire for a hierarchy, primary a military one. This increases his chances in a military campaign. That's how successful empires work; they who go to conquest also believe that this struggle can give them new freedoms.

5. Protests and demonstrations in democracies are often a sign that people are perceived to fall into less freedom, because of falling incomes or growing legal constraints. Rarely it can be and a sign of an increase in intelligence or a need for fun.

In all these and similar situations, when we say "greater" intelligence or hierarchy, we mean higher intensity (modulo, norm) of that vector. Thus, and vice versa, the "reduced" vector means its intensity is reduced.

Example 1.4.2. *Explain male-female relationships as a model effect.*

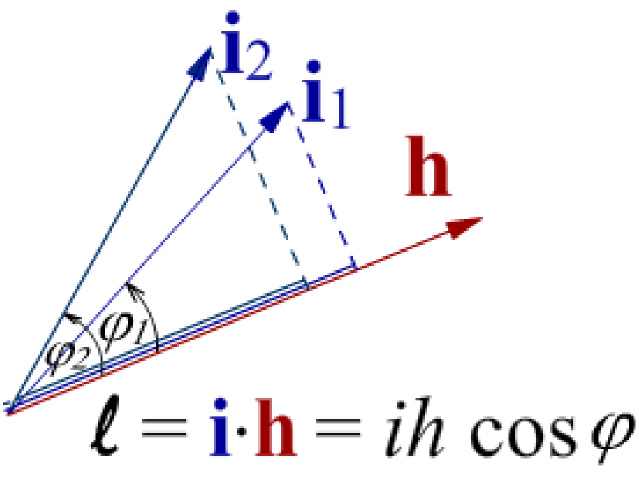

Figure 1.3: Male-female effect.

Solution. The vector **h** and two vectors $\mathbf{i}_1$ and $\mathbf{i}_2$, respectively men's and women's, of equal intensities i are given in the figure 1.3, but whith different angles ($\varphi_1 < \varphi_2$) to hierarchy. That's why they have different projections to the hierarchy ($\cos\varphi_1 > \cos\varphi_2$) and different scalar products, males larger than females. In the case of the same perception information, ℓ, the female corresponds to more hierarchy.

What that could happen? The social (external) hierarchy **h** has evolved over the past thousands of years under the domination of men $\mathbf{i}_1$, especially politics, war, wealth. We got an environment more inclined to the capabilities of men, with a smaller angle $\varphi_1 = \angle(\mathbf{i}_1, \mathbf{h})$. As the cosine of such a smaller angle is larger, the right side of the equality is greater in the case of men, $\ell_1 > \ell_2$, so women in such a society have less success and bear more authority. □

The above example is instructive, of course, not only because of male-female relationships, but in general, for example, for understanding different cultures. An alien can only look dumb because he is unadjusted. It can therefore be easier to deal with the rules that may be too strict for domestic.

However, in the long term, the group that is subordinated to a given society, precisely because of the hardship conditions, can become more capable, which would also eliminate the need for an additional hierarchy. For example, left-handed in a society of predominantly right-handed may be a little more intelligent, precisely because they and the society do not go hand in hand. The oldest child in the family has often a few of IQ points more than the next younger.

When we return to the male-female effect, we may now understand the greater representation of women in the judiciary, or their success in education. Thirdly, if the society were changed (feminized) so that the organization (vector **h**) becomes closer to a feminine than to a male vector, so the angle φ_2 becomes less than the male's angle, which is possible in three or more dimensions, women might be more successful than men in many other areas.

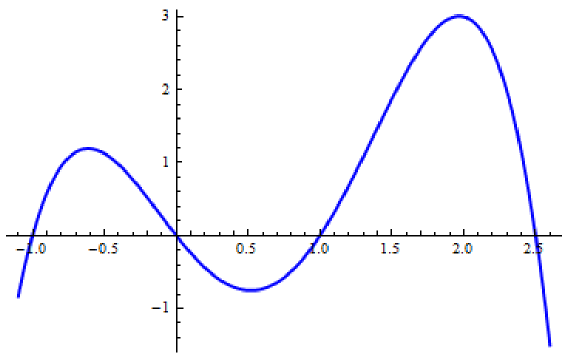

Figure 1.4: Local maximum, for $x_1 \approx -0.6$ and $x_2 \approx 2.0$.

However, these vectors have many components, that is, there are many hidden variables that determine the course of evolution. Society can be feminized in the way that bees have done. Striving for the safety and stability of species can evolve into so perfect that any later change should begin to decline. Like with the local maximum $y_1 \approx 1.1$ and $y_2 \approx 3.0$ functions in the image 1.4. In order to reach the second higher maximum from the first, it passes through the lower ordinates of function, and in the case of evolution step-by-step, a society that overestimates security and stability becomes unacceptable. If the rest of the living world evolves further, the bees could be found to be unhygienic in an environment that is no longer suited to them because of their (local) perfection.

1.5 Adaptation

The given properties are $\omega_1, \omega_2, \dots, \omega_n$ for $n = 1, 2, 3, \dots$ from the environment of some speace of coefficient of intelligence i_k and the hierarchy h_k in relation to the property ω_k for all $k = 1, 2, \dots, n$. Therefore, we have vectors:

$$\mathbf{i} = (i_1, i_2, \dots, i_n), \quad \mathbf{h} = (h_1, h_2, \dots, h_n). \tag{1.5}$$

These vectors are represented by the *components* in the n dimensional Cartesian right-angle coordinate system $O\xi_1\xi_2\dots\xi_n$. The intensities of these vectors are respectively:

$$|\mathbf{i}| = \sqrt{i_1^2 + i_2^2 + \dots + i_n^2}, \quad |\mathbf{h}| = \sqrt{h_1^2 + h_2^2 + \dots + h_n^2}, \tag{1.6}$$

so

$$\ell = \mathbf{i} \cdot \mathbf{h} = i_1 h_1 + i_2 h_2 + \dots + i_n h_n \tag{1.7}$$

is its scalar product.

In order to better understand the use of the representation of the vectors of intelligence and hierarchy by the components, let's look at the following examples.

Example 1.5.1. *The two sources produce* 50 *and* 40 *liters of water in the unit of time, from which we take respectively 20 and 10 percent of the liquid. How much water do we take in total?*

Solution. Mark $a = 0.2$ and $b = 0.1$ then $x = 50$ and $y = 40$. Total we take:

$$\ell = ax + by = 14 \tag{1.8}$$

liters of water in the unit of time. □

The word "water" in this example can be replaced with "information". What is even more important now is to note that the percentages of water consumption were *in line* with the capacity of the sources: from a stronger source we took a higher percentage. If it were inversely, the result would be

$$\ell' = bx + ay = 13 \tag{1.9}$$

liter, and that is the less water. So, when the arrays are matched, when the larger coefficient of the first row corresponds to the larger one of the second, then the result is maximal. This is true in general.

Example 1.5.2. *Prove that from $a \geq b$ and $x \geq y$, follows:*

$$ax + by \geq bx + ay, \tag{1.10}$$

where equality applies if and only if $a = b$ and $x = y$.

Proof. The claim follows from:

$$(ax + by) - (bx + ay) = (ax - ay) - (bx - by) = (a - b)(x - y) \geq 0.$$

□

From these examples, we see for vector $\mathbf{i} = (a, b)$ and $\mathbf{h} = (x, y)$ the scalar product $\ell = ax + by$ is greater if the vectors are "harmonized" in the sense that the larger component of one corresponds to the larger of the other and vice versa, the smaller goes with the smaller. We call the increase in such compliance in terms of greater adherence of the vectors the *adaptation*. It can be shown that this alignment, i.e. adaptation, explains *evolution*.

Example 1.5.3. *Prove that the adaptation of the species to the environment is actually its increase in freedom.*

Proof. Let's exclude only one property S from the environment of an arbitrary type. Let x be probability that the individual will survive if it has the property of S, and $y = 1 - x$ probability that it will not survive. In addition, let a' is probability that the first generation of individuals will behave in accordance with S, and $b' = 1 - a'$ that will not, and then a'' probability that the next generation of individuals will respect S, and $b'' = 1 - a''$ will not.

In the first generation, the freedom is $\ell' = a'x + b'y$, in another $\ell'' = a''x + b''y$. By subtracting we obtain $\ell'' - \ell' = (a''x + b''y) - (a'x + b'y) = (a'' - a')x + (b'' - b')y$, then

$$\ell'' - \ell' = (a'' - a')x - (a'' - a')y = (a'' - a')(xy) \geq 0. \tag{1.11}$$

This means that the alignment of the species with the environment, the "adaptation" of the species, increases its freedom, i.e. $\Delta\ell = \ell'' - \ell' > 0$ iff[12] $(a'' - a')(x - y) > 0$.

Namely, from $x > y$ and therefore $x > \frac{1}{2}$, follows $a'' > a'$. It is precisely this adaptation to a given feature followed by an increase in freedom ℓ. We have the same increase in freedom if there is $a'' < a'$ in the pair with $x < \frac{1}{2}$ (which means $x < y$), and this is again the "evolution into safer features" expressed by different inequalities. Conversely, if the species does not evolve to a greater chance of survival, then it is $x > y$ or $a'' < a'$, so the freedom is smaller, $\Delta\ell < 0$. Also, from $x < y$ and $a'' > a'$, there would be a fall of freedom, $\Delta\ell < 0$. □

Hence, the important conclusion that evolution, adaptation and the perceptions are going together. Evolution is the alignment of the properties of the individual ($\mathbf{i}$) to the

[12] iff – if and only if

environment ($\mathbf{h}$) so the product of the right side of the equation $\ell = \mathbf{i} \cdot \mathbf{h}$ is increasing, and this is an objective increase of the actual amount of opportunity in relation to the given species. Life tends to be free!

Laws do not have to be just natural, but can also be artificial, as we can see from the following example. Let's say that on a road there is a traffic sign of a speed limit on 60 km/h, for which traffic engineers have estimated that it guarantees a safe ride (in the case of driving no faster than stated) with a probability of $x = 0.95$. In other words, out of 100 drivers who adhere to restrictions, only 5 of them could have a problem driving, for example, the traffic accident. Then we have the vector $\mathbf{h} = (x, y)$ where $y = 1 - x = 0.05$ is probability that the ride will not be safe in the event of a violation of the prohibition.

Adaptation of the driver to the set ban can be measured as the frequency of the number of drivers who complied with the restrictions. If in the first measurements, the probability of compliance with the driver's limit would be $a' = 0.7$ (which means the probability of non-maintenance $b' = 1 - a' = 0.3$), and in the second measurement $a'' = 0.8$ ($b'' = 0.2$), then the freedom of the first and second group of drivers were:

$$\ell' = a'x + b'y = 0.68, \quad \ell'' = a''x + b''y = 0.77. \tag{1.12}$$

So again, the better adaptation gives the more freedom.

In artificial conditions, for which the evolution is too slow, this creation of excessive freedom by adaptation of the species does not necessarily lead to greater information of perception, but to an increase in the hierarchy. We are witnessing that in contemporary democracies the legal systems are growing, and now we know why and see it as a spontaneous process. The disadvantage of the excess of freedom, which is constantly being made by adaptations to new laws, always lead to further search for more order and security by the people. The people are who is asked there. In the periods of the rule of European monarchies, from the Middle Age to the present, the previous conclusions do not have to be valid, because the hierarchies in relation to the average inhabitants then may have been declining.

That we really can analyze the separate items, the components of the vector (1.5), is seen from their following algebraic properties. Let there be two pairs of vectors with a pair of coordinates: the first $\mathbf{a}' = (a_1, a_2)$ and $\mathbf{b}' = (b_1, b_2)$, and the second pair $\mathbf{a}'' = (a_3, a_4)$ and $\mathbf{b}'' = (b_3, b_4)$. Then

$$\ell = (a_1, a_2, a_3, a_4) \cdot (b_1, b_2, b_3, b_4) = a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4 =$$

$$= (a_1 b_1 + a_2 b_2) + (a_3 b_3 + a_4 b_4) = (a_1, a_2) \cdot (b_1 b_2) + (a_3, a_4) \cdot (b_3, b_4),$$

$$\ell = \ell' + \ell''. \tag{1.13}$$

This is the additivity of the component pairing, as opposed to the previous additivity of only one of the vectors in the scalar product. In this way, the preceding examples can be generalized, but it can be done also on a different way.

Example 2.3.1 can be generalized to the n-dimensional vector space mentioned at the beginning of this section. Before that, let's recall a few combinatorics concepts.

Various orders of a n-tuple are called permutations. For example, $(1,2,3)$ has six permutations:

$$(1,2,3) \quad (1,3,2) \quad (2,1,3) \quad (2,3,1) \quad (3,1,2) \quad (3,2,1).$$

In general, the n-tuple has $n! = 1\cdot 2\cdot 3\dots n$ permutations[13]. What happens to the sum of products (1.7) when the components of the hierarchy vector are ordered by size, but the components of the intelligence are not?

Assume that the arrays (1.5) are arranged from left to right, from the largest to the smallest. From the example 2.3.1 it follows that any substitution of the $i_k \ge i_j$ pair in front of the factor $h_k \ge h_j$ reduces the total sum (1.7). Thus for $n = 3$ we get:

$$i_1h_1 + i_2h_2 + i_3h_3 \ge \underline{i_2}h_1 + \underline{i_1}h_2 + i_3h_3 \ge i_2h_1 + \underline{i_3}h_2 + \underline{i_1}h_3.$$

In this way we prove the general, the following statement.

Theorem 1.5.4. *If $i_1 \ge i_2 \ge \dots \ge i_n$ and $h_1 \ge h_2 \ge \dots h_n$, then:*

$$i_1h_1 + i_2h_2 + \dots + i_nh_n \ge i_{k_1}h_1 + i_{k_2}h_2 + \dots + i_{k_n}h_n, \tag{1.14}$$

where $(k_1, k_2, \dots, k_n)$ is permutation of the n-tuple $(1,2,\dots,n)$.

Proof. The last index (if $k_n \ne n$) on the right side of the product (1.12) search (among i) on the left, then permute that factor with the adjacent on right, and again with the adjacent right until it reaches its extreme right position. Repeat the same procedure with the next right-hand endpoint (if $k_{n-1} \ne n-1$). With finitely many (less than n) repetition of such procedures, we obtain the required inequality. □

Vectors with harmonized coefficients in the manner in the given theorems, represented by the coordinates by oriented lengths, are closing the minimal angle (figure 1.1). Therefore, their scalar product is maximal. That is another proof of the same theorem.

Because of the commutative addition, the items do not have to be arranged to apply inequality (1.14). It is sufficient that the arrays **i** and **h** are "harmonized", in the way that the greater coefficients of one vector corresponds to the larger one of the other.

In order to generalize (1.11) to arbitrary n-tuple, from which the vectors (1.5) follow, you can observe couples similar to the proof of the previous theorem. In this way, a general conclusion is reached that by the adaptation to the environment of the individual its liberty is increased. In doing so, adaptation means making use of the rules of behavior that imposes the environment, and the environment is presented by laws that, if respected, provide individuals with a greater chance of survival.

In this, there is an unusual link between adjustments in order to survive and increase freedom for adaptation. Strangely enough, in evolution, "survive" means the same as "freeing" in terms of increasing the number on the right side of the equation $\ell = \mathbf{i}\cdot\mathbf{h}$. We said that this number is expected to be equal to the perception's information of the

[13] $n!$ read "n-factorial"

individual, on the left side of the equation. This result today (2016) is so unexpected that we need to try to understand it betters.

Let the properties $\omega_1, \omega_2, \dots, \omega_n$ are random events, with the information:

$$h_1, h_2, \dots, h_n. \tag{1.15}$$

Without worrying, for now, about definitions of the information, we see that these characteristics are the same those possibilities, the choices we have so far talked about. Let p_k be the probability of using the ω_k property of a given type, so it is

$$p_1, p_2, \dots, p_n \tag{1.16}$$

distribution of probability, i.e. $p_1 + p_2 + \dots + p_n = 1$. Maximum freedom

$$\ell = p_1 h_1 + p_2 h_2 + \dots + p_n h_n, \tag{1.17}$$

is gained for the harmonised coordinate components of the vectors $\mathbf{i} = (p_1, p_2, \dots, p_n)$ and $\mathbf{h} = (h_1, h_2, \dots, h_n)$. In other words, the individual is freer if it chooses more informative traits more often.

This at first glance seems to have nothing to do with survival, but we have seen that evolution means survival which corresponds to freedom. However, if instead of preserving the species, for the essence of evolution we take the fight for freedom, these connections become clearer. By increasing the information the individual increases its freedom together with the chances of survival!

This means that the information of the perception ℓ of a living being is a maximal function whose minimum value is the "information perception" ℓ_0 of the non-living being. The difference between these two is a positive real number $\Delta\ell = \ell - \ell_0 > 0$. Already from this follows that ℓ_0 could be well-known to theoretical physicists Lagrange[14] function, because the motion of the non-living matter is subjected to the "principle of the least action", according to which the trajectories of the physical movements are the solutions of the Euler-Lagrange equations. I wrote about this in more detail in the second book (see Cite RastkoPV), which was printed a year after this.

At the end, we have to see what is it the (mathematical) information.

For two ($m = 2$) equal opportunities, as in the case of throwing fair coins, the probability of the realization of each of them is $p = \frac{1}{2}$. However, two is not the number of information of such a random event (throwing coins), because finding out that one outcome happened we find that the other didn't happen. That is why Hartley in 1928 gave to the information "fell the Tail" (the same as "fell the Head") give value one. That is the number of the logarithm base two of the number two, $\log_2 2 = 1$.

For four ($m = 2^2$) equal opportunities, let's say four choices of one of the numbers $1, 2, 3, 4$ the probability is $p = \frac{1}{4}$. It is enough only two so-called binary questions to find out the result, the realization of one of such four equal coincidences. Let's divide the numbers into two groups of two, say $\{1, 2\}$ and $\{3, 4\}$. The first question is: is the

[14]Joseph-Louis Lagrange (1736-1813), Italian mathematician.

requested number in the first group? When the answer is “no”, the requested number is in the second group. The second question is: is the number 3 required? If the answer is “yes”, that is the required number, if the answer is “no” the required number is 4. Accordingly, the information brought about by the realization of one of the four random outcomes is 2. Note, $\log_2 4 = 2$, i.e. $\log_2 p = -4$.

For eight ($m = 2^3$) equal coincidences, say $\{1, 2, \dots, 8\}$, the probability of choosing one is $p = \frac{1}{8}$. In order to find it, we divide eight numbers into two subgroups of four, and by the first question we determine the subgroup with the asked number. Then again divide the chosen subgroup into two, and again. There are only two (binaries) questions for four numbers, so the total information for the eight is three. Note, $\log_2 8 = 3$.

In general, for $m = 2^h$ random issues of equal probabilities $p = \frac{1}{m}$ the information is:

$$h = -\log_2 p. \tag{1.18}$$

This is Hartley's [15] definition of information that is valid for $m = \frac{1}{p}$ equally random outcomes. Accordingly, freedom (1.17) becomes:

$$\ell = -p_1 \log_2 p_1 - p_2 \log_2 p_2 - \dots - p_n \log_2 p_n. \tag{1.19}$$

This form of information was discovered by Shannon[16] in 1948. Today's information is the mean value (mathematical expectation) of Hartley's information (1.18), whereby the basis of the logarithm, here $b = 2$, can be any real number $b > 0$ and $b \neq 1$.

If the base of logarithm (1.18) is the number $b = 2$ we have the binary information whose unit is *bit.* If $b = 10$ we have decimal information with the unit *decit.* However, the most common logarithm base is the Euler[17] number $e = 2.718281\dots$, the base of *natural logarithm*, with the unit information *nat.* The multiplication of Hartley's information (1.18) with the constant $\lambda = \log_b 2$ is changing the base logarithm into b, since:

$$h' = \lambda h = -(\log_b 2)(\log_2 p) = -\log_b p. \tag{1.20}$$

Then Shannon's information goes to units with the base b, but always remains defined over probability distributions (the sum of all p is one). However, it will not be enough for us.

Our body has thousands of processes all at the same time that are almost all unconscious. These are the processes of managing blood flow, various cellular exchanges, liver function, and digestion. So do our subconscious and consciousness. That's why we can talk, walk, and eat at the same time. This is called *multiprocessing*, as opposed to *multitasking* that indicates the operation of the processor processing one by one data alternately of multiple.

Shannon's information well defines multitasking, but it, unfortunately, is very complicated for the multiprocessing we need here. In fact, the emph consciousness itself, for which we want to believe is multitasking, is the result of thousands of unconscious

[15] Ralph Hartley (1888-1970), an American electrical engineer.

[16] Claude Shannon (1316-2001), American mathematician.

[17] Leonhard Euler (1707-1783), Swiss mathematician.

processes (which we cannot manage) and which form an illusion. When we think to made a decision, our subconscious sent us messages, prepared a few seconds earlier that we think that we would make that decision. This "scam" of the subconscious is the result of a multitude of processes, the good organization of low-intelligence cells that make up one collective, us personally.

1.6 Experimental evidences

Unlike previous considerations that were largely original and completely new, the text in this section is not. Only two known discoveries in the domain of human perceptions are detailed here, which should confirm the intuition of the founders of the mathematical theory of information, primarily Hartley and Shannon. First, it is the *Weber Law* for the threshold of the sensitivity of our senses, and the second is the way in astronomy is the classification of stars by their splendor.

1. Within modern psychology there is a discipline called *psychophysics*, which deals with the study of the perception. The term "absolute sensitivity" refers to the ability to detect environmental stimuli. Absolute *limen*, threshold of sensitivity, the smallest amount of energy a person can feel. This is the amount of light in sight, hearing volume, touch pressure.

Differential sensitivity is the ability to differentiate between stimuli. A measure of such sensitivity is called a differential limen or threshold of feeling, which in the classical psychophysics primarily concerned the dimension of intensity. This is the smallest difference in the intensity of two stimuli of the same type that the respondent can only differ in 50 percent of cases. The first discoveries in this area were made by Weber[18], on figure 1.5, measuring the differential sensitivity of the weight.

In 1834, Weber revealed the (statistical) concealment of the sense threshold:

$$\frac{\Delta W}{W} = \kappa, \tag{1.21}$$

where ΔW is needed increase in the intensity of the stimulus so the difference of experience is just noticed, W is the level of irritation, and κ is a constant. In other words, if the respondent at the weight of 50 grams could notice the variation of its weight only by increasing it by 1 gram, then at a weight of 100 grams he would be possible to notice a weight variation of about 2 grams.

Weber's constant κ, right in equality (1.21), for electric stimuli is 0.01; for weight is 0.02; for line length 0.03; for a volume of 0.04; odor 0.05; light and flavor (salty) 0.08. As these are small numbers, the sum of f in all of the thresholds of the lowest, absolute sensitivity W_0, to the given amount of W, can be estimated with an integral caclulation. We get:

$$f(W) = \frac{1}{\kappa}\ln\frac{W}{W_0}. \tag{1.22}$$

[18]Ernst Heinrich Weber (1795-1878), German physician.

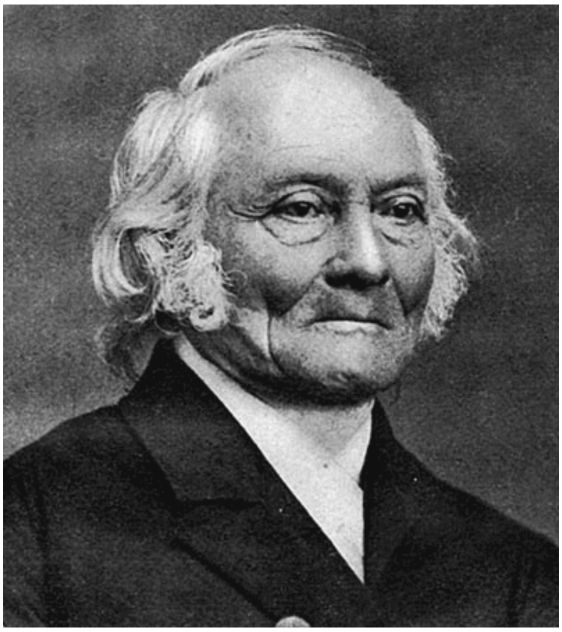

Figure 1.5: Ernst Heinrich Weber

This is Fechner[19] law of 1860. Note that the amount of perception $f(W)$ is formally equal to Hartley's information, and that it also represents the amount of freedom, the freedom of the given senses.

Because of the additivity of freedom, now with the different abilities $\mathbf{i}$ of the individual in the constant hierarchy of $\mathbf{h}$, we estimate the perceptions of our senses are also added by a simple summation into the total freedom ℓ. For $n = 1, 2, 3, \dots$ various senses, the total observations would be $f_1, f_2, \dots, f_n$, and their mean value:

$$f = p_1 f_1 + p_2 f_2 + \dots + p_n f_n, \tag{1.23}$$

where p_k is the probability of using k-th sense, in the order of $k = 1, 2, \dots, n$.

2. The first classification of the stars *glow* was given by *Hipparchus* of Nicaea in the second century BC. He divided the stars visible in the naked eye into six classes or *magnitudes* so that the brightest stars were in the first, and the weakest in the sixth magnitude. After Weber's and Fechner's law for all senses, in the 19th century, a mathematical expression for Hipparchus classification was set:

$$U = C^R, \tag{1.24}$$

that the geometric progression of the stimulus (the cause U) corresponds to the arithmetic progression of the sensing of the senses (R), where C is a constant. This general law is Pogson's[20]. In 1856 he applied it in astronomy.

[19] Gustav Theodor Fechner (1801-1887), German physicist and philosopher.
[20] Norman Robert Pogson (1829-1891), an English astronomer.

Let us mark with E_m the brightness of the star of the apparent size, magnitude m. By measuring it was found that the brightness of the star of the first magnitude is 100 times greater than the brightness given by the star of the sixth magnitude, so that $\log C = 2.5$ was obtained. Then we find:

$$m_1 - m_2 = 2,5 \log \frac{E_2}{E_1}, \tag{1.25}$$

where E_2/E_1 is illumination ratio measured for two light sources of magnitude m_2 and m_1. This is Pogson's law.

In astronomy, they tried to incorporate new results into old ones, so that the stars of the sixth apparent size were at the limit of visibility by the naked eye. However, many of the first-size stars differ significantly in their brightness, so the Hipparchus scale is extended to negative numbers. For the apparent size of Sirius, the brightest star not sky, $m = -1.4$; full Moon $m = -12.6$; Sun $m = -26.8$.

That the amounts of perceptions are logarithmic sizes, such as information or freedom, is no longer a novelty. However, this idea in astronomy goes on, on estimates of "pure" physical quantities, such as *luminescence*, i.e. the total amount of energy the star emits from its surface in the unit of time:

$$L = 4\pi R^2 F(R), \quad L = 4\pi R^2 \sigma T^4, \tag{1.26}$$

where on the left $F(R)$ is flux of the star radius R and the right is the same term with the assumption that the star radiates as the absolute black body of the temperature T. Using (1.26) we get:

$$M_1 - M_2 = 2,5 \log \frac{L_2}{L_1}, \tag{1.27}$$

where M_1 and M_2 are absolute star sizes. The most of the stars have absolute star sizes in the range of $-6 < M_v < +10$. So we get, for the Sun $M_v = +4.8$ ($m_v = -26.8$), Sirius $M_v = +1.5$ ($m_v = -1.4$), Vega $M_v = 0.5$ ($m_v = +0,1$), Alfa Centauri $M_v = +4,7$ ($m_v = +0,3$).

Chapter 2

Democracy

Democracy is a system of government that involves the people in choosing their rulers. According to the US President Abraham Lincoln words: "Democracy is a government of the people, from the people and to the people". In a narrower sense, democracy has a society in which a given group of people is given the same right to elect and be elected to the organs of political power.

The word "democracy" was derived from the Greek *demos* – people and *kratein* – rule, which would mean the rule of the people. The term was created in the 5th century BC when in Athens about 16 percent of the total society had the right to vote. The Demos, or the people who had the right to vote, made decisions in Athens straightly, by referendum. Every member of the demos had the right to one voice in each of such decisions. It was the so-called *direct* democracy.

Different forms of democracy appeared in history. The Athenians themselves, from 487 - 486 BC, improved their democracy by means of magistrates who were equated with the bearers of power, but were chosen by chance. However, democracy has so many bad sides that the desire for the power of demos has been wasted over time and the power of the equal is left to the authorities.

The Roman Republic (about 300 - 50 BC until the Roman Empire) was a democracy. In Western Europe, in the 12th and 13th centuries, various authorities were often elected by the vote of the people from the local deputies, through the head of the authorities to the kings, and even the pope. This lasted until the extreme strengthening of the authority of the Roman Catholic Church and the Inquisition, after which the European Monarchies dominated. The French Revolution in 1789 to Emperor Napoleon was carried by the ideas of freedom and liberalism. During the 19th and 20th centuries, on the theses of Marx[1], but also the interpretations of Engels[2] and the practice of Lenin[3], appeared communist countries with a democratic election within the members of the Communist Party. All communist rules ended with a dictatorship.

The first modern democracy was created in 1789, but in the United States. Modern

[1] Karl Marx (1818-1883), the German philosopher.

[2] Friedrich Engels (1820-1895), the German philosopher

[3] Vladimir Lenin (1870-1924), Russian communist revolutionary.

democracy is a system of government in which ultimate political authority, sovereignty, belongs to the people, either directly or through elected representatives. In modern democracies, "demos" are represented by all adult citizens, who equally (one man - one vote) elect their representatives, who for a several years rule on behalf of the people. This is *indirect* democracy.

Below we will see how the idea of equality leads to some desired consequences of democracy, but also of its deviations. I note that in the years ahead of writing this text, such deviations in society were generally considered as problems that should be removed from more democracy, not its consequences. We will also highlight some difficulties in the idea of equality of people in the conditions of the impossibility of their actual levelness.

Particularly, we'll emphasize the immediate consequences of the formula $\ell = \mathbf{i} \cdot \mathbf{h}$. On the left side of the equation is the information of perception ℓ which is the measure of the quantity of options, and on the right is the scalar product of the information vector $\mathbf{i}$ (the ability of the individual to use the options) and the hierarchy vector $\mathbf{h}$ (environment abilities to deny these options). The inequality of the left and right numbers is the cause of the urge for balance, some of the emotions like the ultimate: love - hatred, or pain - pleasure.

For example, the barely stirred instinct during the leisure in nature, when we are free from the obligations of civilization, urge us to make some order in the environment, or to fishing, or reading. The greater value on the right side of the equality means the excess of freedom for the perceptions of the given person, which is equivalent to the surplus of uncertainty. It forces us to organize ourselves, to be more efficient, to control, or to construct and advance. That is why equality, which is the state of the greatest probabilities and the least uncertainty of given random events, the best starting position for maximum changes. In economics, equity competition is the best environment for society's finances. In sport, by competitions in categories of equal, promise the greatest efforts.

From it follows the dualism of freedom and security, the dualism of progress and efficiency. Less of the first one means more for the second and vice versa.

Infatuation is the commitment of the blinded by the love that is created by lowering the value on the right side of the given equality reducing the vector $\mathbf{i}$, which influence on increase of the vector $\mathbf{h}$. This attachment can also go backward after "unblinding". Fear of imprisonment, mutilation or death is a form of rebellious urge that occurs even before the denial of freedom occurs. Archimedes' euphoria, when he jumped naked out of the bathing tub and ran through the streets of Syracuse, yelled "Eureka", came after the release of a surplus of uncertainty due to the discovery of the law that "everybody immersed in the liquid loses as much weight as the weight of the leaked liquid".

The application of the new formula to the history of Europe could go as follows. Let the average European's perception of information during the last millennium was about constant as well as the clamp of the hierarchy of the monarchies (it may have even declined). If the more grasping and smarter who had acquired more properties had more children, then the average intelligence of Europeans grew, so the beginning of the scientific revolution of 1500, and the French Revolution of 1789, is more understandable. If the similar reverse conclusion is valid, a decline of Western cleverness in the last century would happen, because the smarter while studying and working have fewer offspring. In the analogy we may understand the rapid growth of contemporary legal systems.

In this way, it is understandable that liberalism, the ideology that considers the protection of personal freedoms as the basic purpose of the existence of the state, is starting to shift from the initial need for freedom to the need for security, which is transfer of personal freedoms into the hands of the state regulations. On the other hand, the strengthening of the rule of law, as well as the evolution of simpler forms of life through colonies into increasingly complex forms, is also explained in the manner of the mentioned adaptation here.

2.1 Liberalism

Liberalism is usually the common name for political ideologies of state order which aspires to the greater freedom of the individual often through democracy and under the protection of the rule of law. The word "liberal" comes from the Latin word *liber* (free). Liberals of all schools see themselves as protectors of freedom.

From the previous text and the fact the theses set in these considerations are quite new, we can assume that the first theorists of liberalism had to come from the midst of a too high hierarchy (on the given intelligence), without understanding that the lack of the bondages could also be a problem. When we recall what is considered "liberalism" in philosophy, we will notice that the same repercussions have all the movements of a society in which is blindly striving for freedom with the wrong conviction that more freedom always means more happiness to people. On the other hand, it is shown that the ideologues of liberalism were naive, giving the state the right to control private freedoms, thus launching the process of alienating freedoms.

It is believed that the first true liberals appear in *Enlightenment*. It is the European intellectual movement of the 17th and 18th centuries who has synthesized ideas about God, reason and nature in the fields of art, philosophy and politics. The Enlighteners celebrated the mind of man, with his main qualities – knowledge, freedom and happiness. Among the liberals, the British *Whigs*, opponents of absolute power in the parliaments of the sixties of the 17th century, stood out. The Whigs took full control of the government, remaining dominant until King George III 1760, who forbid them and returned *Toryism*. The independence movement in American colonies is also considered liberal.

Liberals opposed absolute monarchy, mercantilism, and some forms of religious orthodoxy and clericalism. For the first time, they established the concepts of the rights of individuals and the rule of law, and the importance of managing the state through elected representatives. Liberalism as a conscious ideology that freedom is not merely a supplement, but a fundamental foundation of rights within the political entity, and later on the state, has begun to take on a clearer form in response to absolutism, especially in the United Kingdom. Two discussions about the government of John Locke (1632-1704) have established two fundamental liberal ideas: economic freedom (the right to own and use property) and intellectual freedom (including freedom of conscience). However, Locke did not extend his views on religious freedom to Catholics.

At the same time, French philosophers come to the idea of necessary limits of monarchs by law, and reversely, the economic liberals put forward the opposite idea of the "harmony" of the market, holding that market has to be on its own course, left besides the law. In continental Europe, liberalism as an ideological era emerges with France in the revolution and ideas of a minimal state, a strong state, but with the small domain of activity, and is reduced to support peace and security.

In general, liberalism is the original ideology of civil society, created in the 17th and 18th centuries, in the pursuit of that class to realize its political rights, to abolish the privileges and self-will of the authorities. The French Revolution proclaimed freedom to the highest ideal of society, and similarly to other western liberals. At the center of liberalism there is an individual who realizes his civil freedoms in the economic, political and cultural sphere of society. Unlike socialism or conservatism, liberalism is based on the ideas of personal freedom and personal choice.

Here we talk about liberalism as a political philosophy based on freedom and equality. While classical liberalism emphasizes the importance of freedom, social emphasizes the importance of equality to the right of the state to oversee the private life of an individual. A broader, liberal one is considered openness to new ideas and social experimentation versus conservatism that advocates adherence to traditional principles. Economic liberalism is an advocacy of as little state intervention in the economy as possible.

If we were even more comfortable, we could call the "liberal" every movement for freedom that overestimates the personal rights on choices and underestimates the need of an individual for the hierarchy. Any such liberalism, even the well-known one, must end up with some hierarchy tightness as stronger and faster strengthening as it was more sincere and more energetic. A similar conclusion applies to all living beings, but here we look only at examples of our (human) history.

It is less known that in Europe in the Middle Ages was a common practice of genuine democratic elections. After the fall of the Roman Empire (476), until the fall of Constantinople in 1453, Europe followed Christian ideas about the people who were all created equal. Consequently, it was not a rare phenomenon that various leaders were occasionally elected by voting on equal terms. So the firsts of the city, the region and even the pope were elected. On the other hand, we know that this was a period of massive strengthening of religious morality, religious dogmas and norms, and, finally, the Inquisition. The period of the monarchy was perhaps the answer to the fear of the Catholic religion, which was prevailed.

The sincere liberalism of French revolution from 1789, at high speed ended in 1804 with imperial Napoleon. Basically, liberalism was also fascism and Nazism, which ended with the terrible dictatorships of Mussolini and Hitler. In this broader sense, and communism was liberalism, and almost each quickly grabbed toward the dictatorship. Today's American democracy, which increasingly suffices the term "liberal capitalism", also rushes under discipline of powerful firms, corporations and banks, so that the masses are managed by few.

Liberalism is “a road to hell paved with good intentions”, but not only in Christianity. *Caliphate* is an Islamic form of power that represents political unity and the leadership of the Islamic world. In the past, there were many (Rashidi – a righteous caliphate, the dynasty of Umayyads, then Abbasid, Fatimid Caliphate, to Osmanli, which lasted until 1924). On June 29, 2015, Abu Bakr al-Baghdadi, the leader of the then Islamic State of Iraq and Levant, and since then the Islamic state, declared the world’s caliphate. It is the form of an Islamic state led by the caliph, which means the supervisor, the one who cares about the household of the ruler (God).

The original way of choosing a caliph (for the Sunnis) was extremely democratic and was not inherited, since the caliph was chosen by the *shura*, i.e. the supreme Islamic council, which was again on its side elected by lower councils that were chosen even lower to the bottom of the pyramid. And this is the same problem as in the democracy. Liberalism (in a wider sense) sooner or later produces some kind of autocracy, often evil, such as today’s Daesh, the Islamic State (ISIS).

Liberal projects from the past are one-sided. They collapsed, because the freedom of ℓ was not seen through the whole formula $\ell = \mathbf{i}\cdot\mathbf{h}$, but unknowing it they only magnify the intelligence $\mathbf{i}$ (ability of an individual to use options), and ignored the significance of that other factor, the hierarchy $\mathbf{h}$ (the ability of the environment to block the possibilities). As a child who puts the burden on just one side of the seesaw, without understanding the concept of equilibrium. That’s why it have happened to them that the other side bounced strongly, pinning up the scandalous liberals who would blindly believe in their understanding of freedom often again trying the same.

However, at the transition from the 20th to the 21st century, the power that through banks and corporations governs primarily America is that they have noticed the benefits of liberal movements in the world. As if they recognized this absence of a hierarchy in democracy and its advantage in its inability to oppose them. It was easy to manipulate the liberalized environments by the hierarchies of these powers, so American doctrine has become “spreading democracy by force or force” to rob. Let us analyze this phenomenon briefly.

It is known that two persons can do more work than just one, but the two heads are not always smarter than one. Namely, Fromm[4] noted in 1941 that the smartness of a democratic mass can be less than the average person’s in the mass, which was then the subject of many debates and experiments. Here is a theoretical proof of the Fromm state.

Dunning–Kruger effect (D-K) is a cognitive disorder when people with a lack of skills and knowledge suffer from illusory superiority in a certain area, mistakenly believing that their skills are much greater than they actually are[5].

In the image 2.1 it is seen (red line) that with the increase of experience (knowledge in the field) a sense of competence diminishes, so with the knowledge close to expert this feeling begins to grow slowly. The following is an example of the impact of this effect

[4]Erich Fromm (1900-1980), German philosopher.

[5] Dunning–Kruger effect: `https://en.wikipedia.org/wiki/Dunning%E2%80%93Kruger_effect`

on the intelligence of a group of people as a whole in relation to the intelligence of the individual.

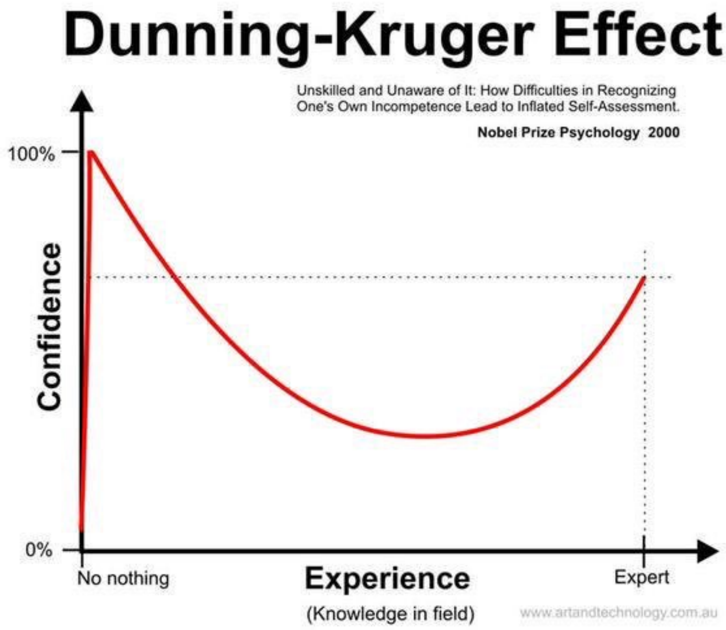

Figure 2.1: Dunning–Kruger effect.

Example 2.1.1. *Show that the D-K effect makes two people duller than one.*

Solution. Let's look at two people A and B separately, and then as a group AB on an intelligence test of just five questions.

Let each person give the same answers to the last three questions, two of which are correct ($\top$) and one incorrect ($\perp$), as in the picture 2.2. In the first two questions, A replied correctly in a row, and the person B was incorrect and correct. Thus, each of the individuals A and B individually has three of the five correct answers.

After a single response, these individuals agree and give another answer that represents the answer of the group AB. Optimal, for the success of the group, would be to propose the same common response to the last three test questions, but about the first they must be agreed.

There comes the D-K effect. On the first question, B has less knowledge and a higher power of persuasion, and they, as a common response, accept the (wrong) proposal of a person B. On the other hand, the person A has less knowledge and is more convincing, so the group for the second question also gives the wrong answer. That's why the overall score of the group is exactly five, which is worse than the results (3 out of 5) of each individual. □

The reason for the less intelligence of the group of the individuals was the Dunning–Kruger effect and the liberal decision-making. If the group was hierarchically ar-

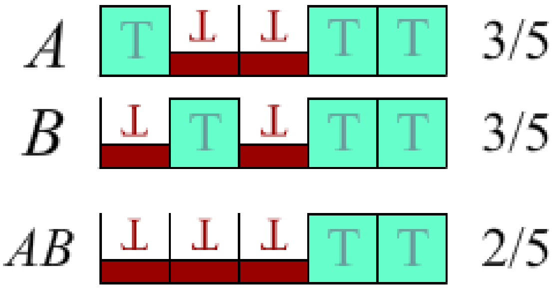

Figure 2.2: Test results.

ranged, under the pressure of the authority other members would agree with the leaders' opinion, and the D-K effect would fail. In the event that the leader is at least average, the result of the group would be at least average. That's why companies and armies are organized hierarchically, because they need efficiency.

Efficient corporations in the democracies are power as sharks among the small fish. They first mastered the US administration, and then began to use it as parasites to gain more money and more power. Hence the strength and domination of America in the 20th century, from the symbiosis of its initial democracy and emerging corporations (banking is implied). So amplified, America recognized the benefits of advocating democracy in the rest of the world and as every successful empire it began to use the combination of this new utopia and, of course, its military forces.

The alleged American struggle for human rights and the introduction of democracy "by will or by force" has become the most horrible nightmare for the countries that could no longer resist that power. If the countries designated as the prey try to cocoon or established some efficient hierarchy for better defense, it would be insufficient under the strikes of ruthless Western military machinery, and if they give up and switched to the "transition countries" of democracy, they would only become one more in a row of the powerless under the manipulation of soulless corporations' interests.

Democracy has become the most important weapon of war. At the end of the 20th century, America combined two of the most important prerequisites needed for every successful empire: it had an attractive utopia and had a strong army. However, at the beginning of the 21st century, there were cracks in American doctrine. After the "introduction of democracy" to Iraq (against Saddam Hussein), Libya (against Muammar Gaddafi), in Afghanistan, there was no improvement. The world began to notice that the "Yellow Revolution" is monitoring by the American predators and that in a series of similar interventions before and after the aforementioned, America bring to them only the destruction of the state and the misery to the population taking the values selfishly. And then the profit became questionable.

Utopia is an imaginary place, a state, or a system in which everything is perfect. The word "utopia" was coined by Thomas More of the Greek "ou" - not and "topos" - the place, for the ideal land in his book "Utopia" from 1516. After the publication of the More's book about the best organization of the state, the notion of "utopia" became synonymous with all the ideas that explore the possibility of an ideal solution, whether the organization of the state, relations between people or the end of any conflicts and wars, but have not been realized in practice so far, regardless of the attempts.

Democracy is a contemporary utopia. Like the earlier idyllic buildings, it will sooner or later become ruinous because its foundations are fake. The basic principles of democracy: freedom, equality, security, all about people of the collective, as we shall see, are not in accordance with the formula $\ell = \mathbf{i} \cdot \mathbf{h}$.

As we already seen form this formula, all the living beings are striving to the freedom, each as well as its perceptions allows. This is the opposite of non-living matter, whose freedom is minimal. The difference of freedoms between living and non-living beings, of the same matter, is positive number which grows with say "vitality" of the being. Let's try to understand the same conclusion at one more way.

What increases our chances and makes progress are options. With greater freedom, the chances of change are greater. Reducing hierarchies here means weakening any organization, security, efficiency, and increasing chances for unpredictable, truly original changes and real development. Consistently, as the greater use of freedom means more physical changes, real changes in some movements, this literally opposes inertia. Freedom and power go together, so that the bacteria under the microscope can be distinguished from dead particles by motion, and the same would, through the telescope, distinguish the spontaneous movement of the celestial body from the one controlled by the living being. The mentioned difference between the freedom of the living and the non-living grows along with the living physical force. If "vitality" is proportional to force, then it is proportional to both energy and impulse (momentum) as well.

There is not much difference between what lawyers call aggressive behavior and this "vitality". Aggression is a physical attack on someone, the movement of glass on the table, and the doing of good works. That is why legislation could, from decade to decade, more and more forms of aggressive behavior declare as illegal. And this is what legal systems do, protected by the ideology of liberalism. Hence paradoxically, although liberalism striving for freedom, it little by little of the freedom of the individual transfer to the law and order.

Communism did the same, turning freedom in security, in a similar but much faster way. It did not take advantage of the development that can bring opportunities of the real freedom, because it did not want to endure their unpleasantness and unpredictability. Great, real freedoms are annoying. They are "wild", primitive and have uncertain outcomes, and therefore we want to replace the development that they can bring as soon as possible for security. Such a rush is the fundamental mistake of the Communists. Communism did not recognize the chances of developing that offers initial equality, which happened in the slowed liberalism, but rushed into a regulated society – and lagged behind.

Liberalism itself, on long-term duration, easily becomes harmful to society, because the excessive glorification of the individual's values that in time leads to the society of ever-increasing selfish, egoist, or hedonist people, less responsible for the next generations. Liberal societies spontaneously are becoming prone to biological degeneration of people. For example, with the development of medical care, the risk of greater illnesses of the next generations is increasing. They who achieve a better status the children are obstacles, making them more capable to have fewer offspring.

Finally, liberal capitalism is facing a crisis because an increasingly small percentage of people have more and more wealth of the world. That is also the consequence of the ideology of protecting the rights of the individual by the power of the state. Many freedoms remain the privilege of only the richest.

2.2 Equality

People have different traits: height, weight, hair color, character, and some of these values can be found in two or more individuals. Alice and Bob can celebrate their birthday the same day, although they may not have the same number of years, are not the same sex, and they do not have to know each other.

When two people A and B have a property of "ω" equal, then we can say that these two people are *equal* in relation to the given property. We can formally write $\rho(A,B)$ or $A\rho B$. Both modes resemble the mathematical *relation* equality, which is why we can write $A = B$. Using mathematical forms, we get an extreme possibility of precise expression.

After selecting an arbitrary but still fixed property ω, each equality is a *relation of equivalence*. It is commonly referred to as "$\sim$" and always has the following three properties valid for any A, B or C:

1. Reflexivity: $A \sim A$,
2. Symmetry: if $A \sim B$ then $B \sim A$,
3. Transitivity: if $A \sim B$ and $B \sim C$ then $A \sim C$.

For example, if the property ω is "nationality", then $A \sim B$ is read, "A and B are of the same nationality".

Why do you need extreme precision? Due to the high sensitivity of the consequences of certain dynamic systems (which "evolve" over time) to the initial conditions. It was discovered by Lorenz[6], the founder of mathematical theory of chaos. This sensitivity due to the title (that the movement of butterflies in Brazil may result in a tornado in Texas) of its annex (see [7]) of 1972 popularly called the *butterfly effect*.

Contrary to the *instability* of butterfly effect, Kuramoto[7] describes the *stable equilibrium* model. He studied (see [8]) systems that lead to synchronization that occurs in certain situations when chaos spontaneously develops into a stable pattern.

Extremely precise expression in the use of equality among people is needed because there are not exactly equal people, and the consequences of even the slightest inequalities can become very significant over time.

[6]Edward Norton Lorenz (1917-2008), American mathematician.
[7]Yoshiki Kuramoto, 1940, Japanese physicist

Each person, even any individual living entity, has some own characteristics different from all others. The multitude of the properties represents one biological entity, the basic biological system. While the properties of different individuals can be the same, two individuals cannot have all the properties equal. Nature does not like that equality!

In 1925, physicist Pauli[8] discovered the *exclusion principle* in quantum mechanics, which states that two identical material particles, fermions, cannot be in the same quantum system, that is, they cannot occupy the same quantum state. In the case of an electron, this means that in the same atom two electrons cannot be equal in all four quantum numbers (n, ℓ, m_ℓ and m_s). The result of this is Mendeleev[9] elements table. Even if are of the same composition, the group of material particles cannot at the same time be in the same place and, accordingly, they must enter into different interactions with the environment.

Starting from its smallest contents, nature avoids equality. It will not make two identical snowflakes, although each will have exactly six sides. There are no two identical tree leaves, although they all do photosynthesis. No matter how two men are similar, they are never exactly the same.

When we advocate for the equality of people, we lie and this "trampling" of truth has its consequences. Consistent insistence on "equality" leads society's science into already noted phenomena that are still considered to be local deviations, and for which a solution is sought by improving democracy. Aware of this problem we escape from the statement "people are identical" in the statement "people are equal".

Equality here means equality in equity, equality before the court. This is the equality of all members of a particular community against the prescribed rules or laws of that community. The need for equality comes from the desire for freedom, justice and righteousness for every human being, fed by the belief that a society that gives equal opportunity to every individual for success in life, whatever is considered a success is possible.

The theory of society knows difficulties in establishing equality and justice together, as demonstrated by the image 2.3. Three unequal figures are presented in the picture on the left and right. To the left, the positions of the fans behind the fence are equally divided, and to the right, these positions are divided justly.

Demonstrating the disagreement of the concepts of equality and fairness, some professor in the classroom, in front of the table, in front of the students sitting in their benches, set up a basket in which they were supposed to throw balls in the basket. Better success will have those who in the same number of attempts with the same type of balls in the same basket score more hits. Already after the first attempts, it became clear that they closer to the basket had better chances. This is not fair! – Students from the last bench protested – because we cannot as easily shoot as can they ahead. That's right, and so is in life – the professor answered – because even when there is equality of goals and rules of the game, that does not mean the equality of your chances!

People are unequal in their needs and desires, and therefore in the experience of what

[8]Wolfgang Pauli (1900 - 1958), Austrian physicist.
[9]Dmitri Mendeleev (1834 – 1907), Russian chemist.

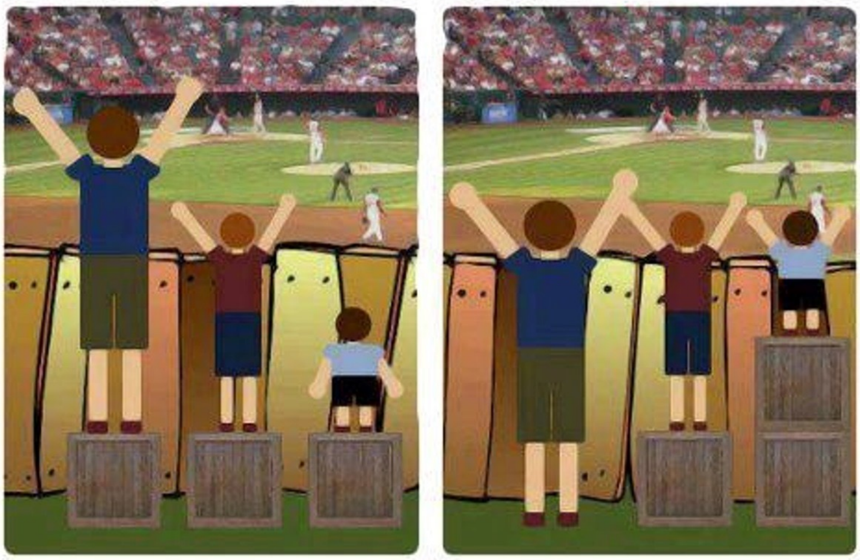

Figure 2.3: Equally and justly.

we call "fair play". When we declare their equality then we "push under the carpet" their differences that later escalate. In the end, we are again faced with the consequences of the previously stated impossibility of full equality of people.

Let's assume a higher priority of *righteousness* instead of the priority of equality in society. It is also a thought experiment.

Let's imagine an isolated society of unequal persons in conditions of limited supplies of food and water. Bigger people or people with higher, faster metabolism need more food and water than smaller people. They are larger, faster consumer inventories. In order for everyone to live equally long before they die of hunger or thirst, food and water must be distributed unequally. But, the duration of life of this population will be longer if that society sacrificed those individuals who are the largest consumers, giving biggest of them equal amounts of food as to the smallest.

This example demonstrates the loss of *efficiency* for righteousness, which is sometimes unacceptable, even for the greatest righteous. Like when one say it's not okay to "slaughter the ox for a kilo meat". Efficiency is acceptable when the military has a legal way to sacrifice a few to save the majority. Contrary to the fluctuation of people between the choices of justice or efficiency, for evolution, the development of the living world on the planet, efficiency is more effective than equity or equality.

Similar inefficiency of equality also exists in all cases of generalization. Say, when "equality" is generalized to humans, animals, and even other living beings[10], in the sense that they are "equitable" when they put the same "right" on the resources around them. When two predators feel equally strong or hungry in front of the prey, then we can

[10] A living being chooses options, unlike the non-living which is left to the empty laws of probability.

consider them as "equal" in relation to hunting that might follow. When given the soil is equally suitable for the growth of two cultures, then these cultures can be considered "equitable" in a given situation.

The announcement of the notion of "fairness" could go towards the feelings and solutions that the subject was satisfied with, then towards voluntarism, then in the direction of spontaneity, respectively from conscious to unconscious living beings. Such "generalized justice" would again be inconsistent with efficiency.

For example, we mentioned the saying that the road to hell can be paved with good intentions. Another example: when an animal follows its wish for food, it can fall into the trap. Third: species that evolved over time into a safe and stable and reaching to (local) perfection can be found in such a state of stagnation that any genetic change for them will mean (temporary) decline, until the surrounding species develops so much with their development, so they can be extinct.

Therefore, we should not take the righteousness we seek to be too serious, if for nothing else, then for the future in which this tendency will take us, which we may not like. Another important feature of the notion of justice is its instability, at least when it comes to seeking the leading principles of society.

My opinion is, that is why we found separate male and female sexes, especially in complex biological species in dynamic environments. Assuming choices and options, and then the existence of the free will of a living being, we assumed that the nature has objective coincidences too, that the material world is not completely deterministic, although it is subject to strict abstract laws. For similar reasons, the non-smoothness of the soil, today we do not have species that would have wheels instead of legs.

Accordingly, reality can make unpredictable jumps. The role of male sex is the survival of the species in such interruptions. For example, in all primates, males more often perish than females; males are prone to suspicious initiatives and risk, ventures and ride for a fall, females to certainty and stability. With its rushing and sacrifice males bridge the inevitable unpredictability, making the male-female species more successful in the evolution. This means that females and offspring can remain committed to safety, balance, and non-risk, which means that the collectivity has better chance to lasts.

The consequences of proclaiming the coincidence real has more. The non-living matter spontaneously pursues "equality", it is more physical to say to amorphous, uniformly dispersed states, that is, to the scales of uniform probabilities, as if it were hiding something. This concealment of information is general occurrence[11] from which follows the spontaneous growth of entropy (the second law of thermodynamics). On the other side is a life that strives for freedom, that is, to options, but alas, making choices and striving for an organization, as if limiting these possibilities. The living being is limited by its own biological abilities, by the information of perception, which is why it comes out again that nature is reluctant to provide information.

Let's again go back to the nearer past of the human societies. While righteousness is sloppy, efficiency is inhuman. It is the all essence of the constant change of social systems throughout history. In doing so, the reasons for which the community is ineffective are

[11] it is elaborated in the next book [2]

changing in history.

Social organization is made up of individuals with a sense of collectivity who want to satisfy their own selfish interests. Even in the struggle for an equal society, there are basically personal interests, as well as the insights of the rebellions that the compromise is beneficial, that it should not need to look for herself more than can be given to another. At the heart of this compromise is efficiency for winning.

Efficiency is at the root of the struggle for equality. However, the idea of equality is utopia, which in turn must therefore be replaced by something else. Whatever else, it is not equality and leads[12] into the hierarchy[13].

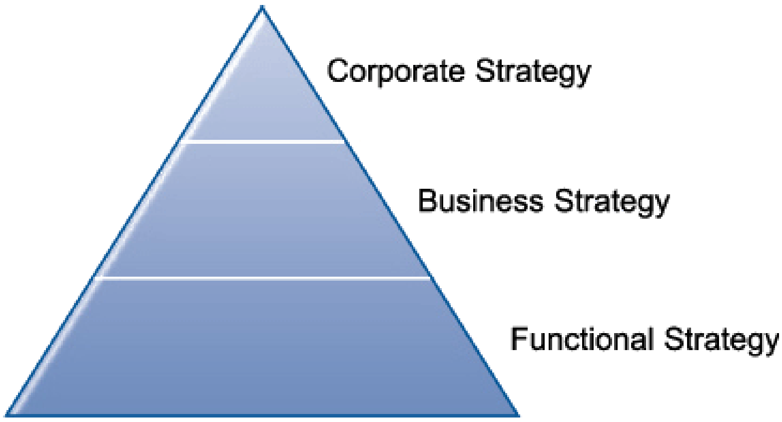

Figure 2.4: Hierarchy in the business.

A hierarchy is an organization or social system whose members are ranked by status or authority in levels of equal (see figure 2.4). At the root of each inequality of groups of the same type are similar desires, i.e. needs, which can be linearly ordered according to the degree of power and domination. This comes from living in groups of the same species, which during the evolution of a series of generations succeeded in finding food, overpowering the enemy's environment and multiplying. Hence the negation of a straight organization leads to a hierarchy.

According to the D-K effect, democracy has a minimal hierarchy ($\mathbf{h} \to 0$), which according to the formula for information of perception ($\ell = \mathbf{i} \cdot \mathbf{h} \to 0$) on long-term is unsustainable. Hence the conclusion, nature, regardless of ours opposition, will find ways to violate the ideal of equality and will produce some authority and hierarchy. Therefore, insisting on the equality of teachers and students, for example in American public schools, leads to the domination of usually the worst grade students and the "unexplained" juvenile violence. Insisting on the equality of women in some[14] jobs, say office, lead to women's dominance in those jobs. Insisting on an equal society of liberal capitalism leads to great differences between the rich and the poor.

[12]inequality + order = hierarchy
[13]It would be too much to reduce the existence of the hierarchy to Banach fixed-point theorem.
[14]Insisting on equality is always selective.

We said that equality is unstable because it is difficult to tolerate a surplus of freedom that comes with too many opportunities or their good balance. Therefore, in the conditions of equality, we are first looking for an organization, explaining it with the need for efficiency, which is actually a reflection of our inability to govern the offered variety of options. By organizing we maim the opportunities, deciding to make some of them the best, and declaring the result as efficient.

Efficiency is a kind of utopia. It can be proved that there are no "best" criteria for the elections[15], whether it is a democratic selection of representatives, whether is about choosing the best candidate for a job based on the "best" points of the faculty and "best" grades. It is paradoxical that identifying for one, any search algorithm always excludes many options, and with them maybe better solutions, which we ignore in practice. The inability to have the best criteria is the recent discovery in mathematics for which was awarded the Nobel Prize for Economics in 1972. The best chance to get to (statistically) the best leader would be to set up a thick sieve for separating three to six candidates, then choosing one in a random way. This last complete coincidence "frees" additional opportunities and proves the breadth of the choices from equality.

When we look around a little, we will see the positive and negative consequences of this impossibility of the best choice everywhere. History is full of geniuses who have indebted the world without having been educated in the "best" way. The best of democracy today can be considered the worst tomorrow, when it turns out to be the cause of the physical and mental degeneration of the population. This entire modern fine story about the development of security, stability and feminization of society falls into the water if it turns out that such a society is being trapped in, or perhaps, a collapse of civilization. Some of the consequences are not obvious, such as the following.

We know that in the sport of team competitions we sometimes see that the A team loses against B, then B against the C team, and then C loses from A. This is usually attributed to a change in the form of a player and a team, which in fact is not always true. That a similar phenomenon is mathematically possible (timeless) shows the next example.

On the figure 2.5 we see the so-called magic square, which has nine fields with different digits (1 - 9), whose sums are 15 in each row, each column, and both diagonals. Let the points of the teams A, B, C in their mutual competitions be presented in the columns. Thus, the first player of A lost against the first player of the team B by $4:9$. The second lost with $3:5$, but the third won with $8:1$. However, the team A lost from the B team by the number of players with $1:2$. Thus, the number of winning teams is $B:C = 1:2$, but $C:A = 1:2$.

There is, therefore, a logical possibility of circular development that can ultimately lead us back to a similar start. This explains the circular flows of history. Indicates on relativity[16] of the term "hierarchy". Thirdly, the example demonstrates that on the basis of "good" fixed criteria it is possible to choose the leader A that is better than the leader B, which is better than C, which is still better than A.

[15] Arrow's Theorem: `https://plato.stanford.edu/entries/arrows-theorem/`

[16] In the book [2] has been shown that probability, information and entropy are relative.

4	9	2
3	5	7
8	1	6

Figure 2.5: Magic Square 3×3.

2.3 Authority

Authority is the power or the right to order, decide and impose obedience. Authority is the ability or decision-making criterion too. Also, authority is a person or organization that has the power of control, especially in the field of politics or administration. The opposite of authority is powerlessness or inability to command, rule or decide. Words with the opposite meaning of authority are impoverishment, then indifference, equality, and anarchy.

From a broader point of view, the entropy of the gas in the room spontaneously increases because the gas molecules try to take as much as possible a "more equitable" position. Because the air intends to distribute uniformly in a room, to become "impersonal" with equally probable positions of the molecule, the Second Law of thermodynamics acts on the gas particles in the room: "The total entropy of the isolated thermodynamic system is increased to its maximum value". In this way we come to the conclusion that nature is trying to hide information. So things are in relation to the macro observer, in this case to someone who sees the whole room. However, if we accept the hiding of information as the main principle, then from the point of view of a living being, we have their spontaneous transition from equality to the hierarchy. The individuals want the state of maximum equality of all options and, accordingly, the maximum perception information, change to an organization.

In the conditions of equality, hierarchies come for another reason, to resolve the conflict. Where great information is there are great uncertainties that easily pass into the struggle for domination. Under the conditions of a large number of options and great opportunities, instead of originality and genuine development, when this excess of freedom is perceived as a nightmare, we are happy to choose the path of calming, direction, and canonization. Instead of freedom, we then choose order and security.

To better understand the discovered side of authority, consider the situation with two or more individuals, entities $X, Y, \dots$, and an indivisible goal A that they want to achieve. This goal can be the leading position of the group, winning in a competition, a unique object, or anything that can be attributed only to one of the subjects. When we say that two or more persons $X, Y, \dots$ are equal, it means that they do not have the criterion for partitioning A. They can then decide on *conflict*. Valid is the vice versa too – if two or more people are confronted, it means that they are equal in the given situation.

The conflict arises because of the absence of another decision and judgment. This trait of conflict, to be the kind of criteria, i.e. the ability of the conflict to be a substitute for the decision-making authority, is universal for all living beings. If equal individuals in the living world of the conflict could be equally identical, constantly pursuing their goal, then the conflict could last and last, with no outcome. Then the significance of the conflict in decision-making would be lower. However, there are no identically equal living beings, and that is why the conflict is so common in the living world in general.

Equal groups of individuals follow the same form of conflicts. Different political parties $X, Y, \dots$ can claim the same right to an indivisible political goal A. Individuals, party members, or voters themselves, do not have to be equal (in any sense). However, the conflict of a group is transferable to individuals, group members. The experience tells it to us.

For example, the *nations* has become visible with the emergence of modern democracy. National sentiments existed before, but over the last two centuries they have become significant, reinforced. They gain importance because of the declared equality of political interests, that is, the equality of parties fighting for power. The contested goals of the parties are the cause of "tearing at the seams" the demos and the formation of national leagues.

There is also a conflict between the unequal, but due to a wrong estimate, as it is sketched on the figure 2.6. The left opponents X and Y (blue) are not at the same level, belonging to the I and II league, respectively. The first, X, overestimates the opponent Y and sees it as Y' (gray) and starts a conflict. If the other returns, after the dilemma, to run or to fight, the opponent Y underestimates the opponent X and sees it as X'. Then the other loses. Like catch Y against the hunter X.

$$\begin{array}{ccc} X \to Y' & X' \leftarrow Y & I \\ X' \leftarrow Y & X \to Y' & II \end{array}$$

Figure 2.6: Conflict of the unequals X and Y.

Another, similar situation is on the same image 2.6 on the right. The first opponent is in the lower league of the opponents Y (red), whom he underestimates and sees him as Y'. If he returns, then Y usually win. For example, when the desperate X attacks, Y loses.

An example of the first situation is the US attack on the state of Iraq in the summer of 2003. The attacker deliberately and covertly overestimates the opponent or underestimates himself to justify some evil reason for the attack (let's say the robbery), but by working with the conflict to increase its significance and come out with a heroic victory. We know that America then defeated Iraq, but that the Austro-Hungarian Monarchy in 1914 similarly attacked Serbia and lost.

The first type of attack can be internal state terror. In 1941, the Independent State of Croatia overestimated the importance of Serbs in order to do genocide (to kill a third, to Catholicize a third, to expel a third). The Ustashi then killed about 1.4 million innocent Serbs, Jews and Roma. This exaggeration of the danger from the Jews was done by the Nazis too. Leaders have changed the perception of their citizens X, increasing the danger of national minorities Y, building a conflict with Y'.

In the case of Croatia, the attacked Serbs were the citizens with nation-state fillings, with an overwhelming sense of their power, and they could be the bearers of the guerrilla resistance against the new state. But then, the Jews were not a people with a national state and could not experience themselves as equals with powerful Nazis.

We note that the terms "overstating oneself" and "underestimating the opponent" here have the same meanings. Therefore, the error of perception by understatement can be reduced to the first case. And vice versa is valid too – when a hunter becomes a persecuted. Of course, both conflicts can happen with conscious or unconscious misconceptions.

We have already seen that the idea of equity as well as the idea of equality is false. In mathematics, as well as in exact science, there would be its end, but things do not stand in the same way in politics. People like to lie, as they like literature, film and general fiction. We have risen above other species not only because we understand the algebra, but also because we can lie, because we can persevere in something even when it becomes clear to us that it is half-truth. Half-truth is actually an untruth, which is exactly why we find it attractive, because it contains both irreconcilable worlds. That is why we love to listen to the predictions of the prophets, the promises of politicians, or the deduction of lawyers.

For example, if we consider people equal, we should point out that we will equally judge them for equal work. However, if people are truly equal, it is unnecessary to point out that they have to be equally judged for equal work. In mathematics, we would note that from the same assumption we received both the assertion and the negation of the claim, from which follows the unmistakable conclusion that the assumption is incorrect. This is the basic method of mathematical proof, by reduction to the contradiction, since it is only from the incorrect assumption possible to deduce both the truth and untruth.

In the judiciary or social sciences, a good polemic can, from the same assumption, prove both the thesis and the antithesis. When you point to some contradictions, he will defend himself arguing that mathematics is inapplicable in social sciences. There are movements in philosophy (see [6]) which "determine" it is so. Because of this stubbornness to confront the truth, the rule of law, together with the theorists of society, is going deeper into deceptions.

The magnitude of equality diminishes the authority of the individual, and the need for some higher superhuman authority arises. In Christianity it was God, and in modern democracies it is mainly a legal system. The first was unprovable and in practice (after a long time) bad, and the other is especially paradoxical primarily because it comes from the tendency of being above everything, and then puts man under this artificial system silencing the fact that natural laws are now considered insufficient and even incorrect. Sooner or later, the second also will come out as bad.

Legal System is the highest authority of democracy. It maintains the utopia of the system and controls the conflicts that are constantly arising from equality. That the conflicts are generated by equality is a discovery that opens many questions, but before all: Why we so much believe in lies? Why we cannot believe to leaders, but can to justice? Where does that lead?

Why we believe so much in the lies? The answer is in the escape from inconvenience that brings too much freedom. Excessive options, excess opportunities that create better chances for true development – cannot be tolerated. We therefore run into limitations, into the legal system, to the organization of security or stability or efficiency. By increasing the organization that reduces freedom, the above mentioned "principle of information" is fulfilled, according to which nature is austere by giving information. We are lying because it is principally on the wider plan, which would be fulfilled anyway, if we would give up of the artificial hierarchies.

When we say that something is *civilized*, it actually means that it is unnatural, that is, out of the flows of a long series of generations to which we are today. Then we easily arrive at the conclusion that the institutions of society, and just a democratic society, are somewhat unnatural. In other words, they are somewhat uncoordinated with our genetic heritage that many, much longer lasts from contemporary problems.

The women who left the offspring, whose characteristics were reaching our times, preferred the dominant men. In fact, this should jet be established, but it seems logical that those mothers who live in the vicinity of the tribal and resources of leaders in times of shortage would have greater chances to transfer their heritage to us. They did not have time for the outside world (outside of the family circle) as their partners. That's why our leaders are more often men, selected among more efficient. Whatever this "more efficient" meant, from the previous analysis follows that they are less fair ones, because the efficiency does not always go with fairness. That's why, we today, wanting justice do not trust to our leaders! But, we want to believe in the judgement of the community.

With the proclamation of equality, law, administration and bureaucracy become more and more important. Because equality creates conflicts that need to be regulated, channeled or condemned. By insisting on the equality of citizens of a democratic state, they have a constant need to resolve the conflict. Over time, they become overwhelmed by legal doctrine, legal institutions, and legal bureaucracy and burdened with legal costs, with omnipresent lawyers, by governments, and assemblies. Equality, the foundation of democracy, results in lawyers being the most frequent participants in government and state institutions.

Democracy cannot live without tendency towards better legislation, towards logical laws without contradictions and those that are not missed or inapplicable. It maintains its conviction by mass participation and aspirations to paragraphs that intelligently, consistently and fairly cover all spheres of life and all possible types of conflicts. It strives but never touches a system of perfect lawyers and trials in terms of accurate and honest implementation of legal acts, without any corruption, arbitrariness or incompetence. The first question is how would such a country look if it were possible?

First of all, it would be a non-confrontational society. For that reason, it would be static and boring. The state would have a problem of muted or completely stopped competition, which would cause difficulties with motivation and economy. Once we made a non-conflicting society, many would then realize that they do not actually want it. Conflict is the backbone of the appeal of art, sport, and life. Imagine informative shows without shocking news about conflicts. In such a perfect country, people would be the biggest problem [17]. Our unstable feelings of justice and order would be much uncivilized.

Communism was an accelerated attempt to create one such "ideal" society, but as we know, it collapsed in collision with the efficiency of *capitalism*. What really happened here?

The communists ignored the objectivity of chance and its power. Stifling unpleasant freedoms they quickly passed over to the comfort of security and stability, and lagged behind those who had longer "suffered" surplus of possibilities. By deciding on Statism, the state control of the economy, they did not use the power of all options in the economy, but were firmly at the point that they "know on what the world is standing" and how to make progress. If there was no competition (capitalism), the Communists would have lasted longer, though at a lower level and at a slower pace of development.

Let me repeat it again. The difference between the "information of perception" of (the total) living being and its non-living matter is a positive real number that is a growing function (physical) force. We conclude this also by observing the movements of living and non-living beings[18]. The use of more freedom increases the action (physical product of momentum and length, or product of energy and time). The consumption of energy grows, and the force, what leads to greater disturbance of inertial movement. I'm literally talking about Newton's law of action and reaction. Because the state of rest is easier, that is why safety and stability are more pleasant than the excess of freedom. Note that the aforementioned entropy has some secret relationship with these observations, which we will discuss on some other occasion in more detail.

Capitalism is a blend of equality and economic efficiency (in an attempt). The extreme form of this is liberal capitalism. Equality offered to the people at the bottom of the hierarchy of democracy is a constant cause of conflict, and competition. Especially this last, competition in the areas of economy, but also the conflicts that are at the root of the feeling of freedom, are the main causes of the success and attractiveness of capitalism. Communism that has not been economically successful, and which have impeded the freedom of the entrepreneurial conflict, for example by subtracting or diminishing private

[17] With boosting $\mathbf{h}$ and weakening $\mathbf{i}$.

[18] Non-living move according to the laws of physics, as the solutions of Euler-Lagrange equations, derived from the principle of the least effect expressed by Lagrangian, in contrast to living beings whose movements have more freedom.

property, are examples that confirm these allegations.

Successful capitalist states support conflicts in the market because monopolies[19] reduce their economic efficiency. For this, there are exact reasons. Namely, in mathematical theory of games it is easy to prove that duopoly[20] realize not only larger so-called Consumption surplus from the monopoly, but also the total surplus. In other words, the state profits are less from monopoly than from free competition.

Capitalism, with the principle of equality, must allow the struggle of various hierarchies for domination, and because of the efficiency of hierarchies against equality, democracy is becoming overwhelmed. The part of capitalism that is economically efficient, in this struggle for power, favors firms, stock companies, and banks, to become the "secondary powers" of the state. This is the reason why today *corporations* are the most successful in America.

Corporations tend to control the US administration, then their army, other democratic states, NATO, and then non-democratic states. We already mentioned that. For the winning empire it is not enough to have a strong army, but it needs a good utopia. Therefore, the war campaigns of the conqueror are called liberation, the introduction of democracy, prosperity, and rarely the struggle for profit or domination. In arrogance, they do not care what the subordinates could recognize them as predators or that the stratification of the few who possess more and more becomes objectively, unstoppable by will or desire of individuals.

Corporations successfully destroy undemocratic systems, but not because that is better for the people there, but because of the liberal capitalism which feel more powerful in democracy as real predators on its field. They make wars in which they profit, because American domination provides them with an unprovoked plunder of "uncivilized" and allows a military monopoly on the world arms market. The side effect of corporate economic efficiency is pushing capitalism into inhumanity.

Thus the idea of equality coupled with economic prosperity gradually in front of our eyes is developing into inequality. Capitalism slides slightly and becomes an evil system that treads everything in front of itself for profit, for the sake of a growing number of more and more selfish and odious people.

When they grasp power, such individuals no longer want the legal system as originally thought, but rather strive for personal rule over laws, at the same time advocating the rule of law over all the rest of the world. And power allows them to manipulate legal systems. The lobbying and corrupting of the judiciary and the somewhat saving world (by obstructions the transformation of our $\mathbf{i}$ into our $\mathbf{h}$) from the terrible dictatorship of the law in fact delay the collapse of liberal capitalism.

It is also paradoxical, but the society after turning to democracy slowly is denied of freedom. From the D-K effect (example 2.1.1) we see the components of the $\mathbf{h}$ fall, which follows from the fall of the components of the group's intelligence and dualism. That's why we expect to see the decline of $\ell = \mathbf{i} \cdot \mathbf{h}$, especially when this happens to the same society with approximately unchanged intelligence of individuals.

[19]Monopoly is the exclusive right to control the supply of goods or services to the marketplace

[20]Duopoly are two firms that dominate the supply of markets, goods or services.

Example 2.3.1. *What happens to the hierarchy when transit to democracy?*

Answer. Democracy allegedly favors freedom through equality. History teaches us that the aspirations of the people for democracy were forms of aspirations for freedom. On the other hand, here we see that among equal individuals happened a lower intensity of $\mathbf{h}$, and that democracy should have a lower product $\mathbf{i} \cdot \mathbf{h}$.

Because the democracy is a kind of liberalism seems to us it goes to increase ℓ, but due to the D-K effect, it reduces $\mathbf{h}$, and asks for lower ℓ. Under the conditions when the individual's mind $\mathbf{i}$ remains approximately the same, it seems to be a paradoxical formula $\ell = \mathbf{i} \cdot \mathbf{h}$?

However, in democracy the state of law is arising! When all people are equal, there is inflation and a decrease in their value, that is, the need for non-human authority increased. Because equality generates conflicts, and conflicts are often unpleasant, legal regulations come with wider approval, by stretching society hierarchy. □

Therefore, the legal system is a new power of hierarchy ($\mathbf{h}$) in democracy. We also encourage it by "knowing" that civilization can and should improve nature, and that natural laws are not enough for us. However, from the dualism of intelligence and hierarchy it follows that the legal system must begin to show signs of one's own life. Here's how this can be noticed.

It is possible to have a parliamentary majority and to pass a law that will be dissatisfied with the majority of citizens, and those who voted for the law. When charging a penalty for speeding, will grumbled and the deputy who might have approved the law that he just broke off. Apparently, the law on a maximum of one child in China would also be bothered by party members who approved it when it began to apply to them personally. These are the first signs of the independence of the rule of law towards the behavior of the organization as a separate being.

Other signs of the revival of legal systems (in democracies) come from the deeper nature of democracy (the principle of equality generates the need for superhuman authority, abstract legal authority). When juridical is above each individual interests, then the principle of humanity is weakened (man is no more above all), which by itself contests the foundation of modern democracy. However, the judicature overcomes this contradiction and wins.

The third indication also comes from the nature of democracy: it is mediocrity, but in terms of superficiality, which pure heart cannot overcome. Because democracy is the dictatorship of those (slightly below) average, who need a little higher $\mathbf{h}$, we have a constant pressure for a bit too many laws (for the average, and especially for the above-average individuals), and again we get more and more of the hierarchy. This is supported by the following example.

Example 2.3.2. *Explain the growth of the number of laws with the formula* $\ell = \mathbf{i} \cdot \mathbf{h}$.

Solution. When a new law is passed (command, regulation, provision), then the user's adaptation period occurs. Adaptation (example 1.5.3) increases the freedom, at constant ($\mathbf{i}$), which creates a void to add authority ($\mathbf{h}$).

Emptiness due to a surplus of freedom hardly falls to less intelligent, whose appeals to "lack of order" are louder. However, the surplus of opportunities for ℓ feel the average too, and democracy is the rule of average, so the fear of freedom becomes the concern of many, from which the need for new regulations is born. After the increasing the hierarchy the majority is happier and the circular process of legalization is again on the way of adoption. □

In this discussion, we do not take into account the good-bad line, because for consistency it should be looked formally, and then it would lose the expected effect. "Good" should be what is "universally acceptable", and such could only be seen as "generally accepted". Then the manipulations to the institutions of rights by the powerful, if they are widespread, are probably "good"? Looking a little longer in the future, we think that the law is "evil" if we notice that the strengthening of the rule of law leads us into a social system that we do not want now (but perhaps by the evolution of people it would become acceptable).

In a very long-term, the same manipulation of rights-holders by the power tycoons can be viewed as "bad". Increasing hierarchies, organizing individuals through evolution into colonies and increasingly complex and more compact organisms, is a natural and is therefore a possible process. For us, now, it seems repulsive that, for example, Western civilization, by strengthening the organization, smoothness, efficiency, shortly speaking by feminization, could become like a plant whose tissues are individuals of small rights, incapable of living outside the system, unable to reproduce in the former extent . However, evolution can make the Westerners find it as a happy world.

Also, the terms good and bad were different in different periods of our history. What happened in times of monarchy would not be good at the time of democracy. The meaning of good and evil is neither the same for individuals living at the same time. Whatever the zebra was thinking about it, the lioness would have died of starvation if she did not kill.

In a random way, but also because nature saves information, the evolving processes of evolution are developing the ability to control options, those skills we here call intelligence. For the same reason, they sometimes run in the direction of $\mathbf{i} \to \mathbf{h}$, and then a little more often the such survive because of the efficiency. Units are organized into collectives, insignificant cells become tissues and organs of a more powerful organism, and in the same context the obstruction of the legal system (the pillar of the hierarchy of our society) seems to be a slowdown in spontaneous development. In the end, human civilization with already significant $\mathbf{i}$ of the individuals and for now poor $\mathbf{h}$ has two options. To maintain or increase $\mathbf{i}$, by increasing the diversity of our perceptions and our environment, as is happening today by spreading democratic freedoms and technological capabilities, which means maintaining or increasing ℓ. Or go to fall of $\mathbf{i}$ due to the growth of $\mathbf{h}$. In the case of the second, from the point of view of the subsequent super hierarchy of stupid people, the disturbance of the evolution of the "justice" of today's powers will be rated "bad".

However, the legal system is not the only authority; it is not the only hierarchy. It is only now being imposed as an invincible force, moreover, as a monster that slowly conquers society, not saving any other hierarchy. Although it is only a recent phenomenon, we can track its progress towards the amount of remaining less powerful hierarchies, including religion and the family. It is evident that the religion and family in contemporary Western civilization are losing the battle with the legal systems.

Religion is an expression of the human need for authority. Regardless of the faith in God, children love benevolent authority, just as they have inherited trust in adults, until such a thing is blamed. Similar feelings in a variable form keep us in the course of our entire life. It is one of the "little things" that is easily overlooked in democracy, but from which religious institutions live.

We aspire to the society of equality and vice versa – we respect the righteous authorities. Popular religions thus emphasize the equality of men before God and His righteousness. Great religions are successful because they praise equality and fairness among people, but also ethics and spirituality, and in general all positive virtues that are deeply inherited, but on the other side, they neither escape the hierarchical organization of their internal order nor efficiency.

On the other hand, due to declaring equality within the very religions, antagonisms can remain, which remain under the control of the authority of the chief and the institution, until eventual recognition of the equality of the institutions themselves, which can lead to external conflicts, which is then easily spread inward. It is a mechanism of conflict that we know have occurred during the long history of religions.

From the previous considerations, it follows that religious institutions also strive for domination with other authorities, including the state. They have bigger opponents in effective hierarchies than in democracy itself. Their bigger opponents are powerful US corporations than local governments. That's why the religion is in the shadow of the corporations, as the "equal" people themselves. Of course, the greatest opponent of religion is the legal system, which many of us are still not aware.

Thus, the power of Catholicism in the Middle Age grew on the propagation of Christian equality, and on the absence of strong competition. At that time, there were no favors of economic efficiency, but the priorities were honesty, justice and spirituality that escalated to the Inquisition and the immorality and rampage of Pope Borja[21] in Spain. As the salvation of such a deviation of God's equality has come from the strengthening of European monarchies, this means that religion is weaker than even the monarch. It is so unusual heroic, yet futile, today's resistance of Islam against the legal systems. Aware of or not of the dangers of a democratic legal system, Islamists hold sharia law.

Only the third great enemy of religion is science. It may sound strange, but social sciences are in that category now in front of the natural. God is someone who is more often sought in the social and psychological sphere, while the exact sciences remain aside. However, I consider that social issues are out of order, not because a more precise study of social relations is not possible, but because social truths are unpleasant and undesirable. Mediocre like half-truths and politicization, and on the other hand serious, genius searchers for the truth are not clinging to the court of mediocrity and do not want to be said "it's politicization" or "conspiracy theory" for some proof similar to

[21]Pope Alexander VI, Rodrigo de Borja (1431-1503) was the Pope from 1492. until 1503.

Pythagoras's theorem. Because lies in social relations are "sweet" it is believed that it has no truths there, that the world of natural sciences is something very far from social phenomena. It is therefore believed that God is someone who does not interfere with natural laws, and this is also transmitted to believers.

People were separated from primates by those parts of their intelligence that enabled them to make better use of the truth, but also to lie. The development of the lying and manipulation skills made them more successful in using the possibilities of the environment, but the dialectically, sweetness of these same lies sometimes can stops the development of **i** in favor of **h**.

Notwithstanding all such criticisms, which will only grow in time, modern democracy will long be considered the best and most successful system of government to date. The leaders were elected by voting of the people and they in turn protect the interests of the people better than any representatives in the past, historians say. It promotes equality of people before the law and beyond, which gives them a great sense of freedom and opportunity. Promotes change without the use of violence. It is believed that democracy has its positive weight in a stable, dedicated and responsible government, even in the *administration*.

However, there are still some other difficulties in democracy. For example, it reflects in the inability of the administration to purposefully manage time and public funds. After each election, it seems that the people have chosen the wrong, incompetent and irresponsible leaders. Democracy is dominated by an average irresponsible representatives; the quantity often wins quality, not just on the market.

In autocracy, power is centralized in one leader and limited to a smaller number of subordinates. Corruption is also limited. On the contrary, in a democracy in which the ruling body is divorced on a broad, massive administration, management becomes costly and slow, with an unbearable feeling of numbness, which encourages cheating and theft. This again creates the need for additional control, which again requires further growth of the administration body, i.e. bureaucracy. People love benevolent authorities, and tolerate the wealth of one king, but not the attempts to fortify many, especially those who chose to represent them to power. They are irritated by thievery of "everyone in the administration as soon as they get a chance".

Corruption proves to be the inevitability of democracy, which capitalism partly hushes up by the legalization of lobbying. Lobbying is any attempt to influence on a voting body or to decisions of the authority by individuals or interested groups. This increases the impact on elections or government decisions, which opens the possibility of manipulating system administration, which is exactly what gives added strength to corporations today. Corporations, as experienced hunters in the hunt for small pray (stealth), are causing chaos or at least socialist movements in the world, knowing that it will ease their catch.

The administration lives in symbiosis with the legal system. These two are two sides of the same body, which slowly takes over all the other hierarchies of our society by joint forces, and consequently it has signs of its own autonomy. The bribery and the outlawing of bureaucracy are very reminiscent of the similar features of the person who took part and began to lead his life, abusing the position. These are the same lines that we find sometimes in the legal system in general.

With all its bureaucracy, in the service of citizens, the administration does not actually protect an ordinary man. It is too slow. Equality encourages the antagonisms and selfishness of individuals, creating the feeling that "no one cares for anyone". Even lower officials are defying their superiors, although both are corrupt or immoral, and they of above try to persuade the public only to retain the power.

To all this, we come from the basic as much inaccurate and even more attractive principle of democracy: equality. The imbedded principle of equality is justified by the needs of individuality and personal freedom. On the other hand, it forces us to declare poorer masses decisions as better, encouraging our stupidity and our desire to yield to a stronger hierarchy. So we actually come to the opposite, to collectivism and personal unhappiness. At the state level, a similar mechanism binds hands to countries in transition and allows already established hierarchies (corporations) to govern more easily. That is why those who have the power, even when they notice a weakness in democracy, no longer want to fix it.

2.4 Coexistence

Coexisence is a shared life at the same time or in the same place. It is a way of living in peace with others in spite of important disagreements. However, from the interpretation of the formula $\ell = \mathbf{i} \cdot \mathbf{h}$ we now know that even in the basis of such a defined coexistence lies the need for a hierarchy, whose mechanisms here we will try to unravel.

The need for coexistence is similar to the need for authority. Both are essentially a social phenomenon arising from the need for a hierarchy. We will find them in addition to the intelligence of the individual, its ability to control the given possibilities in order to satisfy the capacity of its perceptions. This is not possible if we view them merely with rational measures, on "this side" of individual wisdom, but we must understand them in accordance with the principle of information (stinginess of information). The coexistence should try to place it close, for example to the blindness of the mind created by faith in the current widely accepted authority of societies, within the framework of modern democracy, but also in a fashion.

The principle of equality must be maintained hypocritically and artificially because it is utopian. For the sake of preserving utopia and its consistency, assisted by faith in the authority of the state, "equality" is spreading to everything and everywhere and democratic debates lead superficial and naive. We are forced to consider the truths in social sciences as apparent and without the possibility of being exact. It is already noted on the previous pages, but here we continue to underline that so many different forms of coexistence are common, which in turn they are actually impaired.

Recognizing the family's advantages in the biological maintenance of the species is opposed by the popular belief that the family is an arbitrary community of two people (sex is irrelevant) and that the definition of marriage is a matter of daily politics, human rights and legislators. We said that this was primarily a struggle for the domination of hierarchies, here the legal system against the family. Religion wisely stays back in the tradition, but democracy goes towards “open minded” people.

Liberty is made up of options, and with options there is a growing chance of development. Maximum development chances would have options of equal probability, and among other things, due to the desire for bigger opportunities, we overlook natural limitations. For example, evaluating the unequal (there are no equal people or conditions) equally, the better is damaged and worse rewarded. It’s an obstacle to chances, but we blindly believe that democracy will somehow overcome it.

By protecting equality, we blindly believe that artificial science is in many ways on a same level as the natural one. If the law states that men and women are equal, then we ask to not distinguish them, even if they are talking about marriage. Then we do not choose the new definition, a “marriage II”, but generalize the old so it can include the new one. The family is banalizing the “community of two people”, which is irrelevant to sexes. In such ignoring the reasons for survival of the family is hidden the minority fear of the majority, then the idea of “advanced” civilization, the powered lobbies, the emancipation of women. I will explain the last (the “liberating” women by transferring the child care into the society) in more detail.

Emancipation is the release from a dependent position and the acquisition of the freedom of an individual, social group, or institution. Emancipation is the attainment of equal rights with someone or something. Note, this is a typical democratic process that we will consider by focusing on the contemporary emancipation of women, but so that the background of the previous conclusions is seen, and that the conclusions can be applied wider.

If emancipation was a struggle to improve the lives of women, then its important goal would be to recognize the work of motherhood or at least a range of social, health, and retirement benefits to mothers with children. However, then the state would celebrate diversity, which is not inherent in democracies. On the contrary, the emancipation of women is equalizing their rights and obligations with men, characteristically democratic. It is interesting that in this (said spontaneous or objective) process, participants believe that it is most important for them to have a ”fighting spirit” in order to achieve “victory”.

Feminist, the woman that fights for woman right, by the mid-20th century primarily demanded equal pay for work and the right to vote (suffragette) for women in European countries who did not have this rights. Then their struggle focused on gender equality. By achieving the main goals, from 1990 to the present, feminism advocates for the Queer (an umbrella term for sexual and gender minorities who are not heterosexual and/or not cisgender). So they come to the theory that gender identity is not fixed, then to postmodernism (which accepts the globalization of the world, decentralization and pluralism), ecofeminism (partnership and cooperation of women, men and nature) or transfeminism, merging with the transnational (a man and a woman, both or none) movement.

Let us note that all the lines of this feminist struggle go towards equality in the rights of women and men, them with the global world, the global world with nature.

It is essentially a "struggle" for legalizing non-discrimination and drowning everyone into a higher average. It is the alleged struggle of the activists, and in fact the masked spontaneous, natural and often necessary course of democracy.

However, the feminization is not the discovery of modern democracy. It has often appeared in history as a crown at the end of a period of successful development of states. For example, by accepting Christianity, the Roman Empire, in the 3rd and 4th centuries, accepted the idea of equality of people before God, along with short-term forms of feminization. The process of feminization of the once powerful British Empire has not yet been completed, but Britain continues. Such is the case of Empress Wu (Wu Zetian, 624-705), who came to power at the height of the most successful Chinese Tang Dynasty and was the only woman who managed to maintain the unity of the Chinese Empire with the title of Huangdi (Royal Deity) for a time. After its reign, China was feminized for centuries, first in the form of a dozen separate states, and then under the (partial) rule of Mongol, all the way to the Ming dynasty when China became the empire again. The French Revolution was also a powerful but short (until the reign of Napoleon) a period of feminization. The West has not yet reached the level of "equality" of women in the time of Lenin (Soviet Union to 1924).

We see similarities with the rest of the living world. The importance of sex poles grows by complicating the type and dynamics of the environment. The closest species are primates, the most successful of which are the feminized species of lemurs living in Madagascar, in isolation, protected from the rest of the world. There are 13 species classified into five genera. They are predominantly plant-eaters that feed on flowers, fruits and leaves. The favorite dish is Tamarind seed, and sometimes they eat some insect or bird egg. The nearest to us are chimpanzees and bonobos. The former live along Equatorial Africa in groups led by alpha males, all of them are very powerful, use tools and are among the most effective predators (several times more effective than lions). Contrary to them, bonobos are peaceful and living isolated by the Congo River in some kind of democratic decision-making and feminism.

It is difficult to resist to the impression that feminization is a response of the nature on endangering the sudden unexpected changes in the conditions of life. It is the transition to the phase of smoothness, the cessation of the need for initiative and risk, which happens in the conditions of security and stability, when the importance of the mentioned principle of information is growing. Society is formed, matured, it becomes more efficient and become to lag behind the environment that goes further. All living beings go through similar phases. In the phase of youth and ignorance, they are full of initiatives, which are becoming less and more needed by maturation, so that in the end, the absence of the need for options will announce the death of the given individual, to support life in the environment.

In general, in the event of a lasting peace on Earth, feminization is welcome; otherwise it can be a trap, a period of degeneration of our species before sudden changes.

In addition to all positive aspects, dialectically, democratic emancipation of women also has its disadvantages at the moment. This is, for example, the overemployment of a modern woman, who, in addition to her new male role in the public world, still has her biological functions. Some of these functions are related to her offspring and family, who are now suffering. As in the previous considerations, now in the third way we come to the same conclusion, to the weakening of the significance of the family. The characteristic of successful democracies is the decline in birth rate.

Another example is female innate altruism[22] which is oriented towards children and family, unlike a little weaker male's who is facing the outside world. It is also the reason that a successful business woman, who due to her external obligations, does not arrive to have a family, feels uncompleted and somewhat frustrated. All this leads us to the conclusion that the emergence of democracy is a major crossroad, a turning point in the evolution of our society.

I note that the decline in birthrate can be explained in parts because we are non-adapted in evolution to new accelerated changes. Some species cannot reproduce in the cage, some are multiplying there more difficult, and our cage is a legal system. The fall in birthrates in some species is also due to the excessive density of the population, even in the conditions of plenty of food. The reason for the reduction in the need for sex and offspring comes from the very men because of their new (feminized) role. Finally, equality itself makes men less interesting to women (he seek female everywhere, she only among the most popular males), which is an argument that is rarely mentioned in the explanation of modern divorce and marriage frauds. Note that these arguments are consistent with the previous considerations.

Here we actually see the "degeneration" that comes to avoid the truth, its masking to maintain utopia. The word "degeneration" literally means decay or spoilage. Statically formally we use it for the decay of values of deduction due to poor assumptions[23]. Also, in describing long-term development processes, it will mean to us a bad evolution, but also a development into some new value.

As we know, the implication "if A then B", or $A \Rightarrow B$, can be incorrect only if the assumption A is correct, and the consequence B is incorrect. In all other cases, the implication is correct. Hence, with a bad assumption we always have an exact implication, regardless of the consequence. This practically means that mathematics allows ramble that can look quite good, but only if you start from falsehood.

The democratic principle of equality is precisely the precondition behind which can be a perfectly accurate deduction leading to worthless results. That is why debate and competition on the subject of "Democracy and Human Rights" are interesting. There compete two teams against the thesis that has to be accepted or rejected, with the participants who do not care which of these two roles will be assigned to them. This absurd competition with the participant develops the feeling that the truth does not have the exact value in the society science. Give me any thesis, I'll prove it! Give it to me again, I will deny it! – could boast a champion of such debates. It is the degeneration of understanding the truth.

For the same reason, in a democratic legal system it is possible to have such a good lawyer who at the trial of his client can prove what he wants, whether suspected is guilty

[22] Altruism is unselfishness and concern for others.

[23] In mathematics this is meaningless, because the statement can only be true or false.

or not guilty, regardless of whether the person concerned is really guilty or not. Let's say, if we are consistently adhering to equality. It looks like a degeneration of the principles of law, but it actually reflects the need to protect democracy.

At the end, we come to the already mentioned genetic and biological degeneration of our species that threatens us, absurdly, because of the benefits of democracy. Democracy encourages freedom by its principle of equality, and these support the development of society. Freedom at the bottom of the hierarchies generates by democracy is responsible for the launch of the whole society, and on the other hand, for the building of efficient its hierarchies. The combination of development and efficiency in itself brings benefits to society, then selfishness, hedonism and irresponsibility towards the "distant" future. And setting up a man of the highest value in the universe has its price.

Chapter 3

Formalism

The title of this chapter could be the Introduction to the story of the inanimate world, the mathematics of evolution or the physics of information. Anyway, in 2016, a reader in any of the similar names could not easily understand what this is about, because there are not many known things about that at the time. That's why I cut one unique theme into two parts, in two books, the other one being under preparation only after this. Nevertheless, for this I have set apart here a few simple mathematical discussions that do not reveal much of the future text, and they were the basis for the previous considerations, first rejected and then again actual.

It's always surprising how much of the rejected text can again be reissued. For example, by accident, I find the only copy of a long typed notebook with reviews from my colleagues (Rodoljub Bavrlić and Andjelko Glušac), professors of physics at Gimnazija Banja Luka, from which I draw a detail[1]. This detail is about the conservation of information in the world of physics, which was otherwise considered a mathematical abstract concept, and for this reason I suddenly noticed its deeper meaning. If the information is a physical matter, then the options are real!

In addition to various other queries that I've left all around (often "privately" on the Internet) or lost, I have been deprived of a lot of time searching for some mathematical form of the ability of *deciding*, first only in the living world, but perhaps later in general, to the very elementary particles of physics. It's quick to notice that it's hard to observe this term in the way we are used to. The decision-making process will become a blind choice, what is called probability theory in the realization of random events. However, a random event in mathematics is often not what we usually think it is.

When we first observe the decimal numbers of $\pi = 3.14159265359\dots$ it looks and behaves exactly as a series of random numbers. The array will pass the "randomness" test of the probability theory, but it can hardly pass a man who tries to simulate randomness by saying arbitrary numbers. Next time we see the same decimals of the number π, it will be clear that even these are not random numbers too, as those that we expect to come out from a lotto drum. Decimals of the number π are exactly certain numbers in a unique order. So we say that mathematics deals only with pseudo (quasi) coincidences.

[1]Conservation law: `www.academia.edu/8004844/`

We of course can still talk about the "intelligence" or "understanding" of the electron, if we assume that this is the "atom" of the intellect from the world of living beings, which in a large crowd (by passing quantity into quality) gets a new meaning. You can say that physics particles have life as much as you can say that living beings are merely mechanistic, chemical, or similar complex assemblies of absolutely dead things. It's formally not important, but to avoid confusion, we'll use other tags.

The choice of the letters ℓ for freedom did not come (only) because of the word "Liberty", but "Lagrangian", because I was not sure at all about the outcome that I intended to write in more detail in the second book (The Nature of Time). When the work on the second book is completed (later named "Space-time"), and the hardest of the initial suspicions are removed, some parts can be "boldly" put forward. None of my colleagues with whom it was sensible to talk about this topic (2012 – 2016), not even one, I could not been able to convince that the freedom we are talking about in democracy has such strong ties to the principle of the smallest action in physics. That is why in the original versions of the text I also used s for freedom, $\mathbf{r}$ for intelligence[2], and then alphabetically $\mathbf{p}$ for the hierarchy. At least that might have seemed, but the real reasons for choosing these letters were again the usual marks for (relativistic) interval, position, and momentum in physics.

3.1 Probability

Put simply, the *probability* of an event is a number that is so much greater that the greater is the chance that in the given circumstances the event occurs. In physics (thermodynamics), this number is a positive real number, in the mathematics this number is a real number from 0 to 1.

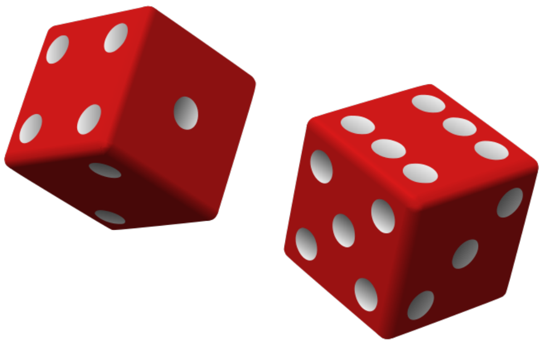

Figure 3.1: Two cubes with 1-6 points.

When we throw one fair-dice (see figure 3.1), the probability of falling of any of its six sides is approximately equal to $\frac{1}{6}$. This is the number we get by knowing that it is

[2]The first letters of "sloboda" and "razum" in Serbian.

one of the six options (e.g., the "three") and that in some order every sixth could be just that possibility. This is a result of numerous experiments. For example, in a series of 100 throws of cubes, only 11 threes happen, and we expect them to be 16 to 17 (sixths of a hundred), but by repeating a series of 100 throws we will notice that the number of "three" really clings around the expected one.

When we throw two dices, expecting two predetermined (different) numbers to fall in the first and second row, the probability is $\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{36}$, in case the fall of one dice is independent of the other. What if the same numbers fall? What if the cubes are different in color and is exactly known which are the "first" and "second"? Already from these seemingly extremely simple examples, we realize that we need clarity in dealing with coincidences. We want by precision to master over nebulosi.

Now let's look at a few examples that will connect us with previous considerations, putting emphasis on the inanimate world.

Let's say we have an event that happens with the probability $p\in(0,1)$ that we can repeat $r=1,2,3,\dots$ times. When we have a n-tuple of similar events with probabilities in terms of the components of the vector $\mathbf{p}=(p_1,p_2,\dots,p_n)$, which we can repeat in the same order $\mathbf{r}=(r_1,r_2,\dots,r_n)$ times, then the total number of realizations of these events is approximately

$$\mathbf{p}\cdot\mathbf{r}=s. \tag{3.1}$$

This is a scalar product of given vectors whose result is a scalar s.

Forasmuch as

$$\mathbf{p}\cdot\mathbf{r}=|\mathbf{p}||\mathbf{r}|\cos\angle(\mathbf{p},\mathbf{r}),\quad -1\le\cos\angle(\mathbf{p},\mathbf{r})\le 1, \tag{3.2}$$

it is

$$-pr\le s\le pr, \tag{3.3}$$

where are

$$p=|\mathbf{p}|=\sqrt{p_1^2+p_2^2+\dots+p_n^2},\quad r=|\mathbf{r}|=\sqrt{r_1^2+r_2^2+\dots+r_n^2} \tag{3.4}$$

modules of given vectors.

Example 3.1.1. *Throw the cube 60 times and count how many the "sixth" fell, then throw a coin 30 times and count how many the "head" had fallen.*

Solution. The first, for the cube we have $p_1=\frac{1}{6}$ and $r_1=60$, so the number of fallen "six" is approximately $s_1=p_1r_1=10$. Then, for coin is $p_2=\frac{1}{2}$ and $r_2=30$, so the number of "head" is approximately $s_2=p_2r_2=15$. At all the sixths and heads is about $s=s_1+s_2=25$. For

$$pr=\sqrt{\left(\frac{1}{6}\right)^2+\left(\frac{1}{4}\right)^2}\cdot\sqrt{60^2+30^2}=25\sqrt{2}\approx 35,4$$

the inequality (3.3) is true. □

Example 3.1.2. *The student correctly answered two thirds of the questions from the first test, and three quarters from the second. At the first test, there were 30 questions, and the second 44. How many were the correct answers in total?*

Solution. The probability of the correct answer to the first test is $p_1 = \frac{2}{3}$, with the number of attempts $r_1 = 30$, and the student had $s_1 = \frac{2}{3} \cdot 30 = 20$ hits. On the second test, $p_2 = \frac{3}{4}$ and $r_2 = 44$, with the number of correct answers $s_2 = \frac{3}{4} \cdot 44 = 33$. In total, both tests have accurate $s = 20 + 33 = 53$ of all responses. Because

$$pr = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{3}{4}\right)^2} \cdot \sqrt{30^2 + 44^2} \approx 53.4$$

again, the inequality (3.3) is true. □

Example 3.1.3. *The position and momentum of a particle are given with the indeterminacy of individual components (abscises, ordinate, and applicate):*

$$\Delta\mathbf{r} = (\Delta x, \Delta y, \Delta z), \quad \Delta\mathbf{p} = (\Delta p_x, \Delta p_y, \Delta p_z).$$

What is the total uncertainty (3.1)?

Solution. We get directly:

$$\begin{cases} s = \Delta x \Delta p_x + \Delta y \Delta p_y + \Delta z \Delta p_z, \\ s \le \Delta r \Delta p \end{cases} \tag{3.5}$$

where are the intensities $\Delta r = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$ and $\Delta p = \sqrt{\Delta p_x^2 + \Delta p_y^2 + \Delta p_z^2}$. □

Note that by applying (3.1) we cannot merge everything with anything. In both the first and the second previous examples, it was intuitively clear that the merger was fine, but in the third example it was not. In the third case the formula (3.5) can be justified by Heisenberg[3] *uncertainty relations*, from quantum mechanics, that we can see later explained with the figure 3.4. In some even more unclear examples, the scalar product (3.1) would be more difficult to explain and even unacceptable.

Independent events are those in which none of them affects the outcome of any other. Such are, for example, different casting of the dice when the probability of the given number does not change while repeating the throw. Dependent events are those who are not independent.

Let's say we have nine boxes (three rows with three) and nine globules. Each of the boxes can fit all nine small balls, but the balls are arranged in a random way so that each of the balls can with the same probability ($p = \frac{1}{9}$) go to any of the boxes.

On the figure 3.2 on the left, all of nine balls is in the box "1a", in the first row ("1") and the first column ("a"). The probability that (any of the nine) balls is in the given box is exactly $\frac{1}{9}$, so the probability of all nine being there is $\frac{1}{9} \cdot \frac{1}{9} \dots \frac{1}{9} = \frac{1}{9^9}$. For any of

[3] Werner Heisenberg (1901-1976), German theoretical physicist

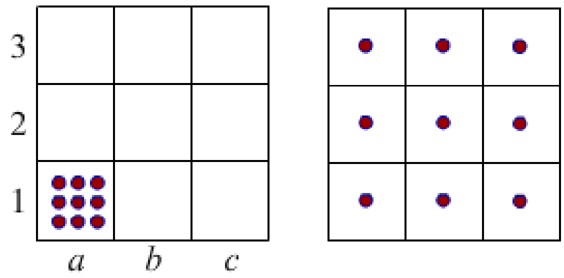

Figure 3.2: Distribution of balls into boxes.

nine boxes, the probability that all nine balls are in one is nine times greater, i.e. $\frac{1}{9^8}$. On the same picture to the right, each of the balls is in a separate box. The probability of this layout is $\frac{9!}{9^9} = \frac{8!}{9^8}$, which is $8! = 8 \cdot 7 \dots 2 \cdot 1 = 40\,320$ times more than the previous one.

In other words, the random arrangement in the given image to the right will be more than 40 thousand times more often than the left, even if all the balls can be in any of the nine boxes. The particles of the air in the room are arranged in the right way, simply because this layout is more likely. Not only in this case, the mess (right) is a more likely event than the order (left).

We will return to this "distribution problem" later on when we consider entropy. Another interesting problem, which is also about probability, is the famous "problem of the secretary".

Let's say we're in the secretary's selection committee. We know that in front of the door there is $n = 100$ candidates, to us completely unknown persons, who will be one at a time entering the interview office, during which we need to make an assessment and make a decision of employment. When we estimate that the candidate will not be received, she leaves the room and no longer can returns, and we cannot revoke our decision. When we estimate that the candidate will be accepted, the decision is definite, the selection process will be canceled without the possibility to see the remaining candidates. Is there an optimal strategy for selecting the best candidate?

For this problem (figure 3.3) the theory of probability has the following solution. We miss about one-third of the candidates (more exactly $n/e = 37$) to get an idea of how many points they can have. Then we choose the first one that has more points than each one from the first group. If none has more points, we choose the last one.

The optimal number n/e, where is the Euler's number $e = 2,71828\dots$, is actually the real measure of those n possibilities, so we can consider it as information, or the measure of uncertainty of the set of n of unknown random outcomes . In such an assessment of information, it is not necessary to have a probability distribution, but the result is equal to that obtained from Shannon's formula (1.19) when we work in the nat units.

Namely, as $-\ln\frac{1}{e} = \ln e = 1$, all n logarithms in the Shannon formula are units, so the total sum is n/e, which is equal to estimated information using a "secretary problem".

This strange probability of $1/e$ often appears, many times in unusual places. Here is

Figure 3.3: The problem of choosing a secretary.

one such example, referred to in the literature as "the postman's problem".

The postman should deliver $n \in \mathbb{N}$ letters to the n address. We know that there is only one way to get every letter exactly where it's sent, and we know that there are a number of ways for those addresses to be mixed up with, it is $n! = 1 \cdot 2 \cdot 3 \dots n$. It's a number that is growing very fast. Therefore, the probability of (random) accurate delivery of all the letters is $1/n!$, that is, the exact random delivery of the even moderate number of letters is an almost impossible event.

However, the inaccurate delivery of all letters has a probability of $1/e$. Namely, the exact delivery of one letter has a probability of $1/n$, so the incorrect delivery of that letter has a probability of $1 - 1/n$. Incorrect delivery of all n letters has the probability $(1 - 1/n)^n \to e$, when $n \to \infty$.

3.2 Information

Space of random events Ω is a set of all possible outcomes of an experiment. Probability is a measure of what will happen to an event $X \subseteq \Omega$. It is a real function $P = \Pr(X)$ of domain Ω and range in the interval [0,1] of real numbers. If the probability that an event will happen does not affect the probability of occurrence of another event, then for these two events we say they are *independent*. The *mutually exclusive* events are if they cannot happen at the same time.

Hartley's *information* or the uncertainty of the random event of probability P is in *nats*:

$$I = -\ln P. \tag{3.6}$$

For example, when throwing a dice the information is $\ln 6 \approx .1.79$ nat. Nat is a unit of information for the natural logarithm of the base $e = 2.71828\dots$, which is used more often than the logarithm of the base 10, whose unit is *decit* or the base 2 of the unit *bit*. The real number P is between 0 and 1, and the logarithm of such is negative, so the information is a positive number, from zero to infinity.

A set of mutually exclusive events with the corresponding probabilities $P_j = \Pr(X_j)$, such that their sum is 1, we call the *complete* set of events. A complete set of events associated with the corresponding probabilities is called *distribution* of probability. For $j = 1, 2, \dots, n$, we write:

$$X = \begin{pmatrix} X_1 & X_2 & \dots & X_n \\ P_1 & P_2 & \dots & P_n \end{pmatrix}, \quad \sum_{j=1}^{n} P_j = 1.$$

When we have the probability distribution of a random variable X, the mean value of Hartley's information:

$$\langle I \rangle = -\sum_{j=1}^{n} P_j \ln P_j, \tag{3.7}$$

it called Shannon's information. Simply said, Shannon's information is the mean value of uncertainty. This is the definition of *discrete* information, which includes an extreme case of $n \to \infty$. Discrete data can only take particular values.

Similarly, the information of the *continuum*[4] is defined. When the infinitesimal probability $\rho d\tau$ of the density ρ is given, where $d\tau$ is the infinitesimal part of the random event from the space Ω, then the average information:

$$\langle I \rangle = -\int_{\Omega} \rho \ln \rho d\tau. \tag{3.8}$$

For example, *normal distribution*:

$$\rho(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \tag{3.9}$$

has average information:

$$\langle I \rangle = \frac{1}{2} \ln(2\pi e \sigma^2). \tag{3.10}$$

Indeed, we calculate in the order:

$$\langle I \rangle = -\int_{-\infty}^{+\infty} \rho(x, \mu, \sigma) \ln \rho(x, \mu, \sigma) dx =$$

$$= -\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \left[\ln\left(\frac{1}{\sigma\sqrt{2\pi}} \right) - \frac{(x-\mu)^2}{2\sigma^2} \right] dx$$

[4]Continuum – the set of real numbers including both the rationals and the irrationals; the number of such numbers.

$$= \ln(\sigma\sqrt{2\pi}) - \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{x-\mu}{2} e^{-\frac{(x-\mu)^2}{2\sigma^2}} d\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

$$= \ln(\sigma\sqrt{2\pi}) + \frac{1}{2}\int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$= \ln(\sigma\sqrt{2\pi}) + \ln\sqrt{e} = \frac{1}{2}\ln(2\pi e\sigma^2).$$

The parameter μ is *middle value* or the expectation of distribution, and σ is its scattering called *standard deviation.*

Comparing (3.10) with Hartley's information we see that $n = \sqrt{2\pi\sigma^2}$ can have the nature of the number of possible outcomes, that is $\frac{1}{n}$ is the probability of these outcomes. This means that wherever we have a normal distribution we can define (3.10) as Hartley's information, and then continue to look for all the consequences of such a definition. Then the question arises, does it make sense to expect that information is a special physical term like speed, energy, momentum? The second question is whether the some conservation law may be valid for information, that its quantity cannot spontaneously arise or disappear, but that it can only go from one form to another, like energy?

Here is an analysis of these questions from the point of view of the special theory of *relativity.* We have two inertial coordinate systems K and K' that move together at speed v along abscise. Let the K' be the reference system with the observer in the outcome. Let's say, we are sitting in the outcome of K' and look at K that slides at speed v along x-axis. In relation to our watch, the one in the system K is lagging behind. It is a known relativistic effect of the dilation of time.

Because the time in K runs slower in relation to us in K', the observer from that system in arrival must see our future so that only at the moment of overcoming our present coincidence. After passing, this observer looks at the departure in our past, because its time is still slowed down in relation to ours.

As is known, the coefficient of this slowdown is $\gamma^{-1} = \sqrt{1 - \frac{v^2}{c^2}}$, where c is the constant light velocity in a vacuum and it can be a number from zero to infinitely, when (constant) velocity is v from zero to c. This means that very fast particles, moving at inertial velocities close to the speed of light, see one another either as a deep past (in departure) or a distant future (in arriving). This means that the past and the future are physical reality, at least as far as inertial movements are concerned, which then means it makes sense to talk about the amount of information they carry.

Namely, there is no information without real uncertainty. For example, if we (always) do not know what a theorem is saying, so one day we find and prove it, the information we get with it is zero. When something is certain that it will happen and that happens, then there is no information in the event.

However, the reality of the time in the way we have just stated it now opens the problem of the existence of uncertainty, and hence the very information itself. If observers from these inertial systems clearly see the past and the future of the Universe, then this past and the future are deterministic, without the possibility of change, without hesitation or arbitrariness. In such a universe there is no room for probability or information! So we got to the paradox of uncertainty.

How to get out of this paradox, and still believe that the theory of relativity is one of the most exacting theories (outside of mathematics)? Here is one solution that I (in vain) advocated as a student of mathematics in Belgrade, around 1980.

Inertial systems are those in which the observer does not feel the effect of acceleration or gravitational force, whether it is a uniform straight line motion outside the gravitational fields or a free fall in such a field[5]. Einstein's special and general theory of relativity applies to such inertial movements. However, there are other, non-inertial movements!

For example, such is a rotation. Newton[6] was at a time take a rotating system to prove the existence of "absolute space". He observed the washbowl filled with the water circulating while hanging from the ceiling and noticed the recess and spillage of the water from the basin. The water turned around and, due to the centrifugal force, spilled out while the room was still, with no motion, concluding that the reverse explanation would not be acceptable (the basin with the spilled water to be still in the swirling room).

This problem struck Einstein for years (in the period from 1905 to 1916) when he occasionally had contradictory views about the rotating system, until the publication of the general theory of relativity bound to inertial systems excluding rotation. In the rotating system, there arises the Coriolis[7] forces that we can measure[8] and conclude that we are in a system that does not stand still. By contrast, a similar physical experiment in the inertial system is not possible to confirm the movement.

Another problem within the rotation would be the inability to align the watches. In inertial systems, such as the mentioned K and K', regardless of the fact that it is slower in one compared to the other time, it is always possible to move a clock from point to point to an uninterrupted time all over the space, within the system K and within the system K', regardless of the fact that two such synchronized times are not mutually simultaneous. Each of these two systems has its own "now" that divides its 4-dimensional space-time into two parts, separating it from the past from the future. The clock synchronization is not possible in a non-inertial system.

In a system that rotates synchronization of time is not possible. When we start on a circle around the center of rotation due to the speed of the movement of that edge, the time is slower in relation to the center, and until we arrive at the end of the circle the discontinuity happened. In a rotating system, there is no mentioned "now" that separates its past from the future!

However, it is precisely this last difficulty for the theory of relativity that will represent a solution to our previously mentioned paradox of uncertainty. In order to understand this solution, we note that the reality of arbitrariness means the possibility of the development of an event in some other dimension. If I can really do something different in a moment, A or B, then these outcomes should be somewhat realistic, without violating the laws of physics, or logic, except that the other outcome I did not do, I even

[5]The official definition for inertial system is a little bit different.

[6]Isaac Newton (1642-1727), English mathematician.

[7]Gaspard-Gustave de Coriolis (1792-1843), the French mathematician.

[8]On the surface of the Earth, therefore, we have a circular motion of a cyclone or water in a drain.

do not see. For the difference of the string theory, we are talking about the dimensions of the time. The question arises: how many dimensions does the physical world have?

We will ask the answer in mathematics, in its branch called topology, which is specializing in dimension problems, since there is nothing more precise about it. One of the topological dimension definitions can be reduced to an inductive[9]:

If a set of points can be divided into two parts by a boundary whose dimension is $n = 0, 1, 2, \dots$, then the whole set is of dimensions $n + 1$. If this partition is not possible, the whole set has more than $n + 1$ dimensions. The point has a dimension of zero, so the final set of points has a dimension of 0.

Sight line, circled lines, and generally curved lines have dimension 1, because one or even many points can be divided into two parts. The final set of lines also has dimension 1. The surface has dimension 2, because it can be divided into two parts by means of lines. The space has dimension 3, because the surface can be divided into two parts. Inertial space-time has dimension 4, because it is possible to synchronize watches that will define the present, 3-dimensional space, which will separate its past from the future. However, in the rotating system, synchronization of watches is not impossible, which means that 3-dimensional space with (own) simultaneous points is not possible, and it is not possible to separate the past from the present using 3 dimensions. This means that the space-time of the rotating system has at least 5 dimensions!

From the same analysis of relativity we will notice something else. Adding to the notion that more likely events carry less information, which we have also called the "principle of information", while noting that nature is stingy with information, it is difficult to hinder the impression that it has something to do with inertia. The body rests in its own inertial system because such states appear to be the most likely, and therefore the least informative. It cannot be without the conclusion that the probability and then the information are relative, depending on the observer and his state of movement. They are the theme of the second book (Space-time, [2]).

The following information problems, which I draw from the texts for later, are also appealing, yet completely invisible to the scientific public (as far as I know). When we already accept that information is a real thing and that it is constantly appearing, because uncertainty is objective and that even in the case of the realization of the most likely events at least the smallest part of the uncertainty must pass to information, then the question arises: where is all this information? The answer is: in the present. What we call "now" is in fact information that is constantly occurring around us!

The last excerpt I mention here is a bit more dynamic. The body does not go spontaneously from its own inertial state K' to another, to relatively K, because the corresponding conditions there seem to it less likely. For that reason, it already sees more relevant information there (the less probable is more informative), which is in line with the principle of information saving, and then with the law of inertia. However, the further places are not the ones on which we are "on our own" and therefore are foreign to us. The space has a perspective that now complicates things because it diminishes the information from distant objects. This is not the topic of the second, but a third

[9] somewhere called Urison's

book, a work on a vacuum in which the relative opening of new information comes with slowing down the time. In order to move the body from one to the other, here in the most puzzled case – the pairs of mutually static points, there is a need of going from the state of (relative) moving, which means a slowing down the relative time, and hence the new relative, previously virtual information.

3.3 Quantum Mechanics

Physical experiments provide information that can be interpreted as interactions. Because this information in the everyday world is negligible, it would be difficult to notice their physical nature if there is no the discovery of quantum mechanics. The discovery of this second nature, the laws of small-length physics, will give a new meaning to information in general.

We know that information is preceded by uncertainty (suspense) which is the state before the realization of a random event. The *amount of uncertainty* of a random event with $n \in \mathbb{N}$ of equally probable outcomes define as the function $s : n \to \mathbb{R}$. Note that the function $s(n)$ depends only on n and it is increasing. Then we note

$$s(m) + s(n) = s(mn), \quad m, n \in \mathbb{N}. \tag{3.11}$$

Namely, let m and n be the numbers of outcomes of two independent random events. The product mn is the number of outcomes of both. This feature has only a logarithmic function.

Theorem 3.3.1. *Let $f(x)$ be a continuous function defined on the set of real numbers $x > 1$. If $f(x)$ is:*

1. *Positive, i.e. for each $x > 1$ is $f(x) > 0$;*
2. *Growing, i.e. for $x_1 < x_2$ is $f(x_1) < f(x_2)$;*
3. *For each allowed x_1, x_2 is $f(x_1) + f(x_2) = f(x_1 x_2)$;*

then $f(x) = C \cdot \log_b x$, where $C > 0$, $b > 1$ are arbitrary real numbers.

Proof. For each $x > 1$ and for every $r > 0$ there are $k = 0, 1, 2, \dots$ such that

$$x^k \le 2^r < x^{k+1}.$$

Hence and because the given function is rising (2.) is

$$f(x^k) \le f(2^r) < f(x^{k+1}),$$

so for the next property (3.) we have:

$$kf(x) \le rf(2) < (k+1)f(x),$$

$$\frac{k}{r} \le \frac{f(2)}{f(x)} < \frac{k}{r} + \frac{1}{r}.$$

The function $f(x) = \log_b x$, $b > 1$ has properties 1 - 3 and is valid for it

$$\frac{k}{r} \le \frac{\log_b 2}{\log_b x} < \frac{k}{r} + \frac{1}{r},$$

and on that basis

$$\left| \frac{\log_b 2}{\log_b x} - \frac{f(x)}{f(2} \right| < \frac{1}{r},$$

for every $r > 0$ and for an arbitrarily large r, for which the expression in the absolute bracket of the last inequality is zero, i.e.

$$f(x) = \frac{f(2)}{\log_b 2} \cdot \log_b x,$$

that is $f(x) = C \cdot \log_b x$. □

If we have a function (3.11), but with arguments positive numbers less than one, then we switch to the reciprocal value of the argument and apply the previous theorem. Therefore, the same theorem applies even after the change of $n = 1/p$ where p is probability of realization of one of the n random events. Then $s(1/p) = \ln(1/p)$, or

$$I(p) = s\left(\frac{1}{p}\right) = -\ln p, \tag{3.12}$$

which is precisely the information (3.6). When the probability p grows from 0 to 1, the information I decreases from $+\infty$ to 0, and vice versa.

That by realization of the random event, the total amount of uncertainty goes into the same amount of information confirm the following example.

Example 3.3.2. *Throwing a Fair Coin.*

Explanation. The fair coin has $n = 2$ outcomes "Head" and "Tail" of equal probabilities $p(2) = \frac{1}{2}$. Uncertainty before throwing the coin is exactly the same to the information after and it is $s(2) = \ln 2 \approx 0,69$ nat. □

Note that by the realization of a random event with two outcomes we resolve two uncertainties. Uncertainty in this case acts as a system of connected vessels with (incompressible) liquid. This is a little clearer in the following example, which Einstein once invited by denying quantum mechanics after discovering the so-called *quantum entanglement*, which will be presented in the next book as an integral part of this theory.

Example 3.3.3. *Pair of gloves, by one in two equal boxes.*

Explanation. Let's imagine two separate gloves out of the pair and put each in two equal boxes, one of which went away to who know where, and the other we received by mail. Before opening the box, we had uncertainty, after opening the information (there is a left or right glove inside). The probabilities for each of the two options are the same, the half, and the amount of uncertainty before opening the box as well as the information after the opening is the logarithm of the number two. On receiving this information, there was no uncertainty more, not only for the open but also for the box that we do not have. □

The following example shows that uncertainty can be taken away from a certain stochastic system part by part and converted into information. The total amount of uncertainty before, as well as the sum of the individual quantities of partitions, is the same as the total information after the outcome of all random events.

Example 3.3.4. *More items by one in more equal boxes.*

Explanation. When we have $n \ge 2$ items, for example numbered with $1, 2, 3, \dots, n$ and put one by one in n equal boxes, the information we get after opening the first, second, ..., $n-1$, is presented in the order:

$$\ln n, \quad \ln(n-1), \quad \dots, \quad \ln 2. \tag{3.13}$$

The last n-th box should not be opened if we carefully monitored which items we previously found. □

Note that the process of subtracting part by part of uncertainty and adding information reminds on adding the energy. Similar to the energy whose quantity is always constant in the closed system, these examples promote the law of maintaining the total amount of uncertainty and information.

Example 3.3.5. *Uncertainty of equal permutations.*

Explanation. The total information obtained after opening all of the above mentioned boxes is equal to the sum

$$\ln n + \ln(n-1) + \dots + \ln 2 = \ln n! \tag{3.14}$$

therefore it is equal to the information we get by choosing one of the $n! = 1 \cdot 2 \dots n$ equally likely permutations of $n \in \mathbb{N}$ elements. □

The particular question is: is the information obtained from the described uncertainties persistent, is the law of maintenance valid for it? A positive answer comes from the very act of measurement, in undisturbed (in the absence of force) "reading" the information. Because measurement can be considered reliable evidence, therefore we believe that the information itself is stable and its transmission from the interaction of the quantum system and the apparatus to the experimenter is sustainable. The unconscious confidence in the information's sustainability was at the very foundations of experimental physics, much before the discovery of the theory of information. What might be questionable here is just the relation between the mathematical information (3.6) and the intuitive.

When we know that something will happen and that happens, the information is zero. A wider definition of information also needs to have the characteristic that a rarer event carries more information, that greater news is “a man has bitten a dog” than news “a dog has bitten a man”. With the similar, these are confirmation of the intuitive correctness of all three (Hartley, Shannon and new) definitions of the information we are working on here. In particular, definitions of uncertainty and information are intuitively acceptable so and the application of their consequences in physics is justified. A different confirmation comes from the way in which it is generalized, that Shannon information is a special case of the new, Hartley special case of Shannon, and that in Hartley it "fits" the amount of uncertainty before and after the realization of a random event.

On the other hand, the adoption of conservation laws for information opens up some new opportunities. Let’s say for explaining the flow of time. What we call the present is a (3-dimensional) space with constant (total) information that comes from the continuous realization of random events. If we notice that this information comes from the future and goes into the past, then we need to adopt two more conclusions. The present goes to the more probable future, and leaves in past the less likely events. The assumption of the relative observation of the flow of time also should not be slurred over.

Namely, the example 3.3.4 testifies that the system is defined by forming information. By transforming a piece of uncertainty into the information, the system becomes better defined, because the probabilities of its remains are increased. It is known Heisenberg’s quotation that “by measuring the electron we define its path”, which is cited as a wonder of quantum mechanics. By opening the box in which Schrödinger’s cat is located and learning that this cat is alive or dead (that are equally probable), it is defined the new “real” condition, which was previously undefined. Increasing the probability of the physical system is always accompanied by loss of uncertainty and formed information, whereby the previous states are defined in addition; the past is changed for the future. So we can understand the course of time to the future, as the transition of uncertainty into information, after which our past becomes clearer. As in saying: tomorrow we will know better what happened to us today.

Time is observed due to the constant formation of information around us. Conversely said, if there were no random events around us, then the time would stop. The meaning of this hypothesis we also found in the *Heisenberg’s uncertainties*.

On the Heisenberg microscope, the figure 3.4, we see more precisely the determination of the particle position Δx which leads to the more undetermined of its momentum Δp, and vice versa. The electron (e^-) is hit by gamma ray (γ). In order to have a higher accuracy of the electron location, we need a smaller wavelength (λ') of the ray, which means a higher momentum (p'), since $\lambda' p' = h$, where $h \approx 6.626 \times 10^{-34}$ m^{2}kg/s is Planck constant. The larger momentum gives a greater range of angles θ and greater uncertainty in the future movement of the electron. When the uncertainty of the position of the electron is $\Delta x = \lambda'$, and the uncertainty of its momentum is $\Delta p = p'$, in the best case is $\Delta x \Delta p = h$. This is a rough estimate. Otherwise, the uncertainty thus obtained is

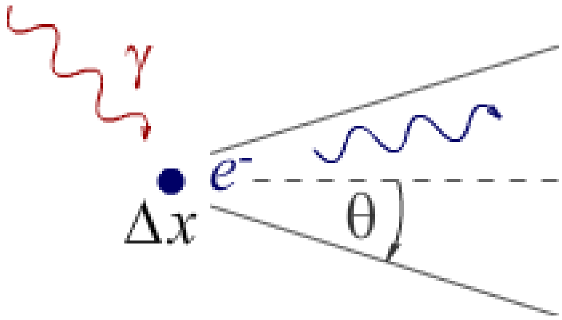

Figure 3.4: Heisenberg's microscope.

overvalued, so we put $\hbar = h/2\pi \approx 1.055$ Js on the right hand side and get:

$$\Delta x \Delta p \geq \frac{\hbar}{2}. \tag{3.15}$$

Precise measurement (see [4]) shows that the right side may even be a smaller number.

In other coordinate axes (x, y, z) we obtain other Heisenberg relations of uncertainty of position and momentum, while the uncertainty along one axis are independent of the uncertainties of the other two. However, there is a similar minimum of time and energy products for $\Delta t \Delta E \geq \frac{\hbar}{2}$ even though the time is not *observable* (dynamic variable that can be measured). All four inequalities can be written together

$$\Delta x \Delta p_x + \Delta y \Delta p_y + \Delta z \Delta p_z - \Delta t \Delta E \geq \hbar, \tag{3.16}$$

which resembles (3.5).

There are various ways of generalizing such expressions and aligning it with information of perception, but one thing is important. The information of perception for living beings is greater than the proper one for non-living. The non-living "information of perception" would be in favor of the theorem reversed to the theorem 1.5.4, with the reverse order of vector coefficients where the smaller is multiplied by larger and larger with the smaller one, in order to obtain the minimal value of the scalar product. The information defined on the basis of (3.16) is Lagrangian, more precisely, proportional to it. Lagrangian is an expression of the principle of the least action in physics.

It is now trivial to prove that the non-living from the living being is different in the degree of freedom. Non-living beings move along geodesic lines and other solutions of the Euler-Lagrange equations which are derived from the principle of the smallest action, and living beings have additional degrees of freedom, which makes them more "vivacious". It is less obvious that the difference between the two freedoms, the difference the information of the perception of the living and the inanimate, is the physical size proportional to the physical *action* (the product of the length and momentum, i.e. the product of time and energy), which is proportional to the force. It is even less evident that reducing the aggressiveness that to people make up the growing legal systems, tend to translates civilization into an increasingly efficient tissue like a plant in which individuals have less and less personal freedoms and more and more social rights and obligations.

Let's go back to the previous physical experiment. The experimenter receives information about the location and the momentum of the electron. When the uncertainty of the position Δx is decreasing, in order to maintain the total amount of uncertainty and information of position-momentum, the uncertainty of the momentum Δp is increasing, and vice versa. Hence, we observe another feature of these strange phenomena that the transformation of uncertainty into information is a process that can go in the opposite direction too, from information to uncertainty. This brings us to the step onto the deeper understanding of communication, the exchange of information between electrons and experimenters (its equipment). Elementary particles are so simple that they can communicate and when their time runs in opposite directions (more details about this in the second book, [12]).

The charged particles interact with photons, but the question is whether photons can communicate with other particles, including other photons. In nature, particles are not that everyone can communicate with everyone, because it is said that nature loves to hide information. Now the basic question arises, are there situations where two quantum systems, two physically real bodies, do not communicate at all?

Secondary communication would be when one sends a message in one language, the translator translates, and the other receives a message in another language that is understandable. Another example, when two particles move away, each of them is simultaneous with the past of the other, so they are realistic together in total with their past. By generalizing this, we will say that the particles A and B are mutually real if there is a C particle that is real with both. These are different realities.

What is then with the realities that can arise from the choice of two different outcomes. According to chaos theory, small differences in the initial steps can lead to large differences at the end of the road. Deciding on a random way to activate or not the explosive device, a terrorist can significantly change the lives of many. Both results have a common past to the moment of decision, which means that we can both be considered in some way mutually realistic. However, they cannot communicate directly. For example, if among the two entangled photons with two different spins (± 1) the first is observed with a spin of $+1$, we know that the second would have spin -1 and vice versa, if the first is observed as -1 the second is $+1$. If further, the opposite spins of the first caused two different realities, the spins of the second could not be cross-perceived, as they would then violate the law of maintaining the sum of spins. There would be no direct communication.

Therefore, the universe we are here talking about would have more (time) dimensions, in "parallel" worlds that do not communicate directly, but they are all realistic in peculiar ways. Summarizing, we note that this (hypothetical) universe is highly relativized. Depending on the choice of the observer, it has different probabilities, information, entropy, and ultimately the immediate reality.

We noticed the law of saving the total amount of "uncertainty + information" applies to each individual closed quantum system. Intuitively is understood that the same law applies in the case of couples that communicate in a special way, but it was mentioned only on the case of pair's position-momentum and time-energy. However, the same can be calculated for any other two states that similarly communicate. Let them have medium information:

$$I_1 = \frac{1}{2}\ln(2\pi e\sigma_1^2), \quad I_2 = \frac{1}{2}\ln(2\pi e\sigma_2^2), \tag{3.17}$$

given by (3.10), then from the sum $I_1 + I_2 =$ const. follows $\sigma_1\sigma_2 =$ const. i.e.

$$\sigma_1\sigma_2 \geq \frac{\hbar}{2}. \tag{3.18}$$

And that should be obtained by measuring these dispersions. The second question is how many quantum states that "similarly" communicate.

The crucial for the physical interpretation of information in quantum mechanics is the probability. We'll look at the familiar things briefly. The state of the sistem $|\psi\rangle$ is represented by a linear superposition of the $|\psi_n\rangle$ of the corresponding operator $\hat{A}$ that represents the observable A we measure:

$$|\psi\rangle = \sum_n |\psi_n\rangle\langle\psi_n|\psi\rangle = \sum_n a_n|\psi_n\rangle. \tag{3.19}$$

By measuring the observable A, the state of the system $|\psi\rangle$ changes to one of its own states $|\psi_n\rangle$ called *eigenvector* or characteristic vector related to the operator $\hat{A}$ and obtains its own value a_n called *eigenvalue*. The only exception to this is when the system has already been in one of its observable states that are measuring. For example, if the system is in its own (eigen) state $|\psi_n\rangle$, measuring the observable A gives with certainty the value a_n without changing the state $|\psi_n\rangle$.

In general, in the *discrete spectrum*, when we measure some observable A of the system $|\psi\rangle$, the probability of obtaining an undegenerated eigenvalue a_n of the corresponding operator $\hat{A}$ is

$$\Pr(a_n) = \frac{|\langle\psi_n|\psi\rangle|^2}{\langle\psi|\psi\rangle}, \tag{3.20}$$

where $|\psi_n\rangle$ is eigenvector of the state $\hat{A}$ with its eigenvalue a_n. For m times degenerate eigenvalue, the probability is

$$\Pr(a_n) = \frac{\sum_{j=1}^m |\langle\psi_n^j|\psi\rangle|^2}{\langle\psi|\psi\rangle}. \tag{3.21}$$

Analogously applies to the *continual spectrum*:

$$\frac{d}{da}\Pr(a) = \frac{|\psi(a)|^2}{\langle\psi|\psi\rangle} = \frac{|\psi(a)|^2}{\int_{-\infty}^{+\infty}|\psi(a')|^2 da'}, \tag{3.22}$$

given by the density of the probability of finding the value of the observable in the interval from a to $a + da$.

Born[10] was the first to interpret $|\psi|^2$ as the probability density, and $|\psi(\vec{r},t)|^2 d^3 r$ as the infinitesimal probability $dP(\vec{r},t)$ of finding the particle in the moment t in the volume element $d^3 r$ of space S located between $\vec{r}$ and $\vec{r} + d\vec{r}$:

$$|\psi(\vec{r},t)|^2 d^3 r = dP(\vec{r},t). \tag{3.23}$$

The total probability of finding a particle somewhere in the space must be one:

$$\int_\Omega |\psi(\vec{r},t)|^2 d^3 r = 1. \tag{3.24}$$

Note, due to the last condition (unit probability standardization) within the given quantum mechanical system is meant multitasking, which is a special case of multiprocessing.

The next important issue is the definition of information, and there is no official agreement. A simple case would be Hartley's information, a logarithm of probability, assuming the same probability of all options. In the case of different probability of distribution of outcomes, we have Shannon information as a kind of Hartley average. On the example of "secretary problem" we also saw that Shannon's information can be unexpectedly approximated by Hartley, when one special average of the outcome is used. This second case inspired us to look for solutions as in the following example, especially because we have to give up of Shannon's definition.

Example 3.3.6. *When is the quantum system information constant?*

Solution. The density of the Born probability is $|\Psi|^2 = \Psi^*\Psi$, where the wave function is in the most general form $\Psi = \Psi(x,y,z,t)$. The information is the logarithm of that number. If it does not change by changing the coordinates $\xi \in \{x,y,z,t\}$, then $\partial_\xi(-\ln|\Psi|^2) = 0$, and hence:

$$\frac{\partial}{\partial\xi}(-\ln|\Psi|^2) = -\frac{\partial}{\partial\xi}\ln(\Psi^*\Psi) = -\frac{1}{|\Psi|^2}\left(\frac{\partial\Psi^*}{\partial\xi}\Psi + \Psi^*\frac{\partial\Psi}{\partial\xi}\right) = 0,$$

$$\frac{\partial\Psi^*}{\partial\xi}\Psi + \Psi^*\frac{\partial\Psi}{\partial\xi} = 0, \tag{3.25}$$

$$\Psi = C_\xi e^{ic_\xi \xi/\hbar}, \quad i = \sqrt{-1}, \tag{3.26}$$

where C_ξ and c_ξ are unknown constants. Choosing the different coordinates ξ we find the wave function:

$$\Psi(\mathbf{r},t) = Ce^{i(\omega t - k\mathbf{r})/\hbar}, \tag{3.27}$$

where $\mathbf{r} = (x,y,z)$ vector of the position. This is the free wave-particle. □

The result is intuitively quite acceptable. The formula (3.25) says that the information of the given system is preserved if the probabilities are unchanged, and the

[10] Born rule: `https://en.wikipedia.org/wiki/Born_rule`

result (3.27) confirms the expectation that the free *wave-particle* does not lose or receive information along the way, until some force begins to act on it.

Note that in a discrete case of probability, the condition that they form a distribution (the sum of the probability is one) and that the information is constant, we also come to the same equation (3.25) and the same solution, a plane wave. The wave function can be written in the following way

$$\psi(\vec{r},t) = Ce^{i(\vec{p}\cdot\vec{r}-Et)/\hbar}, \tag{3.28}$$

where C is a constant, $\vec{p}$ vector of the momentum (impulse), $\vec{r}$ vector of the position, and E is energy. This function satisfies the basic equation of motion of a free wave-particle:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}E\psi = 0, \tag{3.29}$$

which is a (non-relativistic) time independent Schrödinger equation of quantum mechanics, for a wave-particle within a given space (boxes) of potential energy $U(x) = 0$. We calculate that the energy of such a particle is:

$$E = \frac{\hbar^2 k^2}{2m}. \tag{3.30}$$

One particle can have an arbitrary energy value; its spectrum of energy is continuous.

The condition of the probability density norm, $\int_\Omega \psi^*\psi\, d\omega = 1$, leads to the equation of the form (3.25) too, which can also be written as follows:

$$\frac{\partial\psi}{\partial t} = ia\psi,\ \frac{\partial\psi}{\partial x} = b_x\psi,\ \frac{\partial\psi}{\partial y} = b_y\psi,\ \frac{\partial\psi}{\partial z} = b_z\psi, \tag{3.31}$$

where a, b_x, b_y, b_z are real constants. Multiplying each of the equations by arbitrary coefficient and adding we get

$$\lambda_t\frac{\partial\psi}{\partial t} + \lambda_x\frac{\partial\psi}{\partial x} + \lambda_y\frac{\partial\psi}{\partial y} + \lambda_z\frac{\partial\psi}{\partial z} = i\lambda\psi, \tag{3.32}$$

where $\lambda = a\lambda_t + b_x\lambda_x + b_y\lambda_y + b_z\lambda_z$.

It is a linear partial differential equation that represents the physical data system of constant information. By selecting the lambda coefficients so that the equation is invariant to *Lorentz* transformation[11] it is possible to get *Dirac* equation[12] in quantum mechanics.

Taking other derivatives along the same coordinates of the starting four equations, after multiplying with arbitrary numbers and adding, we find:

$$\mu_t\frac{\partial^2\psi}{\partial t^2} + \mu_x\frac{\partial^2\psi}{\partial x^2} + \mu_y\frac{\partial^2\psi}{\partial y^2} + \mu_z\frac{\partial^2\psi}{\partial z^2} + \mu\psi = 0, \tag{3.33}$$

[11]Lorentz transformation: `https://en.wikipedia.org/wiki/Lorentz_transformation`
[12]Dirac equation: `https://en.wikipedia.org/wiki/Dirac_equation`

wherein

$$\mu = a^2\mu_t + b_x^2\mu_x + b_y^2\mu_y + b_z^2\mu_z.$$

For example, choosing the constants

$$\mu_x = \mu_y = \mu_z = 1, \ \mu_t = -\frac{1}{c},$$

where c is the speed of light in vacuum, we get the equation of quantum mechanics known as *Klein-Gordon* equation[13]. It well describes particles without spin (spin 0). When it was discovered in 1926, Schrödinger noticed that it gave poorer results for a hydrogen atom (spin ±1) than his non-relativistic.

Taking the speed of light approximately $c \to \infty$ and selecting the coefficients $\mu_x = \mu_y = \mu_z = 1$, $\mu_t = 0$, the last equation (3.33) becomes:

$$\nabla^2\psi + \mu\psi = 0, \tag{3.34}$$

which we recognize as Schrödinger's, provided:

$$\mu = \frac{8\pi^2 m}{h^2}E.$$

All this indicates that the way and the ease with which we pass from the equation (3.32) to (3.33) and again to Schrödinger's (3.34) comes from some forms of laws of conservation for uncertainty and information.

When a particle has no spin, or when it is in an inertial system, or when we have a flat wave, then the law of information conservation applies. On the contrary, let's say, when the C coefficient for the shape function (3.27) is not constant, it does not have to be. This leads us to the assumption that the overall information of the quantum system can be changed under the external forces, and then the disturbance of the saved information is related to the direction of the time flow. Compare this with notes on the flow of time from the previous section.

On the other hand, the equation (3.1) is on the step from the matrix equations of quantum mechanics. Let's write in line:

$$\mathbf{p}\cdot\mathbf{r} = s,$$

$$p_1r_1 + p_2r_2 + \dots + p_nr_n = s,$$

$$\begin{cases} b_{11}r_1 + b_{12}r_2 + \dots + b_{1n}r_n = r_1 s, & b_{1k} = r_1p_k, \\ b_{21}r_1 + b_{22}r_2 + \dots + b_{2n}r_n = r_2 s, & b_{2k} = r_2p_k, \\ \dots \\ b_{n1}r_1 + b_{n2}r_2 + \dots + b_{nn}r_n = r_n s, & b_{nk} = r_np_k, \end{cases}$$

where the index is $k = 1, 2, \dots, n$. Briefly, we write matrix:

$$\begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ \dots \\ r_n \end{pmatrix} = s \begin{pmatrix} r_1 \\ r_2 \\ \dots \\ r_n \end{pmatrix}, \tag{3.35}$$

[13]Klein–Gordon equation: `https://en.wikipedia.org/wiki/Klein%E2%80%93Gordon_equation`

or:

$$\hat{B}\mathbf{r} = s\mathbf{r},$$

Although it seems to be very far from information, this is in its form a typical equation of Heisenberg's matrix quantum mechanics. Coefficients b_{jk} are complex numbers, and the matrix $\hat{B}$ is Hermitian. This matrix, its own vector, eigenvector $\mathbf{r}$, and its own value, eigenvalue s, represent the states of the quantum system, its variables, and its observables. Finally, here are some examples of the use of these matrices.

Hence the idea that by connecting the living and inanimate world with a unique form, we try to understand the quantum phenomena. The freedom ℓ defined here as a special amount of options, as a number, would have to have intuitive properties of information, but not necessarily exactly equal to those known to date. We have previously seen that this new information could be proportional to Lagrangian, a function that expresses all known physical movements through the principle of the least action. Now we expect that the same information could be proportional to physical observables, in a way particularly relevant in quantum mechanics, simply because they have a similar form.

3.4 Entropy

The word *entropy* (Greek $\epsilon\nu\tau\rho o\pi\eta$ – change within) introduced in physics Clausius in 1865 to present a measure of "bound" energy of some closed material system, i.e. for energy that, in contrast to "free", can no longer be turned into work. The opposite term is *ectropy.* It was not until Boltzman (Lectures on the Principles of Mechanics, 1897-1904) that entropy marked the thermal content of the thermodynamic system, that is, the energy of its molecules.

Entropy S is a measure of system disorder. At the lowest possible temperature, on the absolute zero[14] gas (in theory) could be compacted to achieve a zero volume and had a zero entropy. However, such a low temperature cannot be achieved (Third Principle of Thermodynamics) and so low entropy is impossible. If a body at the temperature T (in Kelvin) is given a small amount of heat ΔQ the body entropy will increase for:

$$\Delta S = \frac{\Delta Q}{T}. \tag{3.36}$$

The second principle of thermodynamics says that heat can only be transferred from a warmer to a cooler body, or that entropy cannot spontaneously drop. A self-abandoned system tries to move from the state lower to a state of greater disorder.

When an object drops its kinetic energy, by bursting into the substrate it, becomes a thermal, slightly heated the substrate at the point of falling. This process is *irreversible.* It is not possible for the object to spontaneously fly back by subtracting the heat energy of the substrate and turning it into its kinetic, and then in potential climbing increasingly in spite of gravity. In the first case, entropy is growing, in the second, it falls. When the ball bounces on the floor, making less and less bows, it moves so that the entropy

[14]Kelvin's degree (K) is equal to the Celsius degree (C), but 0° K = −273.150° C.

increases, while the other way around – the bigger bouncing of the ball spontaneously – is not possible, as the entropy would then fall. When the glass cup falls on the hard surface and breaks, its entropy is bigger, while the other way round – the spontaneous collecting of the heart and the formation of the undamaged glass is not possible, since then its entropy would become smaller.

Spontaneous processes in thermodynamics are called irreversible events that are without external influence. These processes once started are running (with time) until they achieve balance, without the possibility of the opposite flow, except in some cases of continuous feeding by energy or work. However, all melting and evaporation are spontaneous changes, although are consuming energy from the environment!

When physical or chemical change releases heat into the environment, then we say there is a *exothermic* reaction (egzo – out). Such is the extinguishing of lime, wood or coal combustion, neutralization of acids and bases, freezing. If ambient heat is associated with physical or chemical change, there is a *endothermic* reaction (endo – in). Endothermic is cooking a lunch, dissolving a saltire (KNO_3 - potassium nitrate), defrosting.

Exothermic processes are those for which $\Delta Q < 0$, i.e. heat change in formula (3.36) is negative because they lose heat, and endothermic ones with $\Delta Q > 0$, because they take heat from the environment. Spontaneous processes can be both exo and endo-term, so a better spontaneity criterion is given by the Second Law of Thermodynamics, which states that in all cases of spontaneous processes, matter moves from a more orderly to less regulated state, that entropy (3.36) remains the same or grows.

We have announced even better criteria here. The next book (Space-time) arrives to this part of physics starting from the principle of probability that the most likely happened most often. If we accept that information of a more likely event is smaller, we come to the principle of information, that nature with them is not generous. The second law of thermodynamics is the consequence, because the information is hiding from us in this amorphous, uniform, state of equally probable outcomes. Why is such an approach better? It is because the more convincing (mathematically clearer) view and because the same principle of information will include all living beings. Living beings escape from the state of equality of all options, because such a situation is most challenging for them, it opens up the most opportunities and irritates their abilities to the maximum, as opposed to the state of organization when the "amount of options" is lower. This "quantity of options" is actually the new information.

Exploring statistically what the entropy could mean Boltzmann[15] discovered the formula for entropy:

$$S = k_B \ln W, \tag{3.37}$$

where $k_B = 1.38065 \cdot 10^{-23}$ J / K is Boltzmann constant, and W is the number of possible states of the given thermodynamic system. He formulated this formula in the period 1872-1875. In the year 1900, Planck[16] gave it a great deal of significance, using it in 1900 for the discovering the quantization of energy of radiation.

[15] Ludwig Boltzmann (1844-1906), Austrian physicist.
[16] Max Planck (1858-1947), German theoretical physicist,

After the discovery of information (Hartley, 1928), the Boltzmann's formula and entropy get the new meaning. Entropy is information that is missing after a spontaneous event, such as the thermodynamic conversion of some form of energy into heat. Now we can say: the information is hiding in entropy. It is known that this spontaneous event, the increase of entropy, goes towards the shapes of uniform conditions, but it is then somewhat clumsily said that nature strives to increase entropy because it strives for more probable states. It's not wrong, but it's confusingly said.

The less-regulated condition really can have a higher probability. On the figure 3.5 on the left, we see two boxes and two balls, both in the first box. In the case of equal probability of balls per boxes, the probability of this arrangement is $p_1 = \frac{1}{2} \cdot \frac{1}{2}$. On

Figure 3.5: Distribution of two balls in two boxes.

the same picture on the right we have two same boxes but with one ball in each. The probability of such a schedule is $p_2 = \frac{1}{2}$, which is twice of the probability on the left.

With the increase in the number of boxes and pellets, this relationship grows rapidly in favor of dissolution. So already in the case of nine boxes and nine balls (figure 3.2), we have more than 40,000 times the probability of an even distribution than the jellied. The uniform distribution of gas particles in the room is many times more likely than the non-uniform, and therefore they are distributed uniformly. However, it would be too quickly to establish that entropy is growing with probability. The maximum probability P defines the *optimum state* of the gas particles, and the number of multiplicity $W = 1/P$ in this optimal state defines Boltzmann's entropy (3.37).

In the original formula (3.37), the value of W is actually called the probability (German: *Wahrscheinlichkeit*), but the "thermodynamic probability" that is an integer greater than one. It is the number of macroscopic states for a certain distribution of possible micro-states for an ideal gas with N identical particles of which N_j is in the jth microscopic state (range) of the position and impulse. It can be calculated that this number is

$$W = N!/\prod_j N_j! \tag{3.38}$$

where the index j goes through all possible molecular states, $\prod_j$ means the product of all such, and the exclamation mark (factorial) denotes permutations. Using this number we are working here with a "mathematical probability" $P = 1/W$ which is always a real number from zero to one, so Boltzmann's formula becomes

$$S = -k_B \ln P, \tag{3.39}$$

when we can also call it Hartley's.

Let us now discuss the notion "uniform" in Shannon's information. In the first case, we will see that evenly distributed probabilities give maximum information, and in the second, that the result of the Shannon can be obtained and otherwise by equal items.

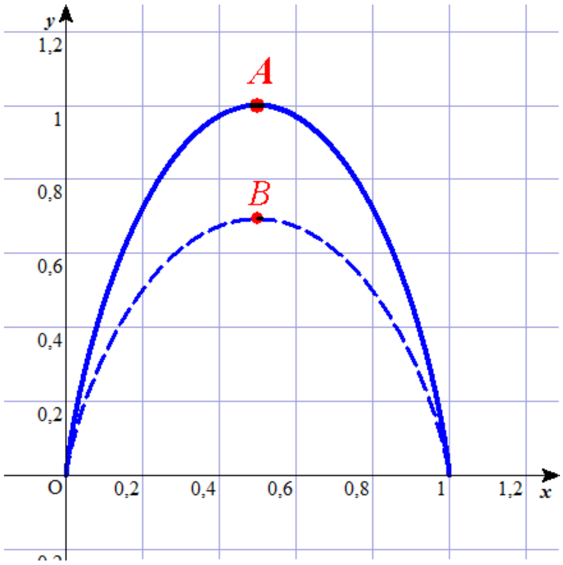

Figure 3.6: Shannon's information.

Example 3.4.1. *Show that the maximum information in the case of two possibilities, $m = 2$, is obtained when the probabilities are equal, $P_1 = P_2 = \frac{1}{2}$.*

Solution. Put $P_1 = x$ and $P_2 = 1 - x$. Shannon's information was then

$$\begin{cases} A: & y = -x\log_2 x - (1-x)\log_2(1-x), \\ B: & y = -x\ln x - (1-x)\ln(1-x), \end{cases} \tag{3.40}$$

expressed in "bit" and "nat", i.e. in the case of logarithmic base 2 and $e = 2.718\dots$.

On the graph 3.6, points $A(\frac{1}{2}, 1)$ and $B(\frac{1}{2}, \ln 2)$, that representing the maxima of the functions (3.40), are seen. By derivative can be proved that these are really the maxima. Note that by increasing the logarithm base, the Shannon curve drops, but it is always symmetric and has minima at points 0 and 1 abscises. □

In the mathematical theory of information is proved the general stand that Shannon's information has maximum when all probabilities of distribution are mutually equal.

Example 3.4.2. *Show that the maximum of the function $y = -x\ lnx$ is the point $T(\frac{1}{e}, \frac{1}{e})$.*

Solution. Let's show it with the derivative. From $y'(x_0) = -\ln x_0 - 1 = 0$ follows $x_0 = \frac{1}{e}$ and then $y(x_0) = \frac{1}{e}$. In the image 3.7 blue is a graph of the function, and the point of the maximum is $T(\frac{1}{e}, \frac{1}{e})$. □

The point T on the given image is the maximum of the function $y(x) = -x\ln x$, the ordinate $y = 1/e$ on abscise $x = 1/e \approx 0.37$. We have already applied the probability $1/e$ to the "secretary's problem", but here we can notice how nature holds optimal states and seek them in leveled distributions, which we than perceive as states of the greatest

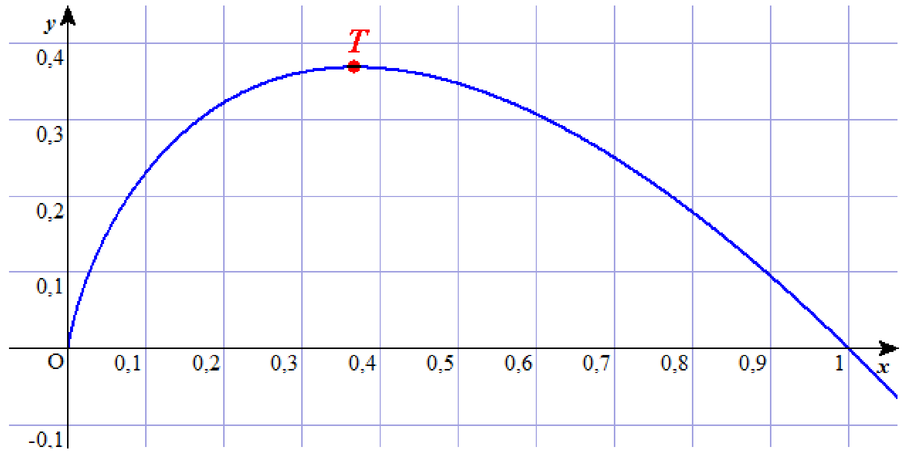

Figure 3.7: Information $y = -x \ln x$.

disorder. So, looking for the optimum, to absorb a maximum of the information, nature tends to increase entropy.

The third thing to discuss here is the extension of the Shannon formula to $s = \mathbf{r} \cdot \mathbf{p}$. This is a scalar product of a vector which (for constant intensities) has a maximum value when the angle between the vectors is zero, that is, when the proportion is valid:

$$r_1 : r_2 : \ldots : r_n = p_1 : p_2 : \ldots : p_n, \tag{3.41}$$

where r_k and p_k are the components of the given vectors. We can easily notice that this is also a kind of "uniformity", which would provide maximum information in the Shannon formula. Regarding the form itself, it does not matter what these r and p are not necessarily probability or logarithms of probability. It is sufficient to note that in the special case, $r_k = P_k$ and $p_k = -\ln P_k$ for $k = 1, 2, \ldots, n$, this formula is reduced to Shannon.

Let's return now to Clausius definition that entropy is spent work. From (3.36) we can observe this also by keeping the temperature constant, since the heat is energy, i.e. work. On the other hand, we know that mechanical work is to overcome resistance over time. The work is positive if the body moves in the direction of the force acting on it, and it is negative in the opposite direction. Therefore, the force does not do work only if it is in balance with resistance.

As we know from physics lessons in school, the work of A of constant force $\mathbf{F}$ on the path $\mathbf{r}$ is equal:

$$A = \mathbf{F} \cdot \mathbf{r}, \tag{3.42}$$

that is, the work is a scalar product of the force and path vectors. Recall, the unit of work in the International System unit is joule (J = kg m^2/s^2, or Nm), and this product of force and length in physics was introduced around 1930 by Coriolis[17].

[17] Gaspard- Gustave de Coriolis (1792-1843), French mathematician.

For example, if the force of 10 N pulls the body on the 3 m path, it performs work $A = 30$ J. A bit more general, a body mass m with a constant acceleration $a = F/m$ at the end of the path $r = a \cdot t^2/2$ has the speed $v = at$, so we find the kinetic energy:

$$A = F \cdot r = m \cdot a \cdot \frac{a \cdot t^2}{2} = m \cdot \frac{(a \cdot t)^2}{2} = \frac{m \cdot v^2}{2} = E_k, \tag{3.43}$$

which means that the work (A) of the force (F) is equal to the kinetic energy (E_k) that the body receives on the path (r). In both cases, the force and the way had the same direction.

The changeable force should be integrated. Then the work is equal to the integral of the scalar product given, for example, by the time parameter t, from the moment t_1 to the moment t_2, or from the position P_1 to P_2 where the body was in those moments. It follows from the properties of the integral that the final work does not depend on the shape of the traversed path, but only from its endpoints.

When the force and the path do not have the same direction, the work is their scalar product, analogous to freedom ($\ell = \mathbf{i} \cdot \mathbf{h}$) of the scalar product of the vector of intelligence and hierarchy. The biggest formal difference here is that the physical force and path are vectors with three components (length, width, height), while the final number of the components of the intelligence and hierarchy is enormous. It is questioned whether this analogy has a deeper meaning?

Mathematicians are used on working with analogies, forms, which they never or rarely give a "deeper meaning". So, for example, it was always strange to me that someone because of Einstein's formula $E = mc^2$ says that energy and mass are the same. If they are the same, then why do we bother asking for Higgs[18] boson? The fact that a certain amount of mass can be converted into an amount of energy according to the given formula means that "mass" is not exactly the same as "energy". And here the "deeper meaning" ceases, in the possibility (not obligatory) of converting one into another.

If we understood, then I can conclude that formula $\ell = \mathbf{i} \cdot \mathbf{h}$ has a "deeper meaning" in the formula $s = \mathbf{r} \cdot \mathbf{p}$ which can represent Heisenberg's uncertainty relations (3.5), as well as the formula for the work of force on the road (3.42). This latter can also be reduced to probability.

Force is a natural accomplice to life, this to change and development, and those to freedoms and options. The most important regulations of the legislation relate to the aggressive behavior, and it is primarily "unauthorized" use of force. We acquire freedom and defend ourselves by force. That is the case of revolutions against dictatorship, the release from the invaders, and also in micro-cases of the struggle against violence in everyday life. Therefore, force is the main culprit for violating the laws in any state and the main instrument of its regulation. Consistent further, when we extend the information of perception to the inanimate world of physics and we recognize it as a Lagrangian factor, then its factors become force, distance, and time.

When in two adjacent rooms with a moving partition between are emitted the gases with different properties, both will exert pressure on the wall between, until the balance

[18] Peter Higgs, born. 1929, the British theoretical physicist.

of force is established. This equilibrium force is the result of pressure that causes the freedom of movement of gas particles. It is the vector that acts on the body that moves in the given path, causing the work in formula (3.42). We will not deal with defining the "force of probability" here, because such a formal treatment in this discussion would be too much. It is sufficient to note that there is a path of strong associations, and also mathematical analogies of the formula $\ell = \mathbf{i} \cdot \mathbf{h}$ to the formula for physical work, and then again back to formal understanding of "force" which causes fear of freedom in people in the event of a surplus of freedom for ℓ, or a rebellion for freedom – in the case of a deficiency.

This kind of thinking has its consequences[19] that have been announced "between rows", which I did not want to say explicitly before finding support for publishing a book. The first is, if the information is real, physical, then the options are objective. Then the world is not only 4-dimensional, with three dimensions of space (length, width, height) and one time, but it has at least one more (in fact even more) dimensions of time. To turn to these other dimensions, force is needed, which from the point of view of options means – freedom. It is not reasonable to assume that, depending on the choice, we fall into some unrealistic physical world, different natural laws. It is logical to assume that all such different streams of time are equivalent concerning the laws of physics, and then again we have a problem with entropy.

By discovery of entropy the thermodynamics has become the first branch of physics in which it is no longer the same which sign has the flow of time. We know that the Earth moves around the Sun along the ellipse, but it is irrelevant whether we look at that film "normally" or "backwards". The laws of mechanics, as well as electrodynamics, remain the same when we replace time variable t with $-t$. This, however, is not so in thermodynamics. A film in which the pieces of a broken window would be assembled into an undamaged glass surface in the opening would not be physically correct, since smashing the glass is a thermodynamic process. It seems that the development of the system to conceal information is one-way.

The exit from this paradox is also indicated in the preceding text. It's the relativity of probability. For different observers, seemingly identical random events do not have the same probability! That is why the law of inertia is valid, because each of the bodies moves on the most likely path for themselves. The same applies to satellites and other celestial bodies moving in the gravitational field. The presence of force means a probability disorder and vice versa, the probability of distortion is observed as a force. This means that the return of the movie back, the glass that fell from the table and broke, is not physically equivalent to the actual flow of time backwards. Such a return flow, an eventual observer, would be a witness to the enormous forces invisible to us, which would lift the glass from the floor and compile it. His experience of what "we see" in our direction would not have been a normal movie strip released backward.

It's absurd, but it's easier to accept alleged travelers going by any time axes, but two that communicate and have opposite observations of the direction of time. What is the question for one that is the answer for another, and vice versa, each one must

[19] added after printing

hit the question in advance and gave the answer beforehand. However, this is possible if they are very elementary particles, so simple and without the ability of complicated "thinking", which would blindly "kick the first ball". For more complex bodies, the law of large numbers of probability theory is valid, but these are already two themes for some other time.

We have a similar inversion of entropy from the point of view of the macro-world, observing the spontaneous rise in the disorder of the micro-world and dual, from the point of view of living beings to the spontaneous escape from freedom. The both spontaneity are the consequences of the principles of information (stinginess of nature), and that is the consequence of the principle of probability (realization of more probable), and these discussions are the themes of the second book "Space-time". Of course, it will take us away from the known bounds of entropy, but not in its denial. This is seen from the following.

Neumann[20] entropy is the extension of classical Gibbs entropy to statistical mechanics (which we will discuss more in the next section). For the quantum mechanical system described by the density matrix $\hat{\rho}$, this entropy is defined by

$$S = -\mathrm{Tr}(\hat{\rho}\ln\hat{\rho}),$$

where $\mathrm{Tr}(\dots)$ means the trace (sum of the diagonal elements) of the parent matrix. When $\hat{\rho}$ is written using its eigen (own) vectors $|1\rangle, |2\rangle, |3\rangle, \dots$ then the density

$$\hat{\rho} = \sum_j a_j |j\rangle\langle j|,$$

so Neumann's entropy

$$S = -\sum_j a_j \ln a_j.$$

Here are some simple examples.

Let the mixed state $|0\rangle$ has probability $1/2$ and $|1\rangle$ probability $1/2$, then:

$$|0\rangle\langle 0| = \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix},$$

$$|1\rangle\langle 1| = \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0 & 1\end{pmatrix} = \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix},$$

$$\hat{\rho} = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \begin{pmatrix}1/2 & 0\\0 & 1/2\end{pmatrix},$$

$$S = -\frac{1}{2}\ln\frac{1}{2} - \frac{1}{2}\ln\frac{1}{2} = \ln 2.$$

Another example, we have vectors:

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle),$$

[20] John von Neumann (1903-1957), Hungarian-American mathematician.

both probability 1/2. Hence:

$$|+\rangle\langle+| = \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1 & 1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix},$$

$$|-\rangle\langle-| = \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}1 & -1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & -1\\-1 & 1\end{pmatrix},$$

$$\hat{\rho} = \frac{1}{2}|+\rangle\langle+| + \frac{1}{2}|-\rangle\langle-| = \begin{pmatrix}1/2 & 0\\0 & 1/2\end{pmatrix},$$

$$S = -\frac{1}{2}\ln\frac{1}{2} - \frac{1}{2}\ln\frac{1}{2} = \ln 2.$$

So, we have obtained two identical density matrices and two equal results for the S entropy, although the initial mixed states are different. This is proof that (Neumann's) entropy conceals information. Yet the density matrix completely reflects the effects of measurement. We see this from the following example.

Example 3.4.3. *Let's give a mixed state in an orthonormal basis $|\alpha_j\rangle$. Show that the outcomes $|\alpha_j\rangle$ have the probabilities $\langle\alpha_j|\hat{\rho}|\alpha_j\rangle$.*

Solution. Let the state $|\psi_k\rangle$ is associated to the probability p_k, where $0 \le p_k \le 1$ and $\sum_k p_k = 1$. Let's mark the probability of measuring $|\alpha_j\rangle$ with $\Pr(j)$. Then:

$$\Pr(j) = \sum_k p_k|\langle\psi_k|\alpha_j\rangle|^2 = \sum_k p_k\langle\alpha_j|\psi_k\rangle\langle\psi_k|\alpha_j\rangle =$$

$$= \left\langle\alpha_j\left|\sum_k p_k|\psi_k\rangle\langle\psi_k|\right|\alpha_j\right\rangle = \langle\alpha_j|\hat{\rho}|\alpha_j\rangle,$$

and it was supposed to show. □

3.5 Statistics

Statistics is a method of collecting, analyzing, interpreting, presenting and organizing mass data. It is not a branch of mathematics. In the applied statistics (in scientific, industrial, social problems), the usual terms are *population* (like "all people living in a given state" or "every atom of crystal") and a representative *sample*. Here we will pay attention to only two statistical results on the samples: coefficient of correlation and linear regression.

Correlation (lat. *con* – with, *relatio* – relationship) is a mutual relationship between two or more things. In statistics the *correlation* is a dependency between two random variables or two sets of data, estimated using the real number r greater than -1 and less than +1, which we call *correlation coefficient*.

Correlation $r > 0.7$ means strong positive relationships (more of one – more another) of variables, while r from -0.3 to zero means bad negative relationships (more of one – less other).

First, we need two sets of data, two random variables:

$$\mathbf{x} = (x_1, x_2, \dots, x_n), \quad \mathbf{y} = (y_1, y_2, \dots, y_n) \tag{3.44}$$

for which the mean values are calculated:

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{1}{n}\sum_{k=1}^{n} x_k, \quad \bar{y} = \frac{y_1 + y_2 + \dots + y_n}{n} = \frac{1}{n}\sum_{k=1}^{n} y_k, \tag{3.45}$$

then variance:

$$S_x^2 = \frac{1}{n-1}\sum_{k=1}^{n}(x_k - \bar{x})^2, \quad S_y^2 = \frac{1}{n-1}\sum_{k=1}^{n}(y_k - \bar{y})^2, \tag{3.46}$$

covariance:

$$S_{xy} = \frac{1}{n-1}\sum_{k=1}^{n}(x_k - \bar{x})(y_k - \bar{y}) = \frac{1}{n-1}\sum_{k=1}^{n}(x_k y_k - \bar{x}\bar{y}). \tag{3.47}$$

They define *Pearson coefficient* linear correlation:

$$r = \frac{S_{xy}}{\sqrt{S_x^2}\sqrt{S_y^2}} = \frac{\sum_k (x_k - \bar{x})(y_k - \bar{y})}{\sqrt{\sum_k (x_k - \bar{x})^2}\sqrt{\sum_k (y_k - \bar{y})^2}} = \frac{\sum_k (x_k y_k - \bar{x}\bar{y})}{\sqrt{\sum_k (x_k - \bar{x})^2}\sqrt{\sum_k (y_k - \bar{y})^2}}. \tag{3.48}$$

Note that r can be calculated directly, as a fraction to the right.

The correlation r is the *cosines* angle between the vectors associated with $\mathbf{x}$ and $\mathbf{y}$ whose intensions are roots in (3.48), and the scalar product is the numerator. The corresponding expression for the freedom of these vectors would be $L = \ell - n\bar{x}\bar{y}$, where ℓ is freedom defined by the earlier "ordinary" scalar product.

The first example: the ice cream store was following the daily temperature ($\mathbf{x}$) and the number of pieces of the sold ice cream ($\mathbf{y}$) during the $n = 12$ days and formed a table:

x:	14.2	16.4	11.9	15.2	18.5	22.1	19.4	25.1	23,4	18.1	22.6	17.2
y:	215	325	185	332	406	522	412	614	544	421	445	408

We calculate in the order, from $\bar{x} = 18.675$ and $\bar{y} = 402.4167$ to $r = 0.9575$. The conclusion is that there are strong positive relationships between air temperature and ice cream sales. The cosine angle between the associated vectors is $\cos\gamma = r$, so the angle $\gamma = 16.7°$.

While the correlation examines the dependence of the random variables, *regression* examines their form using the regression line. At the plane Oxy given the points $T(x_k, y_k)$ with coordinates (3.44), for which we are looking the equation of straight line

$$y = a + bx \tag{3.49}$$

which is statistically the closest to them. We use the *least squares* method and get:

$$a = \bar{y} - b\bar{x}, \quad b = \frac{\sum_k (x_k y_k - \bar{x}\bar{y})}{\sum_k (x_k^2 - \bar{x}^2)}, \tag{3.50}$$

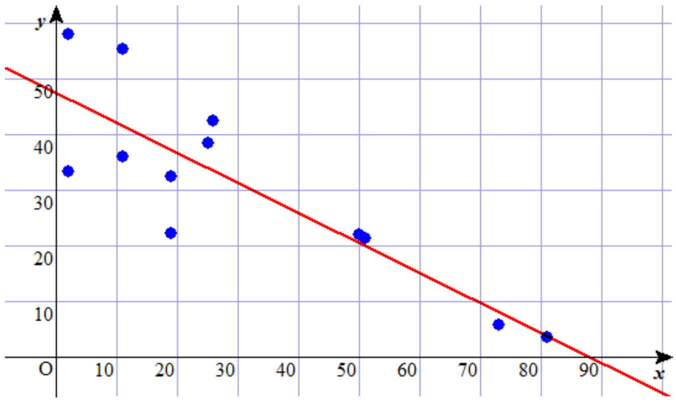

Figure 3.8: Regression line.

with the usual labels.

Another example[21]: the first variable **x** is the percentage of students receiving an allowance in 12 US schools and it represents their socio-economic status. The other variant **y** is the percentage of students who wear a helmet while riding a bicycle.

x:	50	11	2	19	26	73	81	51	11	2	19	25
y:	22.1	35.9	57.9	22.2	42.4	5.8	3.6	21.4	55.2	3.,3	32.4	38.4

Hence, $n = 12$, $\bar{x} = 370/12 = 30.833$ and $\bar{y} = 370.6/12 = 30,883$. Next we calculate:

$$\ell = \sum_{k=1}^{12} x_k y_k = 7195.7; \quad r = \frac{-4231.13}{\sqrt{7855.67}\sqrt{3159.68}} = -0.849266 \tag{3.51}$$

which is the r of strong negative correlation. Regression parameters are:

$$b = \frac{-4231.13}{7855.67} = -0.538609 \quad a = 47.4904$$

then the regression line:

$$y = -0.54x + 47.49. \tag{3.52}$$

It is a red line that, in terms of the smallest squares, best matches the blue spots in the image 3.8. The poorer are rarely wearing a protective helmet.

The wealthier have greater freedoms that come with money, and on the other hand they handle more traffic hierarchies. Intelligence (ability to use options) is plastic. When we build some skills, in order to achieve the maximum effect, it helps if we are stiff at other things that are less important to us. For example, a manager who strictly adheres to his daily schedules and in irrelevant for business creativity suffers a great deal of hierarchy, on the other side, he can expand into success. He does not need a lot of intelligence for efficiency in the jobs that are expected of him, if in other areas he carries a great hierarchy.

[21]SJSU: `http://www.sjsu.edu/`

In the above two cases, ice cream shops and carrying a helmet for a bike, we have seen that the freedom $\ell = \mathbf{i}\cdot\mathbf{h}$ can be approximated by covariance, and by the linear coefficient of the linear correlation in expression (3.49):

$$L = \ell - n\bar{x}\bar{y}, \quad \ell = \sum_{k=1}^{n} x_k y_k. \tag{3.53}$$

In the sequel (Stereometry) we will see the geometric interpretation of these expressions, of which the angle is r cosine, and which proportion is a regression, but for now we are retaining a few tasks from the algebra of these (random) n-tuples. The question is, can matching the correlation with that previous adaptation be "easier" to mathematics?

Example 3.5.1. *Show that the L of expression (3.53) is positive when the intelligence $\mathbf{x}$ and the hierarchy $\mathbf{y}$ are harmonized (adjusted), in the case:*
a) when $n = 2$; b) when $n = 3$.

Solution. a) We have $\bar{x} = \frac{1}{2}(x_1 + x_2)$, $\bar{y} = \frac{1}{2}(y_1 + y_2)$ and $\ell = x_1y_1 + x_2y_2$, so:

$$L = \ell - 2\bar{x}\bar{y} = (x_1y_1 + x_2y_2) - \frac{1}{2}(x_1y_1 + x_1y_2 + x_2y_1 + x_2y_2) = \frac{1}{2}(x_1 - x_2)(y_1 - y_2),$$

which means that $x_1 - x_2$ and $y_1 - y_2$ are the same sign, hence $L > 0$.
b) From $\bar{x} = \frac{1}{3}(x_1 + x_2 + x_3)$, $\bar{y} = \frac{1}{3}(y_1 + y_2 + y_3)$ and $\ell = x_1y_1 + x_2y_2 + x_3y_3$ follows:

$$L = \ell - 3\bar{x}\bar{y} = \frac{1}{3}[(x_1 - x_2)(y_1 - y_2) + (x_1 - x_3)(y_1 - y_3) + (x_2 - x_3)(y_2 - y_3)],$$

which again means the vectors $\mathbf{x}$ and $\mathbf{y}$ are harmonized, then L is positive number. □

The rules of the game imposed by nature are sometimes relatively easy to evaluate. For example, if I do not get enough *vitamin C* so much and so many days, I will have so much chance of get *scurvy.* However, such evaluation can be difficult with social norms.

Example 3.5.2. *Show some example of using regression for law assessment.*

Solution. Let's say, the relying on the behavior of individuals. The value of the law can be approximated by the inclination (adaptability, harmonization) of the users to the law.

There are traffic signs in 12 places, and we count vehicles and determine the frequency of compliance with these regulations. After a certain period of time, we get sequences from which we can draw the regression law and define the "quality" of the law, the item per item. □

I recall once again that statistics do not have the power of mathematics (the conclusion of statistics cannot be used in the proof of a mathematical theorem) nor can mass decisions be considered relevant for the assessment of the (average) individual's opinion.

3.6 Stereometry of statistics

Given are the two n-tuples of random variables $A(a_1, a_2, \dots, a_n)$, $A'(a'_1, a'_2, \dots, a'_n)$ and n-tuple of units $U(1, 1, \dots, 1)$. They define three vectors $\mathbf{a} = \overrightarrow{OA}$, $\mathbf{a}' = \overrightarrow{OA}'$ and $\mathbf{u} = \overrightarrow{OU}$ in rectangle Cartesian coordinate system $O\xi_1\xi_2\dots\xi_n$. Orgonal projections of the points A and A' on the line OU are points M and M' respectively, as in the figure 3.9.

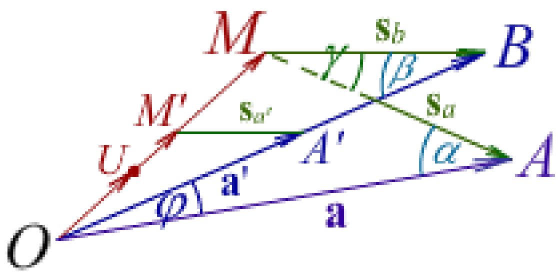

Figure 3.9: Rectangular triangles OAM and $OA'M'$.

As we know, the *arithmetic mean* is the average value of the given numbers. If we define the arithmetic means with $\mu(\mathbf{a}) = \bar{a}$ and $\mu(\mathbf{a}') = \bar{a}'$, where:

$$\bar{a} = \frac{a_1 + a_2 + \dots + a_n}{n}, \quad \bar{a}' = \frac{a'_1 + a'_2 + \dots + a'_n}{n}, \tag{3.54}$$

then we can prove that the triangles OAM and $OA'M'$ are right, with the right angles in the vertex M and M', where:

$$\mathbf{m} = \overrightarrow{OM} = \bar{a}\,\mathbf{u}, \quad \mathbf{m}' = \overrightarrow{OM}' = \bar{a}'\,\mathbf{u}. \tag{3.55}$$

This is the assertion of the following theorem.

Theorem 3.6.1. *The OAM triangle is right angled with $\angle M = 90^\circ$.*

Proof. Calculate the scalar product:

$$\overrightarrow{OM} \cdot \overrightarrow{MA} = (\bar{a}, \bar{a}, \dots, \bar{a}) \cdot (a_1 - \bar{a}, a_2 - \bar{a}, \dots, a_n - \bar{a}) =$$

$$= \mu \cdot (a_1 - \bar{a}) + \mu \cdot (a_2 - \bar{a}) + \dots + \mu \cdot (a_n - \bar{a})$$

$$= \mu \cdot (a_1 + a_2 + \dots + a_n - n\bar{a}) = \bar{a} \cdot 0 = 0.$$

From $\overrightarrow{OM} \cdot \overrightarrow{MA} = 0$ follows $\overrightarrow{OM} \perp \overrightarrow{MA}$, and then $\angle M = 90^\circ$. □

Let the point B be on the line OA', so that the orthogonal projection at the line OU falls to the point M. If for similar regular triangles OBM and $OA'M'$ is valid the proportion $\overline{OB} : \overline{OA}' = \overline{OM} : \overline{OM}'$, then $\overrightarrow{OB} = \bar{a}\,\overrightarrow{OA}'/\bar{a}'$ or $\mathbf{b} = \bar{a}\,\mathbf{a}'/\bar{a}'$. Hence:

$$\mathbf{b} = \overrightarrow{OB} = (b_1, b_2, \dots, b_n) = \frac{\bar{a}}{\bar{a}'}\,(a'_1, a'_2, \dots, a'_n), \tag{3.56}$$

so $b_k = \frac{\bar{a}}{\bar{a}'} a'_k$ in the order of $k = 1,2,\dots,k$. From the *line of middle* OM we see line AB at the angles $\varphi = \angle AOB$ and $\gamma = \angle AMB$, which is shown in the image 3.9.

Example 3.6.2. *Present the triples* $A(11,20,5)$ *and* $A'(10,5,3)$ *in the Cartesian system.*

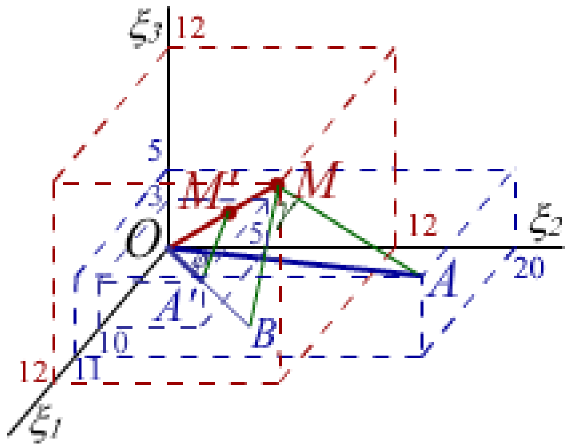

Figure 3.10: Triples $A(11,20,5)$, $A'(10,5,3)$.

Solution. In the figure 3.10 we see the right triangle OAM and barely $OA'M'$. Arithmetic means are $\bar{a} = 12$ and $\bar{a}' = 6$, so the point B has the coordinates $B(20,10,6)$.

The OAM triangle sides are:

$$\begin{cases} \mathbf{a} = \overline{OA} = \sqrt{11^2 + 20^2 + 5^2} = \sqrt{546}, \\ \mathbf{m} = \overline{OM} = \sqrt{12^2 + 12^2 + 12^2} = \sqrt{432}, \\ \mathbf{s}_a = \overline{MA} = \sqrt{(11-12)^2 + (20-12)^2 + (5-12)^2} = \sqrt{114}, \end{cases}$$

and $\overline{OA}^2 = \overline{OM}^2 + \overline{MA}^2$, and Pythagorean theorem holds. The OAM triangle is right.

The $OA'M'$ triangle sides are:

$$\begin{cases} \mathbf{a}' = \overline{OA'} = \sqrt{10^2 + 5^2 + 3^2} = \sqrt{134}, \\ \mathbf{m}' = \overline{OM'} = \sqrt{6^2 + 6^2 + 6^2} = \sqrt{108}, \\ \mathbf{s}_{a'} = \overline{M'A'} = \sqrt{(10-6)^2 + (5-6)^2 + (3-6)^2} = \sqrt{26}, \end{cases}$$

and $\overline{OA}^2 = \overline{OM}^2 + \overline{MA}^2$. The $OA'M'$ triangle is also right. □

Example 3.6.3. *Triplets* $A(-1,2,3)$ *and* $A'(4,-2,2)$ *represent in the Cartesian system.*

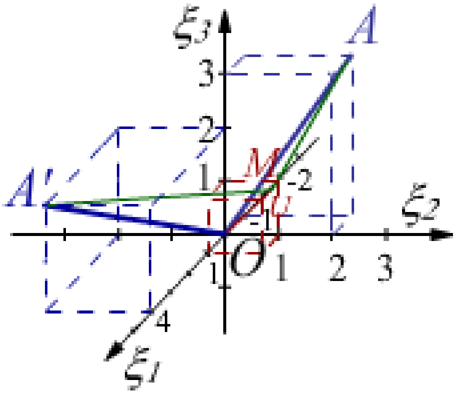

Figure 3.11: Triplets $A(-1,2,3)$, $A'(4,-2,2)$.

Solution. The solution is in the picture 3.11. The arithmetic mean of the points A and A' are equal to $\bar{a} = \bar{a}' = \frac{4}{3}$, so $M(\frac{4}{3}, \frac{4}{3}, \frac{4}{3})$. For the OAM triangle we find:

$$\begin{cases} \overline{OA} = \sqrt{(-1)^2 + 2^2 + 3^2} = \sqrt{14} = \sqrt{\frac{42}{3}}, \\ \overline{OM} = \sqrt{(\frac{4}{3})^2 + (\frac{4}{3})^2 + (\frac{4}{3})^2} = \sqrt{\frac{16}{3}}, \\ \overline{MA} = \sqrt{(-1 - \frac{4}{3})^2 + (2 - \frac{4}{3})^2 + (3 - \frac{4}{3})^2} = \sqrt{\frac{26}{3}}, \end{cases}$$

so $\overline{OA}^2 = \overline{OM}^2 + \overline{MA}^2$.

For another triangle:

$$\begin{cases} \overline{OA'} = \sqrt{4^2 + (-2)^2 + 2^2} = \sqrt{24} = \sqrt{\frac{72}{9}}, \\ \overline{OM} = \sqrt{(\frac{4}{3})^2 + (\frac{4}{3})^2 + (\frac{4}{3})^2} = \sqrt{\frac{16}{3}}, \\ \overline{MA'} = \sqrt{(4 - \frac{4}{3})^2 + (-2 - \frac{4}{3})^2 + (2 - \frac{4}{3})^2} = \sqrt{\frac{56}{3}}, \end{cases}$$

and again $\overline{OA'}^2 = \overline{OM}^2 + \overline{MA'}^2$. □

The ABM triangle is called the *correlation triangle*. The sides of this triangle are vectors:

$$\begin{cases} \overrightarrow{MA} = \mathbf{s}_a = (a_1 - \bar{a}, a_2 - \bar{a}, \dots, a_n - \bar{a}), \\ \overrightarrow{MB} = \mathbf{s}_b = (b_1 - \bar{b}, b_2 - \bar{b}, \dots, b_n - \bar{b}), \\ \overrightarrow{AB} = \overrightarrow{MB} - \overrightarrow{MA} = \mathbf{s}_b - \mathbf{s}_a. \end{cases} \tag{3.57}$$

On the other hand, from the OAB triangle we find:

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (b_1 - a_1, b_2 - a_2, \dots, b_n - a_n) = \mathbf{b} - \mathbf{a}, \tag{3.58}$$

and also $\mathbf{s}_b - \mathbf{s}_a = \mathbf{b} - \mathbf{a}$. The cosine rule[22] gives:

$$\begin{cases} \overline{AB}^2 = \overline{MA}^2 + \overline{MB}^2 - 2\cdot\overline{MA}\cdot\overline{MB}\cdot\cos\gamma, \\ \overline{AB}^2 = \overline{OA}^2 + \overline{OB}^2 - 2\cdot\overline{OA}\cdot\overline{OB}\cdot\cos\varphi. \end{cases} \tag{3.59}$$

From the right triangles OAM and OBM follows:

$$\begin{cases} \overline{OM} = a\sin\alpha & \overline{MA} = a\cos\alpha, \\ \overline{OB} = a\sin\alpha/\sin\beta, & \overline{MB} = a\sin\alpha\cot\beta, \end{cases} \tag{3.60}$$

where $a = \overline{OA}$, $\alpha = \angle OAM$, $\beta = \angle OBM$. Thus:

$$\overline{MA}^2 + \overline{MB}^2 - 2\cdot\overline{MA}\cdot\overline{MB}\cdot\cos\gamma = \overline{OA}^2 + \overline{OB}^2 - 2\cdot\overline{OA}\cdot\overline{OB}\cdot\cos\varphi,$$

$$s_a^2 + s_b^2 - 2s_a s_b\cos\gamma = a^2 + b^2 - 2ab\cos\varphi,$$

$$2ab\cos\varphi - 2s_a s_b\cos\gamma = (a^2 - s_a^2) + (b^2 - s_b^2),$$

$$2(ab\cos\varphi - s_a s_b\cos\gamma) = m^2 + m^2,$$

$$ab\cos\varphi - s_a s_b\cos\gamma = m^2,$$

or using scalar vector products:

$$\mathbf{a}\cdot\mathbf{b} = \mathbf{m}\cdot\mathbf{m} + \mathbf{s}_a\cdot\mathbf{s}_b. \tag{3.61}$$

The marks are from the image 3.9 and $\mathbf{m} = \overrightarrow{OM}$.

In particular, the cosine of the angle between the vectors $\mathbf{s}_a$ and $\mathbf{s}_{a'}$ is:

$$\cos\gamma = \frac{\mathbf{s}_a\cdot\mathbf{s}_b}{|\mathbf{s}_a||\mathbf{s}_b|} = \frac{\sum_k(a_k-\bar{a})(b_k-\bar{b})}{\sqrt{\sum_k(a_k-\bar{a})^2}\sqrt{\sum_k\left(b_k-\bar{b}\right)^2}}. \tag{3.62}$$

We get the same result for gamma if we use $\mathbf{s}_{a'}$ instead of $\mathbf{s}_b$. This is the *Pearson coefficient* r of linear correlation. On the other hand:

$$\cos\varphi = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{\sum_k a_k b_k}{\sqrt{\sum_k a_k^2}\sqrt{\sum_k b_k^2}} \tag{3.63}$$

It is also the cosine of the angle between the given vectors $\mathbf{a}$ and $\mathbf{a}'$.

Example 3.6.4. *Find the angles in examples 3.6.2 and 3.6.3.*

Solution. In example 3.6.2:

$$\begin{cases} \cos\gamma = \frac{-4-8+21}{\sqrt{114}\sqrt{26}} = 0,202048, & \gamma\approx 78^\circ, \\ \cos\varphi = \frac{11\cdot20+20\cdot10+5\cdot6}{\sqrt{11^2+20^2+5^2}\sqrt{20^2+10^2+6^2}} = 0,831829, & \varphi\approx 34^\circ, \end{cases} \tag{3.64}$$

[22] Law of cosines: `https://en.wikipedia.org/wiki/Law_of_cosines`

so the gamma angle is more than twice as high as the angle φ.

In example 3.6.3:

$$\begin{cases} \cos\gamma = \frac{22/3}{\sqrt{26/3}\sqrt{56/3}} = 0,576557, & \gamma \approx 55^\circ, \\ \cos\varphi = \frac{-2}{\sqrt{14}\sqrt{24}} = -0,109109, & \varphi \approx 96^\circ. \end{cases} \tag{3.65}$$

Gamma is nearly two times smaller than φ. □

Example 3.6.5. *The first vector* **a** *has components of the percentage of students receiving social assistance in 12 schools, and the other* $\mathbf{a}'$ *percentage of students using the helmet while driving a bicycle.*

a:	50	11	2	19	26	73	81	51	11	2	19	25
$\mathbf{a}'$:	22.1	35.9	57.9	22.2	42.4	5.8	3.6	21.4	55.2	33.3	32.4	38.4

Calculate the angles γ *and* φ.

Solution. For $\bar{a} = 370/12 = 30,833$ and $\bar{a}' = 370.6/12 = 30.883$, get:

$\mathbf{s}_a$:	19.2	-19.8	-28.8	-11.8	-4.8	42.2	50.2	20.2	-19.8	-28.8	-11.8	-5.8
$\mathbf{s}_{a'}$:	-8.8	5.0	27.0	-8.7	11.5	-25.1	-27.3	-9.5	24.3	2.4	1.5	7.5

Then we calculate cosines[23]:

$$\begin{cases} \cos\gamma = \frac{-4231.14}{\sqrt{7855.68}\sqrt{3159.68}} = -0.849266, & \gamma \approx 148^\circ, \\ \cos\varphi = \frac{7195.7}{\sqrt{19264}\sqrt{14605.0}} = 0.428992, & \varphi \approx 65^\circ. \end{cases} \tag{3.66}$$

The angle γ is more than twice as high as the angle φ, and the correlation coefficient $r = -0.85$ shows a strong negative correlation. The poorer, the students rarely use the helmet. □

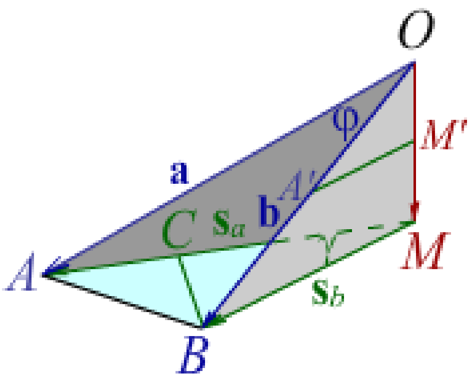

Figure 3.12: Line BC is perpendicular to MA.

In the correlation triangle MAB straight line BC, where $C \in MA$, is perpendicular to the side MA, as seen in the picture 3.12.

[23] see (3.51)

From the previous we know:

$$\overline{MC} = \overline{MB} \cdot \cos\gamma = s_b \cos\gamma, \quad \overline{MA} = |\mathbf{s}_a| = s_a, \tag{3.67}$$

which now gives:

$$x : y = \overline{MC} : \overline{MA} = \frac{s_b}{s_a}\cos\gamma = \frac{s_b}{s_a}\frac{\mathbf{s}_a \cdot \mathbf{s}_b}{|\mathbf{s}_a||\mathbf{s}_b|},$$

$$x : y = \frac{\mathbf{s}_a \cdot \mathbf{s}_b}{|\mathbf{s}_a|^2} = \frac{\sum_k (a_k - \bar{a})(b_k - \bar{b})}{\sum_k (a_k - \bar{a})^2}, \tag{3.68}$$

with arithmetic means $\bar{b} = \bar{a}$. This is the equation of the *regression line*.

Note: At the time of this writing, the observations of the "stereometry" of statistical terms of correlation and regression were completely unknown to the public, and that is why my choice of new names may not have been the most handy.

Afterword

Now, after more than a year has passed since the printed version of this text, and when the basic ideas seem more acceptable to many, I am obliged to give the readers some additional explanation of the time and conditions of their creation. The evidence of what I'll say are the last three bibliographies (see [10], [11], and [12]) and similar others mainly my private articles published not only on Accademia.edu. In these ongoing, the texts of this book and my next book [2] were prepared together, as two branches of the same tree, but not published at the same time.

The fundamental question was whether we, living beings, have free will at all? Is an unknown fate above us, behind everything what we will do, managed by an unknown, elusive or simply too complex network of (deterministic) causes? Although from everyday life, the existence of such an iron destiny may seem excessively, the strictly formal proof of its absence (as well as the presence) does not seem easy.

We know that today it is not possible to create an algorithm for a computer that would actually generate random numbers. For example, the machine could begin getting the first three decimals of the pi number, and with a given formula calculate a new number, from which it would again take three first digits, and again calculating a new three and again, repeating the pattern. We would get a series of pseudo-random three-digit numbers. These series could never be an objectively random, because the resetting the computer and bringing it to the same initial conditions, it should generate the same series. That's why I chose the definition of life as a (fictive) machine that can make decisions, whatever it means.

The problem of the existence of destiny remains open, so I tried to solve it in parallel with information. If an outcome of a "random" event is deterministic, then the produced information cannot be something physical. Also, if the information is allegedly physical, than this world is governed by principles of probability. In physics would be the strangest to explain the classical mechanics by probability, so I dealt with it the most.

I have been dealing with mathematical information and communication theory for decades, but now it became a matter of physics. It became an important place in many of my private works, some of which are available to the public, with some even in English as "Conservation law of information for particles" left at the Academy[24] a few years ago.

About the dilemma that tormented me, the issue of conservation of information is crucial. I found that that law can be considered acceptable, but only in inertial systems. This solves my basic problem, but opens up new inevitable questions. What are the non-inertial systems? What is the force? Then I come into (otherwise for me personally old) the stories about the multi-dimensional time, on the threshold of which I completed the second book mentioned above.

Rastko Vuković,
Banja Luka, October 2017

[24] https://www.academia.edu/8004844/Conservation_law_of_information_for_particles

Bibliography

[1] Rastko Vuković: *INFORMACIJA PERCEPCIJE*, sloboda, demokratija i fizika, Ekonomski institut Banja Luka, 2016. (https://archive.org/details/Informacija)

[2] Rastko Vuković: *SPACE-TIME*, principles physics of chances, Economic Institute Banja Luka, 2017. (https://www.scribd.com/document/349336253/Principles)

[3] David Hilbert: Grundlagen der Geometrie, 1899. (https://en.wikipedia.org/wiki/Hilbert%27s_axioms)

[4] Lee Rozema, Ardavan Darabi, Dylan Mahler, Alex Hayat, Yasaman Soudagar, Aephraim Steinberg: *Violation of Heisenberg's Measurement-Disturbance Relationship by Weak Measurements*, Physical Review Letters, 2012; 109 (10) DOI: (http://journals.aps.org/prl/abstract/10.1103/PhysRevLett.109.100404)

[5] Erich Fromm: *Escape from Freedom*, Farrar & Rinehart, 1941 (https://en.wikipedia.org/wiki/Escape_from_Freedom)

[6] Edmund Husserl: *The Crisis of European Sciences* and Transcendental Phenomenology, first published: 1936 in German, 1954 in English. (https://en.wikipedia.org/wiki/The_Crisis_of_European_Sciences_and_Transcendental_Phenomenology)

[7] Lorenz, Edward Norton (1972). "Predictability: Does the Flap of a Butterfly's Wings in Brazil Set Off a Tornado in Texas?" Address at the 139th Annual Meeting of the American Association for the Advancement of Science, Sheraton Park Hotel, Boston, Mass., December 29, 1972.

[8] Kuramoto, Yoshiki (1975). H. Araki, ed. Lecture Notes in Physics, International Symposium on Mathematical Problems in Theoretical Physics 39. Springer-Verlag, New York. p. 420.

[9] Brownian motion (https://en.wikipedia.org/wiki/Brownian_motion)

[10] Rastko Vukovic: *Liberty, Intelligence and Hierarchy*, The first part: Vectors; May 15, 2016. (https://www.academia.edu/25346912/Liberty_Intelligence_and_Hierarchy)

[11] Rastko Vukovic: *Entropy and Inertia*; August 9, 2016. (https://www.academia.edu/27650508/Entropy_and_Inertia)

[12] Rastko Vuković: *PRIRODA VREMENA*, hipoteze o verovatnoći i entropiji;, Banja Luka, oktobar 2016. (https://www.academia.edu/29346330/)

Index